

Math 115 HW #7
Due Friday, October 30

1. Write the number

$$\frac{3 + 2i}{4 - 3i}$$

in the form $a + bi$.

2. Prove the following properties of complex numbers

(a) $\overline{z + w} = \bar{z} + \bar{w}$

(b) $\overline{z\bar{w}} = \bar{z}w$

(c) $\overline{z^n} = \bar{z}^n$

[Hint: Write $z = a + bi$, $w = c + di$]

3. Find all solutions of the equation

$$2x^2 - 2x + 1 = 0.$$

4. Let

$$z = \sqrt{3} + i, \quad w = 1 + \sqrt{3}i.$$

Find polar forms for zw , z/w and $1/z$ by first putting z and w into polar form.

5. Find all the fifth roots of 32 and sketch them in the complex plane.

6. Write

$$e^{-i\pi}$$

in the form $a + bi$.

7. Use Euler's formula (i.e. $e^{i\theta} = \cos \theta + i \sin \theta$) to prove the following formulas for $\cos x$ and $\sin x$:

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}, \quad \sin x = \frac{e^{ix} - e^{-ix}}{2i}.$$