

Math 115 HW #9

Due Friday, November 20

1. Solve the differential equation

$$\sin x \frac{dy}{dx} + (\cos x)y = \sin(x^2).$$

2. Solve the initial-value problem

$$y' = x + y, \quad y(0) = 2.$$

3. Solve the initial-value problem

$$xy' = y + x^2 \sin x, \quad y(\pi) = 0.$$

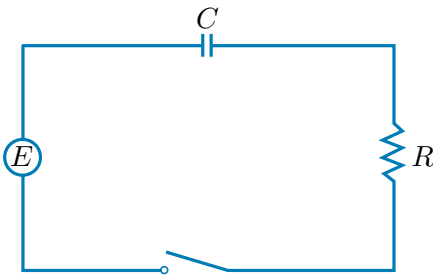
4. The figure shows a circuit containing an electromotive force, a capacitor with a capacitance of C farads (F), and a resistor with a resistance of R ohms (Ω). Ohm's Law says that the voltage drop across the resistor is RI . The voltage drop across the capacitor is Q/C , where Q is the charge (in coulombs), so in this case Kirchhoff's Law gives

$$RI + \frac{Q}{C} = E(t).$$

By definition $I = dQ/dt$, so we have

$$R \frac{dQ}{dt} + \frac{1}{C}Q = E(t).$$

Suppose the resistance is 5Ω , the capacitance is $0.05 F$, a battery gives a constant voltage of $60 V$, and the initial charge is $Q(0) = 0$ coulombs. Find the charge and the current at time t .



5. An object with mass m is dropped from rest and we assume that the air resistance is proportional to the speed of the object. If $s(t)$ is the distance dropped after t seconds, then the speed is $v = s'(t)$ and the acceleration is $a = v'(t)$. If g is the acceleration due to gravity, then the downward force on the object is $mg - cv$, where c is a positive constant, and Newton's Second Law gives

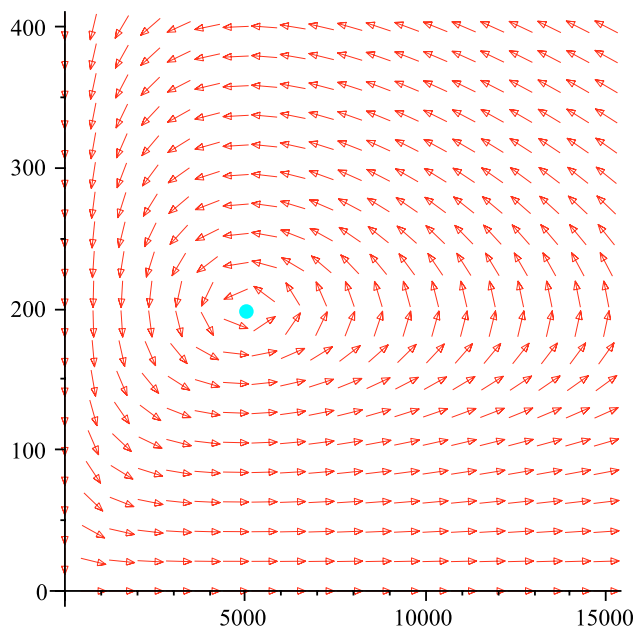
$$m \frac{dv}{dt} = mg - cv.$$

- (a) Solve this for v (note that this is a linear differential equation in v).
- (b) What is the terminal velocity of the falling object?
- (c) Find the distance the object has fallen after t seconds?
6. Each system of differential equations is a model for two species that either compete for the same resources or cooperate for mutual benefit (flowering plants and insect pollinators, for instance). Decide whether each system describes competition or cooperation and explain why it is a reasonable model. (*Hint*: think about the effect an increase in one species has on the growth rate of the other.)
- (a) $\frac{dx}{dt} = 0.12x - 0.0006x^2 + 0.00001xy$
 $\frac{dy}{dt} = 0.08x + 0.00004xy$
- (b) $\frac{dx}{dt} = 0.15x - 0.0002x^2 - 0.0006xy$
 $\frac{dy}{dt} = 0.2y - 0.00008y^2 - 0.0002xy$
7. Populations of aphids and ladybugs are modeled by the equations

$$\frac{dA}{dt} = 2A - 0.01AL$$

$$\frac{dL}{dt} = -0.5L + 0.0001AL$$

- (a) Find the equilibrium solutions and explain their significance.
- (b) Find an expression for dL/dA .
- (c) The direction field for the differential equation in part (b) is shown. Use it to sketch a phase portrait.



- (d) Suppose that at time $t = 0$ there are 1000 aphids and 200 ladybugs. Draw the corresponding phase trajectory and use it to describe how both populations change.
- (e) Use part (d) to make rough sketches of the aphid and ladybug populations as functions of t . How are the graphs related to each other?