Math 113 HW #1 Solutions

§ 1.1

6: Determine whether the curve is the graph of a function of x. If it is, state the domain and range of the function.

Answer: The pictured curve is the graph of a function. The domain and range of the function are:

$$\text{Domain: } -2 \leq x \leq 2$$
$$\text{Range: } -1 \leq y \leq 2$$

23: Given $f(x) = 4 + 3x - x^2$, evaluate the difference quotient

$$\frac{f(3+h) - f(3)}{h}.$$

Answer: Plugging in $x = 3 + h$ to $f(x)$ yields

$$f(3+h) = 4 + 3(3+h) - (3+h)^2 = 4 + 9 + 3h - (9 + 6h + h^2) = 4 - 3h - h^2.$$

Likewise,

$$f(3) = 4 + 3(3) - 3^2 = 4.$$

Therefore, the difference quotient

$$\begin{aligned} \frac{f(3+h) - f(3)}{h} &= \frac{(4 - 3h - h^2) - 4}{h} \\ &= \frac{-3h - h^2}{h} \\ &= -3 - h. \end{aligned}$$

44: Find the domain and sketch the graph of the function

$$f(x) = \begin{cases} x + 9 & \text{if } x < -3 \\ -2x & \text{if } |x| \leq 3 \\ -6 & \text{if } x > 3 \end{cases}.$$

Answer: Since the three pieces in the definition of f account for all real numbers, the domain of f consists of all real numbers. The graph of f is shown in Figure 1.

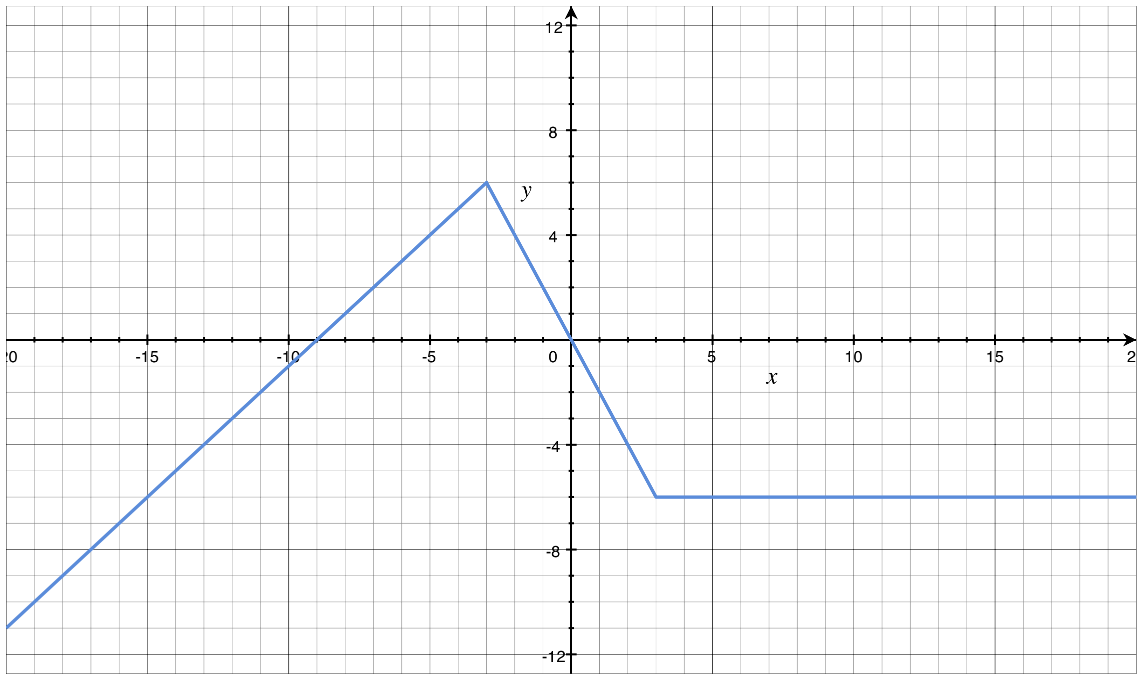

Figure 1: The graph of $y = f(x)$

56: A Norman window has the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 30 ft, express the area A of the window as a function of the width x of the window.

Answer: Let h denote the height of the rectangle. Then we know that the perimeter of the window is equal to

$$x + 2h + \text{outer perimeter of semi-circle.}$$

Since the semi-circle in our Norman window has radius $x/2$, its contribution to the perimeter of the window is half the circumference of a circle of radius $x/2$:

$$\frac{1}{2}\left(2\pi\frac{x}{2}\right) = \pi\frac{x}{2}.$$

Therefore, the perimeter of the window is

$$x + 2h + \pi\frac{x}{2} = \left(1 + \frac{\pi}{2}\right)x + 2h.$$

Since we know the perimeter of the window is equal to 30 ft, the above expression is equal to 30 and we can solve for h:

$$2h = 30 - \left(1 + \frac{\pi}{2}\right)x,$$

so

$$h = 15 - \left(\frac{1}{2} + \frac{\pi}{4}\right)x.$$

Therefore, the area A of the window is equal to

$$\begin{aligned} A(x) &= \text{area of rectangle } + \text{ area of semi-circle} \\ &= xh + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2 \\ &= x\left[15 - \left(\frac{1}{2} + \frac{\pi}{4}\right)x\right] + \frac{\pi x^2}{8} \\ &= 15x - \frac{x^2}{2} - \frac{\pi x^2}{4} + \frac{\pi x^2}{8} \\ &= 15x - \left(\frac{1}{2} + \frac{\pi}{8}\right)x^2. \end{aligned}$$

§ 1.2

2: Classify each of the following functions:

- **(a)** $y = \frac{x-6}{x+6}$ is a rational function.
- **(b)** $y = x + \frac{x^2}{\sqrt{x-1}}$ is an algebraic function.
- **(c)** $y = 10^x$ is an exponential function.
- **(d)** $y = x^{10}$ is a polynomial of degree 10.
- **(e)** $y = 2t^6 + t^4 - \pi$ is a polynomial of degree 6.
- **(f)** $y = \cos\theta + \sin\theta$ is a trigonometric function.

