Math 113 HW #12 Solutions

1. Exercise 5.2.18. Express the limit

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{\cos x_i}{x_i}\Delta x$$

as a definite integral on $[\pi, 2\pi]$.

Answer: This is simply the definition of the definite integral

$$\int_{\pi}^{2\pi}\frac{\cos x}{x}dx.$$

2. Exercise 5.2.34. The graph of g consists of two straight lines and a semicircle. Use it to evaluate each integral

(a) $\int_0^2 g(x)dx$

Answer: Since on $[0,2]$ the graph of $g(x)$ is just a straight line of slope -2 coming down from $y=4$ to $y=0$, the area is just the area of the triangle

$$\frac{1}{2}2\cdot 4 = 4.$$

Since this area is above the x-axis, definite integral equals the area, so $\int_0^2 g(x)dx = 4$.

(b) $\int_2^6 g(x)dx$

Answer: On $[2,6]$ the graph of $g(x)$ is a semi-circle of radius 2 lying below the x-axis. Its area is

$$\frac{1}{2}\pi(2)^2 = 2\pi.$$

Since it lies below the axis, the integral is negative, so

$$\int_2^6 g(x)dx = -2\pi.$$

(c) $\int_0^7 g(x)dx$

Answer: Since

$$\int_0^7 g(x)dx = \int_0^2 g(x)dx + \int_2^6 g(x)dx + \int_6^7 g(x)dx = 4 - 2\pi + \int_6^7 g(x)dx,$$

we just need to determine $\int_6^7 g(x)dx$. Since this is a straight line of slope 1 going up from the x-axis (at $x=6$) to $y=1$ (at $x=7$), it describes a triangle of area

$$\frac{1}{2}1\cdot 1 = \frac{1}{2}.$$

Since this area lies above the axis, $\int_6^7 g(x)dx = 1/2$, so

$$\int_0^7 g(x)dx = 4 - 2\pi + \int_6^7 g(x)dx = 4 - 2\pi + \frac{1}{2} = \frac{9}{2} - 2\pi \approx -1.78.$$

3. Exercise 5.2.44. Use the result of Example 3 to compute

$$\int_1^3 (2e^x - 1)dx.$$

Answer: Example 3 says that $\int_1^3 e^x dx = e^3 - e$, we need to use the properties of the definite integral to express the given integral in terms of $\int_1^3 e^x dx$.

Now, by Property 4,

$$\int_1^3 (2e^x - 1)dx = \int_1^3 2e^x - \int_1^3 1dx.$$

In turn, by Property 1,

$$\int_1^3 1dx = 1(3-1) = 2.$$

By Property 3,

$$\int_1^3 2e^x dx = 2\int_1^3 e^x dx.$$

Putting these together, then,

$$\int_1^3 (2e^x - 1)dx = 2\int_1^3 e^x dx - 2.$$

Plugging in the value we know for $\int_1^3 e^x dx$, we see that

$$\int_1^3 (2e^x - 1)dx = 2(e^3 - e) - 2 = 2(e^3 - e - 1) \approx 32.73.$$

4. Exercise 5.3.14. Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function

$$h(x) = \int_0^{x^2} \sqrt{1+r^3}\, dr.$$

Answer: Make the change of variables $u = x^2$. Then

$$h'(x) = \frac{d}{dx}\left(\int_0^{x^2} \sqrt{1+r^3}\, dr\right) = \frac{d}{dx}\left(\int_0^{u} \sqrt{1+r^3}\, dr\right).$$

By the Chain Rule, this is equal to

$$\frac{d}{du}\left(\int_0^{u} \sqrt{1+r^3}\, dr\right)\frac{du}{dx}.$$

Using the Fundamental Theorem and the fact that $\frac{du}{dx} = 2x$, we see that

$$h'(x) = \sqrt{1+u^3}\,(2x) = \sqrt{1+(x^2)^3}\,(2x) = 2x\sqrt{1+x^6}.$$

5. Exercise 5.3.26. Evaluate the integral

$$\int_{\pi}^{2\pi} \cos\theta \, d\theta.$$

Answer: Since $\sin\theta$ is *an* antiderivative of $\cos\theta$, the second part of the Fundamental Theorem says that

$$\int_{\pi}^{2\pi} \cos\theta \, d\theta = \Big[\sin\theta\Big]_{\pi}^{2\pi} = \sin 2\pi - \sin\pi = 0 - 0 = 0.$$

6. Exercise 5.3.36. Evaluate the integral

$$\int_{0}^{1} 10^x \, dx.$$

Answer: Since

$$\frac{d}{dx}\left(10^x\right) = 10^x \ln 10,$$

we see that

$$\frac{10^x}{\ln 10}$$

is an antiderivative of 10^x. Therefore,

$$\int_0^1 10^x dx = \left[\frac{10^x}{\ln 10}\right]_0^1 = \frac{10}{\ln 10} - \frac{1}{\ln 10} = \frac{9}{\ln 10}.$$

7. Exercise 5.3.40. Evaluate the integral

$$\int_1^2 \frac{4+u^2}{u^3}\, du.$$

Answer: Re-write the integral as

$$\int_1^2 \left(\frac{4}{u^3} + \frac{u^2}{u^3}\right) du = \int_1^2 4u^{-3} du + \int_1^2 u^{-1} du.$$

Then, since $\frac{u^{-2}}{-2} = -\frac{1}{2u^2}$ is an antiderivative for u^{-3} and since $\ln u$ is an antiderivative for u^{-1}, we see that the above is equal to

$$\left[4\frac{-1}{2u^2}\right]_1^2 + \Big[\ln u\Big]_1^2 = \left(-\frac{2}{4} + \frac{2}{1}\right) + (\ln 2 - \ln 1) = \frac{3}{2} + \ln 2.$$