Math 115 HW #10

Due Friday, December 4

1. Suppose $y_1(t)$ and $y_2(t)$ are both solutions of the differential equation

$$P(t)y'' + Q(t)y' + R(t)y = 0.$$

Show that, for any constants C_1 and C_2, the function

$$C_1y_1(t) + C_2y_2(t)$$

is also a solution of this differential equation.

2. Solve the differential equation

$$6y'' - 7y' - 12y = 0.$$

3. Solve the initial-value problem

$$2y'' + 6y' + 17y = 0, \quad y(0) = 1, y'(0) = 5.$$

4. A spring-mass-dashpot system (like the door-closing mechanism in many doors) can be modeled by the differential equation

$$mx'' + cx' + kx = 0$$

where x is the displacement of the object, m is the mass of the object, c is the damping constant for the dashpot, and k is the spring constant. Suppose we have such a system with a mass $m = 20$ kg, a spring with $k = 5$, and a dashpot whose damping constant c we can adjust. What value of c should we pick to get *critical damping*?

5. Solve the differential equation

$$y'' - y' - 6y = e^{2x}.$$

6. Solve the differential equation

$$y'' - 4y' + 4y = e^{2x}.$$

7. Solve the initial-value problem

$$y'' + 9y = \cos 3x + \sin 3x, \quad y(0) = 2, y'(0) = 1.$$