

Math 2250 Final Exam Practice Problems

1. What are the domain and range of the function

$$f(x) = \frac{\ln x}{\sqrt{x}}?$$

2. Find the point on the graph of $y = e^{3x}$ at which the tangent line passes through the origin.
3. Find the equation of the tangent line to the curve

$$xy^3 - x^2y = 6$$

at the point $(3, 2)$.

4. Use an appropriate linearization to approximate $\sqrt{96}$.

5. Consider the function $f(x) = x^2e^{-x^2}$. What is the absolute maximum of $f(x)$?

6. Water is draining from a conical tank at the rate of 18 cubic feet per minute. The tank has a height of 10 feet and the radius at the top is 5 feet. How fast (in feet per minute) is the water level changing when the depth is 6 feet? (Note: the volume of a cone of radius r and height h is $\frac{\pi r^2 h}{3}$.)

7. The function $f(x) = x^4 - 6x^3$ is concave down for what values of x ?

8. Evaluate the limit

$$\lim_{x \rightarrow 0} (1 - 6x)^{1/x}.$$

9. Let $f(x) = x^{\cos x}$. What is $f'(\pi/2)$?

10. For $0 \leq t \leq 5$, a particle moves in a horizontal line with acceleration $a(t) = 2t - 4$ and initial velocity $v(0) = 3$.

(a) When is the particle moving to the left?

(b) When is the particle speeding up?

(c) What is the position of the particle at time t if the initial position of the particle is 6?

11. If $\int_0^6 f(x)dx = 10$ and $\int_0^4 f(x)dx = 7$, find $\int_4^6 f(x)dx$.

12. Evaluate the definite integral

$$\int_{\pi/6}^{\pi/4} \sin t \, dt.$$

13. Evaluate the integral

$$\int \frac{2}{t-3} \, dt.$$

14. Evaluate the definite integral

$$\int_1^4 \frac{2\sqrt{x} + 4x^2}{x} \, dx$$

15. Suppose the velocity of a particle is given by

$$v(t) = 6t^2 - 4t.$$

What is the displacement (i.e. change in position) of the particle from $t = 0$ to $t = 2$?

16. Suppose that

$$\int_0^{x^2} f(t) dt = \sqrt{x^2 + 1} - 1.$$

What is $f(2)$?

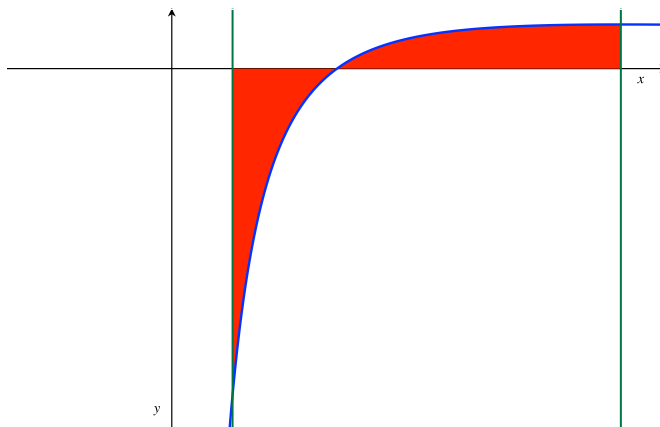
17. Evaluate the integral

$$\int 3e^{\tan x} \sec^2 x dx.$$

18. Evaluate the definite integral

$$\int_0^{\pi/16} 8 \tan(4x) dx.$$

19. What is the area of the red region in the figure? The blue curve is given by $y = 2\frac{\ln x}{x}$ and the vertical green lines are the lines $x = \frac{1}{e}$ and $x = e$.



20. What are the domain and range of the function

$$f(x) = \frac{1 + e^x}{1 - e^x}?$$

21. What is the equation of the line tangent to the graph of $y^3 + 3x^2y^2 + 2x^3 = 4$ at the point $(1, -1)$?

22. Evaluate the limit

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2 + x}.$$

23. The volume of a cube is increasing at a rate of $10 \text{ cm}^3/\text{min}$. How fast is the surface area increasing when the length of an edge is 10 cm ?

24. Find the maximum and minimum values, inflection points and asymptotes of $y = \ln(x^2 + 1)$ and use this information to sketch the graph.

25. Use an appropriate linearization to approximate $e^{1/10}$.

26. What is the absolute maximum value of $f(x) = x^{1/x}$ for $x > 0$?

27. Suppose the velocity of a particle is given by

$$v(t) = 3 \cos t + 4 \sin t.$$

If the particle starts (at time 0) at a position 7 units to the right of the origin, what is the position of the particle at time t ?

28. Evaluate the definite integral

$$\int_0^{\pi/6} \frac{2 + \cos^3 \theta}{\cos^2 \theta} d\theta.$$

29. Evaluate

$$\int \csc r \cot r \, dr.$$

30. Let

$$g(x) = \int_1^{x^2} \frac{\sin t}{\sqrt{t}} dt.$$

What is the derivative of g ?

31. If $f(x) = \frac{x}{\ln x}$, find $f'(e^3)$.

32. At what value(s) of x (if any) is the function $f(x)$ defined below discontinuous?

$$f(x) = \begin{cases} x^2 + 4x + 5 & \text{if } x < -2 \\ \frac{x}{2} & \text{if } -2 \leq x \leq 2 \\ 1 + \sqrt{x-2} & \text{if } x > 2 \end{cases}$$

33. Find the equation of the tangent line to the curve

$$x \cos y = 1$$

at the point $(2, \frac{\pi}{3})$.

34. A particle moves so that its position at time t is given by

$$s(t) = \sqrt{3t^2 + 4}.$$

If $v(t)$ is the velocity of the particle at time t , what is $\lim_{t \rightarrow +\infty} v(t)$?

35. A technical writer is producing a book which must have 1-inch side margins and 2-inch top and bottom margins. If the area of the page can be at most 50 in², what dimensions give the most printed area per page?

36. Suppose $f(t) = 2t + \cos t$. Evaluate the definite integral

$$\int_{-\pi/2}^{\pi/2} f(t) dt.$$

37. Evaluate the limit

$$\lim_{x \rightarrow \infty} \frac{\ln(3 + 2e^{3x})}{6x}.$$

38. As a spherical raindrop falls, it evaporates (i.e. loses volume) at a rate proportional to its surface area. Show that the radius of the raindrop decreases at a constant rate. (*Hint*: the volume of a sphere of radius r is $\frac{4}{3}\pi r^3$, and the surface area is $4\pi r^2$)

39. Suppose $f'(t) = 3e^t - 2 \sec t \tan t$ and that $f(0) = 4$. Give a formula for $f(t)$.