

## Math 113 HW #2 Solutions

1. Exercise 1.2.18. The monthly cost of driving a car depends on the number of miles driven. Lynn found that in May it cost her \$380 to drive 480 mi and in June it cost her \$460 to drive 800 mi.

- (a) Express the monthly cost  $C$  as a function of the distance driven  $d$ , assuming that a linear relationship gives a suitable model.

**Answer:** Since we're using a linear model, we want, first, to find the slope of the line. Clearly, if the line goes through the points  $(480, 380)$  and  $(800, 460)$ , then the slope of the line is

$$\frac{460 - 380}{800 - 480} = \frac{80}{320} = \frac{1}{4}.$$

Now we use the point-slope formula with the point  $(480, 380)$ :

$$C - 380 = \frac{1}{4}(d - 480) = \frac{d}{4} - 120.$$

Hence,

$$C(d) = \frac{d}{4} + 260.$$

- (b) Use part (a) to predict the cost of driving 1500 miles per month.

**Answer:** Plugging in  $d = 1500$  to the expression for  $C(d)$  yields

$$C(1500) = \frac{1500}{4} + 260 = 375 + 260 = 635,$$

so we estimate that it will cost \$635 to drive 1500 miles per month.

- (c) Draw the graph of the linear function. What does the slope represent?

**Answer:** The graph is shown in Figure 1. The slope represents the marginal cost of driving an additional mile.

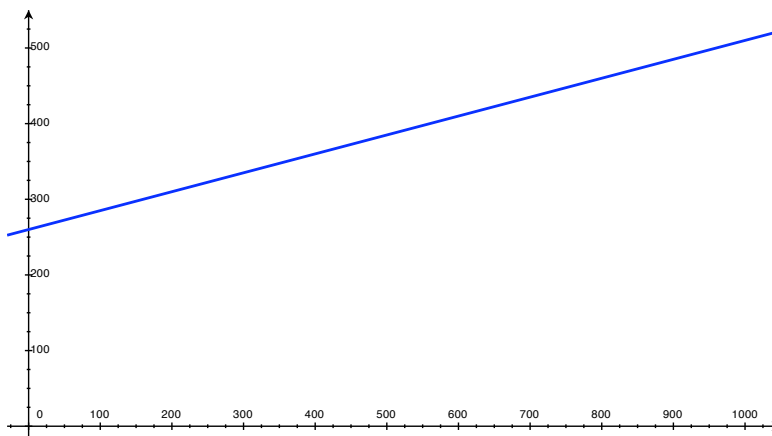


Figure 1: The cost function  $C(d) = \frac{d}{4} + 260$ .

(d) What does the  $y$ -intercept represent?

**Answer:** The  $y$ -intercept, \$260, gives the cost of owning a car which is independent of the number of miles driven (for example, the cost of insurance would be included in this cost). An economist might call this the “fixed cost of driving”.

(e) Why does a linear function give a suitable model in this situation?

**Answer:** A linear function gives a suitable model because we would expect the cost of driving to be more or less proportional to the number of miles driven.

2. Exercise 1.3.14. Graph the function

$$y = 4 \sin 3x$$

by hand, not by plotting points, but by starting with the graph of one of the standard functions given in Section 1.2, and then applying the appropriate translations.

**Answer:** This graph will be like the graph of  $f(x) = \sin x$ , except stretched vertically by a factor of 4 and compressed horizontally by a factor of 3, yielding the graph shown in Figure 2.

