# Math 2250 HW #8 Solutions to WebWork Problems 1 & 8

Everybody is assigned slightly different versions of each problem. The below are representative examples of the assigned problems, but may not exactly match the problem you were assigned.

1. Use implicit differentiation to find $dy/dx$, given:

$$e^{2x} = \sin(x + 7y).$$

**Answer:** First, differentiate both sides with respect to $x$:

$$\begin{aligned} \frac{d}{dx}(e^{2x}) &= \frac{d}{dx}(\sin(x+7y)) \\ e^{2x} \cdot 2 &= \cos(x+7y) \cdot \frac{d}{dx}(x+7y) \\ 2e^{2x} &= \cos(x+7y) \cdot (1+7y') \end{aligned}$$

by the Chain Rule, or, equivalently,

$$2e^{2x} = \cos(x+7y) + 7y'\cos(x+7y).$$

Isolating the $y'$ term yields

$$7y'\cos(x+7y) = 2e^{2x} - \cos(x+7y),$$

so we have

$$y' = \frac{2e^{2x} - \cos(x+7y)}{7\cos(x+7y)}.$$

This would be a perfectly acceptable answer, or I could also split the right hand side into two terms:

$$y' = \frac{2e^{2x}}{7\cos(x+7y)} - \frac{1}{7}.$$

8. Find $dy/dx$ if

$$\ln(xy) = e^{x+y}.$$

**Answer:** Again I use implicit differentiation:

$$\begin{aligned} \frac{d}{dx}(\ln(xy)) &= \frac{d}{dx}(e^{x+y}) \\ \frac{1}{xy} \cdot \frac{d}{dx}(xy) &= e^{x+y} \cdot \frac{d}{dx}(x+y) \\ \frac{1}{xy}(xy' + 1 \cdot y) &= e^{x+y}(1+y') \\ \frac{xy'}{xy} + \frac{y}{xy} &= e^{x+y} + y'e^{x+y} \\ \frac{y'}{y} + \frac{1}{x} &= e^{x+y} + y'e^{x+y}. \end{aligned}$$

Grouping the terms containing $y'$ yields

$$y'\left(\frac{1}{y} - e^{x+y}\right) = e^{x+y} - \frac{1}{x},$$

so we see that

$$y' = \frac{e^{x+y} - \frac{1}{x}}{\frac{1}{y} - e^{x+y}}.$$