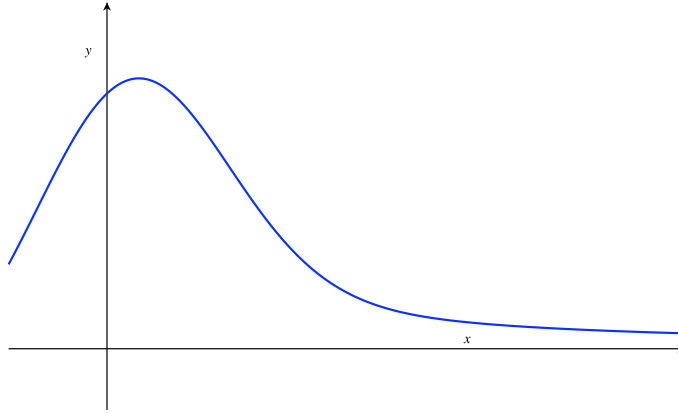


Math 115 HW #8

Due Friday, November 13

1. The function with the given graph is a solution of one of the following differential equations. Decide which is the correct equation and justify your answer. (a) $y' = 1 + xy$ (b) $y' = -2xy$



(c) $y' = 1 - 2xy$

2. Find an equation of the curve that passes through the point $(0, 1)$ and whose slope at (x, y) is xy .
3. Find the orthogonal trajectories of the family of curves

$$y^2 = kx^3.$$

(An orthogonal trajectory of a family of curves is a curve that intersects each curve of the family orthogonally, that is, at right angles)

Sketch several members of each family on a common graph (feel free to use *Mathematica* or any other graphing utility to help create this sketch).

4. A glucose solution is administered intravenously into the bloodstream at a constant rate r . As the glucose is added, it is converted into other substances and removed from the bloodstream at a rate that is proportional to the concentration at that time. Thus a model for the concentration $C = C(t)$ of the glucose solution in the bloodstream is

$$\frac{dC}{dt} = r - kC$$

where k is a positive constant.

- (a) Suppose that the concentration at time $t = 0$ is C_0 . Determine the concentration at any other time t by solving the differential equation.
- (b) Assuming that $C_0 < r/k$, find $\lim_{t \rightarrow \infty} C(t)$ and interpret your answer.
5. A vat with 500 gallons of beer contains 4% alcohol (by volume). Beer with 6% alcohol is pumped into the vat at a rate of 5 gal/min and the mixture is pumped out at the same rate. What is the percentage of alcohol after one hour?

6. Biologists stocked a lake with 400 fish and estimated the carrying capacity (the maximal population for the fish of that species in that lake) to be 10,000. The number of fish tripled in the first year.

(a) Assuming that the size of the fish population satisfies the logistic equation

$$\frac{dP}{dt} = rP \left(1 - \frac{P}{N} \right),$$

find an expression for the size of the population after t years.

(b) How long will it take for the population to increase to 5000?

7. (a) Show that if P satisfies the logistic equation, then

$$\frac{d^2P}{dt^2} = r^2P \left(1 - \frac{P}{N} \right) \left(1 - \frac{2P}{N} \right).$$

(b) Deduce that a population grows fastest when it reaches half its carrying capacity.

8. Consider a population $P = P(t)$ with constant relative birth and death rates α and β , respectively, and a constant emigration rate m , where α, β , and m are positive constants. Assume that $\alpha > \beta$. Then the rate of change of the population at time t is modeled by the differential equation

$$\frac{dP}{dt} = rP - m \quad \text{where } r = \alpha - \beta$$

(a) Find the solution of this equation that satisfies the initial condition $P(0) = P_0$.

(b) What condition on m will lead to an exponential expansion of the population?

(c) What condition on m will result in a constant population? A population decline?

(d) In 1847, the population of Ireland was about 8 million and the difference between the relative birth and death rates was 1.6% of the population. Because of the potato famine in the 1840s and 1850s, about 210,000 inhabitants per year emigrated from Ireland. Was the population expanding or declining at that time?