

Math 2260 Written HW #10 Solutions

1. Consider the sequence

$$\left(\frac{5}{1}, \frac{8}{2}, \frac{11}{6}, \frac{14}{24}, \frac{17}{120}, \frac{20}{720}, \dots\right).$$

What is the n th term in this sequence?

Answer: First, notice that the denominator of the n th term is just $n!$. For the numerator, notice that the numerator of the first term is 5, the numerator of the second term is $5 + 3$, the numerator of the third term is $5 + 2 \cdot 3$, etc. Therefore, the numerator of the n th term will be

$$5 + (n - 1) \cdot 3 = 5 + 3n - 3 = 2 + 3n.$$

Therefore, the n th term of the sequence will be

$$\frac{2 + 3n}{n!}.$$

2. Consider the sequence $(a_n)_{n=1}^{\infty}$ where

$$a_n = \frac{\ln(n)}{\ln(2n)}.$$

Does the sequence converge or diverge? Find the limit if it converges.

Answer: Using the multiplicative property of the natural logarithm,

$$\ln(2n) = \ln(2) + \ln(n),$$

so we see that

$$a_n = \frac{\ln(n)}{\ln(2) + \ln(n)}.$$

Notice that, by L'Hôpital's Rule,

$$\lim_{x \rightarrow \infty} \frac{\ln(x)}{\ln(2) + \ln(x)} = \lim_{x \rightarrow \infty} \frac{1/x}{1/x} = 1.$$

Therefore,

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{\ln(n)}{\ln(2) + \ln(n)} = \lim_{x \rightarrow \infty} \frac{\ln(x)}{\ln(2) + \ln(x)} = 1.$$