Math 113 Final Exam Practice Problems

1. What are the domain and range of the function
$$f(x) = \frac{\ln x}{\sqrt{x}}?$$

2. Find the inverse of the function $f(x) = 1000(1 + 0.07)^x$.

3. Find the point on the graph of $y = e^{3x}$ at which the tangent line passes through the origin.

4. Find the equation of the tangent line to the curve
$$xy^3 - x^2y = 6$$
at the point $(3, 2)$.

5. Use an appropriate linearization to approximate $\sqrt{96}$.

6. Determine the absolute maximum and minimum values of the function
$$f(x) = \frac{x}{1 + x^2}.$$

7. A specialty publisher has typically sold trade paperbacks for \$15, averaging 300 sales per week. The publisher has found that increasing the price by 50 cents reduces sales by 10 per week. If the books cost \$10 each to make, what price should the publisher charge to maximize profit?

8. Water is draining from a conical tank at the rate of 18 cubic feet per minute. The tank has a height of 10 feet and the radius at the top is 5 feet. How fast (in feet per minute) is the water level changing when the depth is 6 feet? (Note: the volume of a cone of radius r and height h is $\frac{\pi r^2 h}{3}$.)

9. Find the inflection points for the function
$$f(x) = 8x + 3 - 2\sin x, \quad 0 < x < 3\pi.$$

10. Consider a bacteria culture that starts with a single, isolated bacterium. If the rate of change of the population of the culture is proportional to its size and if there are 100 bacteria after 1 hour, how many bacteria should we expect to see after 2 hours? [*Hint*: your answer should be a simple, recognizable number]

11. Evaluate the limit
$$\lim_{x \to 0^+} x^2 \csc^2 x.$$

12. Let $f(x) = x^{\cos x}$. What is $f'(\pi/2)$?

13. Given that
$$f'(t) = 2t - 3\sin t, \quad f(0) = 5,$$
find f.

14. Find the absolute minimum value of the function
$$f(x) = \frac{e^x}{x}$$
for $x > 0$.

15. If $\int_0^6 f(x)dx = 10$ and $\int_0^4 f(x)dx = 7$, find $\int_4^6 f(x)dx$.

16. Evaluate the definite integral

$$\int_{\pi/6}^{\pi/4} \sin t dt.$$

17. Evaluate the integral

$$\int \sec 3t \tan 3t dt.$$

18. Evaluate the definite integral

$$\int_1^4 \frac{2\sqrt{x} + 4x^2}{x} dx$$

19. Suppose the velocity of a particle is given by

$$v(t) = 6t^2 - 4t.$$

What is the displacement of the particle from 0 to 2?

20. Suppose that

$$\int_0^{x^2} f(t)dt = \sqrt{x^2 + 1} - 1.$$

What is $f(2)$?