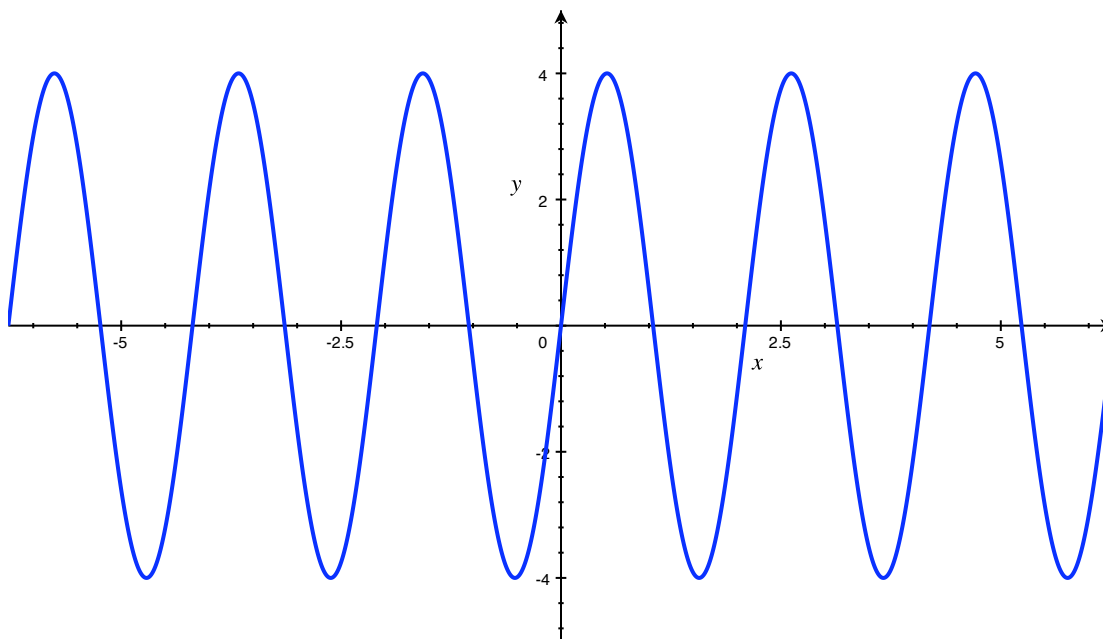


Figure 2: The graph  $y = 4 \sin 3x$

3. Exercise 1.3.26. A variable star is one whose brightness alternately increases and decreases. For the most visible variable star, Delta Cephei, the time between periods of maximum brightness is 5.4 days, the average brightness (or magnitude) of the star is 4.0, and its brightness varies by  $\pm 0.35$  magnitude. Find a function that models the brightness of Delta Cephei as a function of time.

**Answer:** Since the brightness alternately increases and decreases, we should model it by a periodic function, which suggests either the sine or the cosine. Let's try using sine. Then we

want to write the brightness  $B$  as a function of  $t$  (measured in days) in the form

$$B(t) = a \sin(bt) + c.$$

Focusing first on  $a$ , notice that  $a$  controls how *much* the function varies or, in other words, how much it fluctuates around the average value. Since the brightness of Delta Cephei varies by  $\pm 0.35$ , we should let  $a = 0.35$ .

Turning to  $b$ , notice that  $\sin(bt)$  returns to its starting point whenever  $bt$  is a multiple of  $2\pi$ . Since the period of Delta Cephei is 5.4 days, we want the function  $B(t)$  to return to its starting point whenever  $t$  is a multiple of 5.4. In other words, we want

$$b(5.4) = 2\pi,$$

$$\text{or } b = \frac{2\pi}{5.4} = \frac{\pi}{2.7} = \frac{10\pi}{27}.$$

Finally,  $c$  gives the average value of the function. Since the average brightness of Delta Cephei is 4.0, we should let  $c = 4.0$ .

Putting this all together, then, we would model the brightness of Delta Cephei by the function

$$B(t) = 0.35 \sin\left(\frac{10\pi}{27} t\right) + 4.0.$$

4. Exercise 1.3.36. Let  $f(x) = \frac{x}{1+x}$  and  $g(x) = \sin 2x$ . Find the following functions and their domains.

(a)  $f \circ g$

**Answer:** By definition,

$$(f \circ g)(x) = f(g(x)) = f(\sin 2x) = \frac{\sin 2x}{1 + \sin 2x}.$$

Now, this function is perfectly well-defined except when the denominator is zero, which occurs precisely when  $\sin 2x = -1$ .

The function  $\sin x$  is equal to  $-1$  when  $x = 3\pi/2, 7\pi/2, 11\pi/2, \dots$ , so  $\sin 2x$  will equal  $-1$  when  $x = 3\pi/4, 7\pi/4, 11\pi/4, \dots$ . These bad values are precisely those numbers of the form  $(4n - 1)\pi/4$ , so the domain of  $f \circ g$  is

$$\{x \in \mathbb{R} : x \neq \frac{(4n - 1)\pi}{4} \text{ for any integer } n\}.$$

(b)  $g \circ f$

**Answer:** By definition,

$$(g \circ f)(x) = g(f(x)) = f\left(\frac{x}{1+x}\right) = \sin\left(2\frac{x}{1+x}\right) = \sin\frac{2x}{1+x}.$$

The sine function can be evaluated on all real numbers, so the only inputs we have to worry about are those which make the denominator of the fraction equal to zero. In other words, the domain of  $g \circ f$  is the set

$$\{x \in \mathbb{R} : x \neq -1\}.$$

(c)  $f \circ f$

**Answer:** By definition,

$$(f \circ f)(x) = f(f(x)) = f\left(\frac{x}{1+x}\right) = \frac{\frac{x}{1+x}}{1 + \frac{x}{1+x}} = \frac{\frac{x}{1+x}}{\frac{1+x}{1+x} + \frac{x}{1+x}} = \frac{\frac{x}{1+x}}{\frac{1+2x}{1+x}} = \frac{x}{1+2x}.$$

This function is well-defined except when  $f$  is not well-defined (which happens when  $x = -1$ ) and when the denominator equals zero, so the domain of  $f \circ f$  is the set

$$\{x \in \mathbb{R} : x \neq -1/2, x \neq -1\}.$$

(d)  $g \circ g$

**Answer:** By definition,

$$(g \circ g)(x) = g(g(x)) = g(\sin 2x) = \sin(\sin 2x).$$

Since the sine function is defined on all real numbers, the domain of  $g \circ g$  is all of  $\mathbb{R}$ .