4: Match each equation with its graph

- **(a)** $y = 3x$ corresponds to the graph G.
- **(b)** $y = 3^x$ corresponds to the graph f.
- **(c)** $y = x^3$ corresponds to the graph F.
- **(d)** $y = \sqrt[3]{x}$ corresponds to the graph g.

6: What do all the members of the family of linear functions $f(x) = 1 + m(x + 3)$ have in common? Sketch several members of the family.

Answer: Each of the functions in this family is a line passing through the point $(-3, 1)$. By varying the different values of m we can get all such lines except the vertical line (which would correspond to $m = \infty$, if that was a valid choice for m). Several members of this family of lines are shown in Figure 2.

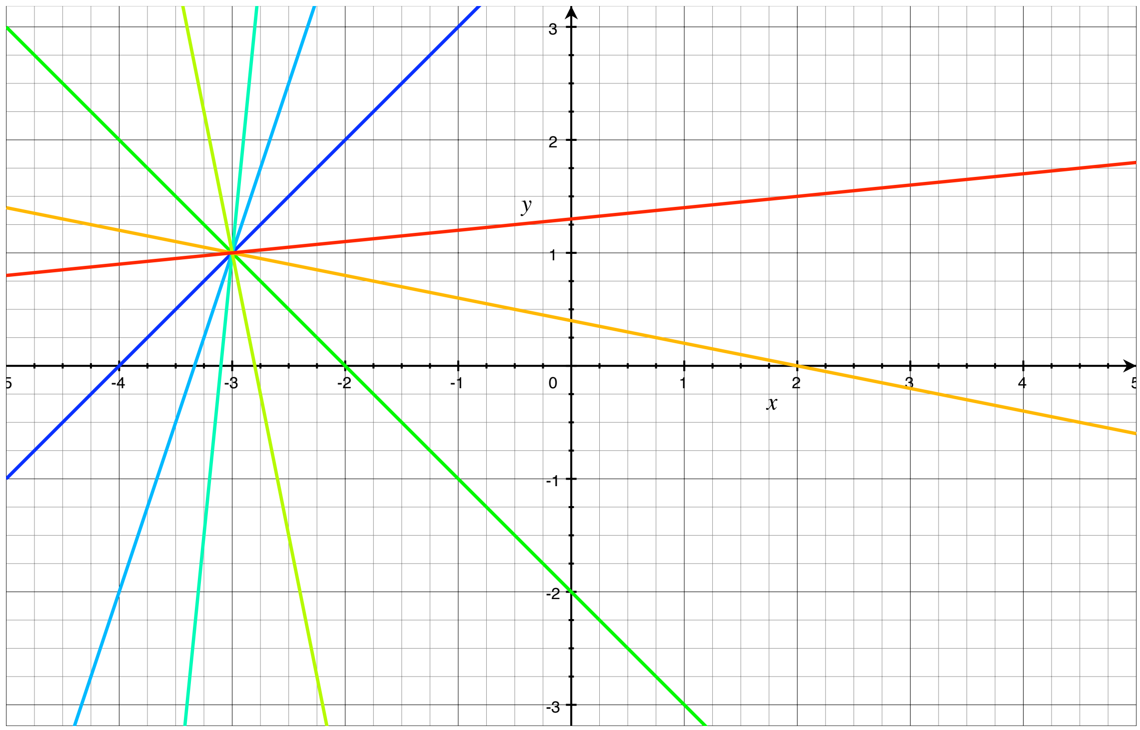

Figure 2: Various lines of the form $y = 1 + m(x + 3)$

16: The manager of a furniture factory finds that it costs \$2200 to manufacture 100 chairs in one day and \$4800 to produce 300 chairs in one day.

(a) Express the cost as a function of the number of chairs produced, assuming that it is linear. Then sketch the graph.

Answer: First, it makes sense to think of the number of chairs as the input and the cost to produce them as the output. Therefore, let $C(x)$ denote the cost of producing x chairs. Assuming $C(x)$ is linear, we want to find the equation of the line passing through the points $(100, 2200)$ and $(300, 4800)$. Such a line has slope

$$m = \frac{4800 - 2200}{300 - 100} = \frac{2600}{200} = 13.$$

Therefore, using the point-slope formula, the equation of the line is

$$y - 2200 = 13(x - 100),$$

so

$$y = 13(x - 100) + 2200 = 13x - 1300 + 2200 = 13x + 900.$$

Thus, we see that

$$C(x) = 13x + 900.$$

(b) What is the slope of the graph and what does it represent?

Answer: The slope of the graph $y = C(x)$ is equal to 13; this represents the cost of producing an additional chair. In economic terms, the marginal cost of production is \$13/chair.

(c) What is the y-intercept of the graph and what does it represent?

Answer: The y-intercept of $y = C(x)$ is equal to \$900; this represents the fixed costs of production.