

Math 113 Final Exam Practice Problems

1. What are the domain and range of the function

$$f(x) = \frac{\ln x}{\sqrt{x}}?$$

2. Find the inverse of the function $f(x) = 1000(1 + 0.07)^x$.
3. Find the point on the graph of $y = e^{3x}$ at which the tangent line passes through the origin.
4. Find the equation of the tangent line to the curve

$$xy^3 - x^2y = 6$$

at the point $(3, 2)$.

5. Use an appropriate linearization to approximate $\sqrt{96}$.
6. Consider the function $f(x) = x^2e^{-x^2}$. What is the absolute maximum of $f(x)$?
7. A movie theater has been charging \$7.50 per person and selling about 400 tickets on a typical weeknight. After surveying their customers, the theater estimates that for every \$1.50 that they lower the price, the number of moviegoers will increase by 30 per night. Find the demand function and find the price which will maximize the theater's revenue.
8. Water is draining from a conical tank at the rate of 18 cubic feet per minute. The tank has a height of 10 feet and the radius at the top is 5 feet. How fast (in feet per minute) is the water level changing when the depth is 6 feet? (Note: the volume of a cone of radius r and height h is $\frac{\pi r^2 h}{3}$.)
9. The function $f(x) = x^4 - 6x^3$ is concave down for what values of x ?
10. Consider a bacteria culture that starts with a single, isolated bacterium. If the rate of change of the population of the culture is proportional to its size and if there are 100 bacteria after 1 hour, how many bacteria should we expect to see after 2 hours? [Hint: your answer should be a simple, recognizable number]

11. Evaluate the limit

$$\lim_{x \rightarrow 0} (1 - 6x)^{1/x}.$$

12. Let $f(x) = x^{\cos x}$. What is $f'(\pi/2)$?
13. For $0 \leq t \leq 5$, a particle moves in a horizontal line with acceleration $a(t) = 2t - 4$ and initial velocity $v(0) = 3$.
(a) When is the particle moving to the left?
(b) When is the particle speeding up?
(c) What is the position of the particle at time t if the initial position of the particle is 6?

14. If $\int_0^6 f(x)dx = 10$ and $\int_0^4 f(x)dx = 7$, find $\int_4^6 f(x)dx$.

15. Evaluate the definite integral

$$\int_{\pi/6}^{\pi/4} \sin t dt.$$

16. Evaluate the integral

$$\int \frac{2}{t-3} dt.$$

17. Evaluate the definite integral

$$\int_1^4 \frac{2\sqrt{x} + 4x^2}{x} dx$$

18. Suppose the velocity of a particle is given by

$$v(t) = 6t^2 - 4t.$$

What is the displacement of the particle from 0 to 2?

19. Suppose that

$$\int_0^{x^2} f(t) dt = \sqrt{x^2 + 1} - 1.$$

What is $f(2)$?