5. Exercise 1.3.44. Express the function

$$G(x) = \sqrt[3]{\frac{x}{1+x}}$$

in the form  $f \circ g$ .

**Answer:** Define the functions

$$f(x) = \sqrt[3]{x}$$
$$g(x) = \frac{x}{1+x}.$$

Then

$$f \circ g(x) = f(g(x)) = f\left(\frac{x}{1+x}\right) = \sqrt[3]{\frac{x}{1+x}} = G(x),$$

as desired.

6. Exercise 1.3.52. Use the given graphs of  $f$  and  $g$  to estimate the value of  $f(g(x))$  for  $x = -5, -4, -3, \dots, 5$ . Use these estimates to sketch a rough graph of  $f \circ g$ .

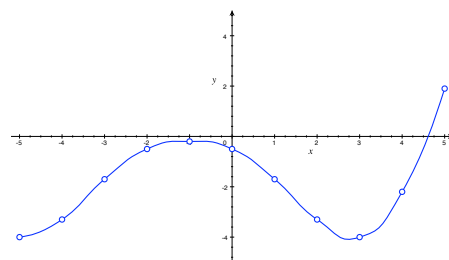
**Answer:** To determine the value of  $f(g(x))$  when  $x = 0$ , we estimate from the graph that  $g(0) \approx 2.8$  and  $f(2.8) \approx -0.5$ . Thus,

$$f(g(0)) \approx f(2.8) \approx -0.5.$$

Similar calculations yield the following values of  $f(g(x))$ , with a possible graph of  $f(g(x))$  shown to the right:

$x$	$g(x)$	$f(g(x))$
-5	-0.2	-4
-4	1.2	-3.3
-3	2.2	-1.7
-2	2.8	-0.5
-1	3	-0.2
0	2.8	-0.5

$x$	$g(x)$	$f(g(x))$
1	2.2	-1.7
2	1.2	-3.3
3	-0.2	-4
4	-1.9	-2.2
5	-4.1	1.9



7. Exercise 1.3.56. An airplane is flying at a speed of 350 mi/h at an altitude of one mile and passes directly over a radar station at time  $t = 0$ .

(a) Express the horizontal distance  $d$  (in miles) that the plane has flown as a function of  $t$ .

**Answer:** Since the plane is traveling at a constant speed of 350 mi/h and since distance equals rate times time, we have that

$$d(t) = 350t$$

is the horizontal distance function.

(b) Express the distance  $s$  between the plane and the radar station as a function of  $d$ .

**Answer:** Using the Pythagorean Theorem, the distance  $s$  from the plane to the radar

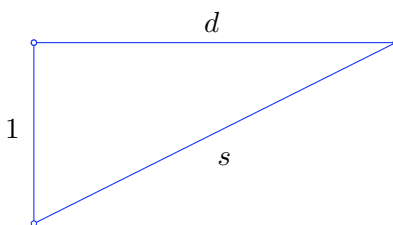


Figure 3: Schematic picture of airplane and radar station

station is given by

$$s^2 = 1 + d^2,$$

so we have that

$$s(d) = \sqrt{1 + d^2}.$$

(c) Use composition to express  $s$  as a function of  $t$ .

**Answer:** If we want to express the distance from the radar station as a function of  $t$ , we simply take the composition

$$s \circ d(t) = s(d(t)) = s(350t) = \sqrt{1 + (350t)^2} = \sqrt{1 + 122,500t^2}.$$

8. Exercise 1.3.62. If  $f(x) = x + 4$  and  $h(x) = 4x - 1$ , find a function  $g$  such that  $g \circ f = h$ .

**Answer:** At first glance, it seems like we could just multiply the results of  $f$  by 4 to get  $h$ . Of course, this doesn't work, since

$$4(x + 4) = 4x + 16 \neq 4x - 1.$$

However, if we subtract 17 after multiplying by 4, we will get the right answer. In other words, if we define

$$g(x) = 4x - 17,$$

then we see that

$$(g \circ f)(x) = g(f(x)) = g(x + 4) = 4(x + 4) - 17 = 4x - 1 = h(x),$$

as desired.