Math 2250 Written HW #2 Solutions

1. Find the following limit, or explain why the limit doesn't exist:

$$\lim_{h\to 0} \frac{\sqrt{6h+1}-1}{h}.$$

Answer: Notice that both the numerator and the denominator go to zero as $h \to 0$, so we can't just use the limit laws to evaluate the limit. In this case, it turns out that the right thing to do is to rationalize the numerator:

$$\lim_{h\to 0} \frac{\sqrt{6h+1}-1}{h} = \lim_{h\to 0} \frac{\sqrt{6h+1}-1}{h} \cdot \frac{\sqrt{6h+1}+1}{\sqrt{6h+1}+1} = \lim_{h\to 0} \frac{(6h+1)-1}{h(\sqrt{6h+1}+1)} = \lim_{h\to 0} \frac{6h}{h(\sqrt{6h+1}+1)}.$$

Canceling h from both numerator and denominator yields

$$\lim_{h\to 0} \frac{6}{\sqrt{6h+1}+1} = \frac{6}{2} = 3,$$

so we can conclude that

$$\lim_{h\to 0} \frac{\sqrt{6h+1}-1}{h} = 3.$$

2. Find the following limit, or explain why the limit doesn't exist:

$$\lim_{x\to 16} \frac{\sqrt{x}-4}{x-16}.$$

Answer: Again, the numerator and denominator both go to zero as $x \to 16$, so we need to get a bit clever. In this case, notice that we can factor the denominator as

$$x - 16 = (\sqrt{x}+4)(\sqrt{x}-4).$$

Therefore,

$$\lim_{x\to 16} \frac{\sqrt{x}-4}{x-16} = \lim_{x\to 16} \frac{\sqrt{x}-4}{(\sqrt{x}+4)(\sqrt{x}-4)}.$$

Canceling $\sqrt{x}-4$ from top and bottom gives us

$$\lim_{x\to 16} \frac{1}{\sqrt{x}+4} = \frac{1}{8}.$$

Hence, it follows that

$$\lim_{x\to 16} \frac{\sqrt{x}-4}{x-16} = \frac{1}{8}.$$

3. Determine each of the following limits, or explain why they don't exist.

(a) $\displaystyle\lim_{x\to -2^+} 3(x+3)\frac{|x+2|}{x+2}$ (b) $\displaystyle\lim_{x\to -2^-} 3(x+3)\frac{|x+2|}{x+2}$

(a) **Answer:** Since we're interested in $x > -2$, we know that $x + 2 > 0$, which means that

$$|x + 2| = x + 2.$$

Hence,

$$\lim_{x \to -2^+} 3(x+3)\frac{|x+2|}{x+2} = \lim_{x \to -2^+} 3(x+3)\frac{x+2}{x+2} = 3 \cdot 1 \cdot 1 = 3.$$

(b) **Answer:** In this part, we're interested in $x < 2$, which means that, for the values of x we care about, $x + 2 < 0$. Therefore,

$$|x + 2| = -(x + 2),$$

so we have

$$\lim_{x \to -2^-} 3(x+3)\frac{|x+2|}{x+2} = \lim_{x \to -2^-} 3(x+3)\frac{-(x+2)}{x+2} = 3 \cdot 1 \cdot (-1) = -3.$$