Math 113 HW #5 Solutions

§2.7

12. Shown are graphs of the position functions of two runners, A and B, who run a 100-m race and finish in a tie.

(a) Describe and compare how the runners run the race

Answer: A runs the race at a constant speed, never speeding up or slowing down. B accelerates throughout the race, starting out slower than A and, by the end, running faster than A.

(b) At what time is the distance between the runners the greatest?

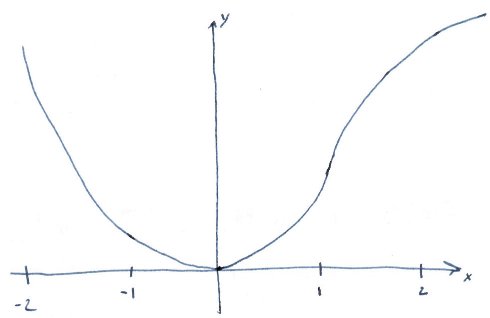
Answer: Based on the graph, it appears that they are furthest apart after 8 seconds, when they are approximately 30 meters apart.

(c) At what time do they have the same velocity?

Answer: The two graphs appear to have the same slope (i.e., velocity) 9 or 10 seconds into the race.

20. Sketch the graph of a function g for which $g(0) = g'(0) = 0$, $g'(-1) = -1$, $g'(1) = 3$, $g'(2) = 1$.

Answer:



26. Let

$$f(t) = t^4 - 5t.$$

Find $f'(a)$.

Answer: By definition,

$$\begin{aligned} f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} &= \lim_{h\to 0} \frac{\big[(a+h)^4 - 5(a+h)\big] - \big[a^4 - 5a\big]}{h} \\ &= \lim_{h\to 0} \frac{\big[a^4 + 4a^3h + 6a^2h^2 + 4ah^3 + h^4 - 5a - 5h\big] - \big[a^4 - 5a\big]}{h} \\ &= \lim_{h\to 0} \frac{4a^3h + 6a^2h^2 + 4ah^3 + h^4 - 5h}{h} \\ &= \lim_{h\to 0} \big(4a^3 + 6a^2h + 4ah^2 + h^3 - 5\big) \\ &= 4a^3 - 5. \end{aligned}$$

30. Let

$$f(x) = \sqrt{3x+1}.$$

Find $f'(a)$.

Answer: By definition,

$$\begin{aligned} f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} &= \lim_{h\to 0} \frac{\sqrt{3(a+h)+1}-\sqrt{3a+1}}{h} \\ &= \lim_{h\to 0} \frac{\sqrt{3a+3h+1}-\sqrt{3a+1}}{h}. \end{aligned}$$

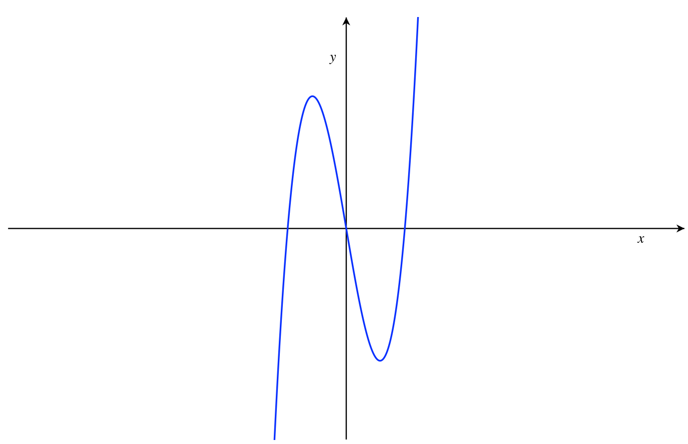
Multiplying numerator and denominator by $\left(\sqrt{3a+3h+1}+\sqrt{3a+1}\right)$ yields

$$\begin{aligned} \lim_{h\to 0} \frac{(3a+3h+1)-(3a+1)}{h\left(\sqrt{3a+3h+1}+\sqrt{3a+1}\right)} &= \lim_{h\to 0} \frac{3h}{h\left(\sqrt{3a+3h+1}+\sqrt{3a+1}\right)} \\ &= \lim_{h\to 0} \frac{3}{\sqrt{3a+3h+1}+\sqrt{3a+1}} \\ &= \frac{3}{2\sqrt{3a+1}}. \end{aligned}$$

§2.8

4. Using the given graph of f, sketch the graph of f'.

Answer:



24. Let

$$f(x) = x + \sqrt{x}.$$

Find the derivative of f using the definition of the derivative. State the domain of f and f'.

Answer: Note, first of all, that the domain of the function is all non-negative real numbers. By the definition of the derivative,

$$
\begin{aligned}
f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} &= \lim_{h\to 0}\frac{\left[(x+h)+\sqrt{x+h}\right]-\left[x+\sqrt{x}\right]}{h} \\
&= \lim_{h\to 0}\frac{h+\sqrt{x+h}-\sqrt{x}}{h} \\
&= \lim_{h\to 0}\left(\frac{h}{h}+\frac{\sqrt{x+h}-\sqrt{x}}{h}\right) \\
&= 1+\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}
\end{aligned}
$$

Multiplying numerator and denominator of the fraction inside the limit by $\left(\sqrt{x+h}+\sqrt{x}\right)$ yields

$$
\begin{aligned}
&= 1+\lim_{h\to 0}\frac{(x+h)-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)} \\
&= 1+\lim_{h\to 0}\frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)} \\
&= 1+\lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}} \\
&= 1+\frac{1}{2\sqrt{x}}.
\end{aligned}
$$

Notice that f' is not defined for $x=0$; hence, the domain of f' is all positive real numbers.

26. Let

$$f(x)=\frac{3+x}{1-3x}.$$

Find the derivative of f using the definition of the derivative. State the domain of f and f'.

Answer: Note that the domain of f consists of all $x\neq 1/3$. By definition

$$
\begin{aligned}
f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} &= \lim_{h\to 0}\frac{\frac{3+(x+h)}{1-3(x+h)}-\frac{3+x}{1-3x}}{h} \\
&= \lim_{h\to 0}\frac{\frac{(3+x+h)(1-3x)}{(1-3x-3h)(1-3x)}-\frac{(3+x)(1-3x-3h)}{(1-3x)(1-3x-3h)}}{h} \\
&= \lim_{h\to 0}\frac{\frac{\left[3+x+h-9x-3x^2-3xh\right]-\left[3-9x-9h+x-3x^2-3xh\right]}{(1-3x-3h)(1-3x)}}{h} \\
&= \lim_{h\to 0}\frac{\frac{10h}{(1-3x-3h)(1-3x)}}{h} \\
&= \lim_{h\to 0}\frac{10}{(1-3x-3h)(1-3x)} \\
&= \frac{10}{(1-3x)^2}
\end{aligned}
$$

Note that f' is defined whenever $x\neq 1/3$, so it has the same domain as f.

36. The graph of f is given. State, with reasons, the numbers at which f is not differentiable.

Answer: The function f is not differentiable at the following numbers:

$x = 0$ because f is not continuous there.

$x = 3$ because the graph of f has a vertical tangent there.

$x = 6$ because the graph of f has a vertical asymptote there.