Math 2260 Written HW #15 Solutions

1. Suppose $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$ are vectors in space such that $|\vec{\mathbf{u}}| = |\vec{\mathbf{v}}|$. What can you say about the angle between the vectors $\vec{\mathbf{u}} + \vec{\mathbf{v}}$ and $\vec{\mathbf{u}} - \vec{\mathbf{v}}$?

   Answer: Notice that

   $$(\vec{\mathbf{u}} + \vec{\mathbf{v}}) \cdot (\vec{\mathbf{u}} - \vec{\mathbf{v}}) = \vec{\mathbf{u}} \cdot \vec{\mathbf{u}} + \vec{\mathbf{v}} \cdot \vec{\mathbf{u}} - \vec{\mathbf{u}} \cdot \vec{\mathbf{v}} - \vec{\mathbf{v}} \cdot \vec{\mathbf{v}} = \vec{\mathbf{u}} \cdot \vec{\mathbf{u}} - \vec{\mathbf{v}} \cdot \vec{\mathbf{v}}.$$

   Therefore, since $\vec{\mathbf{u}} \cdot \vec{\mathbf{u}} = |\vec{\mathbf{u}}|^2$ and $\vec{\mathbf{v}} \cdot \vec{\mathbf{v}} = |\vec{\mathbf{v}}|^2$ and since $|\vec{\mathbf{u}}| = |\vec{\mathbf{v}}|$, it follows that

   $$(\vec{\mathbf{u}} + \vec{\mathbf{v}}) \cdot (\vec{\mathbf{u}} - \vec{\mathbf{v}}) = |\vec{\mathbf{u}}|^2 - |\vec{\mathbf{v}}|^2 = 0.$$

   Hence, $\vec{\mathbf{u}} + \vec{\mathbf{v}}$ and $\vec{\mathbf{u}} - \vec{\mathbf{v}}$ are perpendicular whenever $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$ are the same length.

2. Let $\vec{\mathbf{u}} = 2\vec{\imath} - \vec{\jmath}$ and let $\vec{\mathbf{v}} = \vec{\imath} + 2\vec{\jmath}$.

   (a) What is the vector $\vec{\mathbf{u}} \times \vec{\mathbf{v}}$?

   Answer: By definition,

   $$\begin{aligned}
   \vec{\mathbf{u}} \times \vec{\mathbf{v}} &= \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{\mathbf{k}} \\ 2 & -1 & 0 \\ 1 & 2 & 0 \end{vmatrix} \\
   &= \vec{\imath}((-1)(0) - (0)(2)) - \vec{\jmath}((2)(0) - (0)(1)) + \vec{\mathbf{k}}((2)(2) - (-1)(1)) \\
   &= 0\vec{\imath} + 0\vec{\jmath} + 5\vec{\mathbf{k}} \\
   &= 5\vec{\mathbf{k}}.
   \end{aligned}$$

   (b) Draw the vectors $\vec{\mathbf{u}}$, $\vec{\mathbf{v}}$, and $\vec{\mathbf{u}} \times \vec{\mathbf{v}}$.

   Answer: $\vec{\mathbf{u}}$, $\vec{\mathbf{v}}$, and $\vec{\mathbf{u}} \times \vec{\mathbf{v}}$.