

PFM MATH 104 QUIZ III SOLUTIONS

(1) Evaluate the integral

$$\int \frac{x+2}{\sqrt{x^2+4x}} dx.$$

Answer: Let $u = x^2 + 4x$. Then $du = (2x + 4)dx$, so we can re-write the integral as

$$\int \frac{x+2}{\sqrt{x^2+4x}} dx = \frac{1}{2} \int \frac{2x+4}{\sqrt{x^2+4x}} dx = \frac{1}{2} \int \frac{du}{\sqrt{u}}.$$

Now, we can evaluate the integral:

$$\frac{1}{2} \int \frac{du}{\sqrt{u}} = \frac{1}{2} \int u^{-1/2} du = \frac{1}{2} \cdot 2u^{1/2} + C = \sqrt{u} + C.$$

Substituting $x^2 + 4x$ back in for u yields the final answer,

$$\sqrt{x^2 + 4x} + C.$$

(2) Evaluate the definite integral

$$\int_1^2 x\sqrt{x-1} dx.$$

Answer: Let $u = x - 1$. Then $du = dx$ and $x = u + 1$. Also, $u = 0$ when $x = 1$ and $u = 1$ when $x = 2$, so we can re-write the integral as

$$\begin{aligned} \int_0^1 (u+1)\sqrt{u} du &= \int_0^1 (u^{3/2} + u^{1/2}) du = \left[\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2} \right]_0^1 \\ &= \frac{2}{5} + \frac{2}{3} \\ &= \frac{16}{15}. \end{aligned}$$

(3) Let

$$f(x) = \ln \sqrt[5]{x+2}.$$

What is $f'(3)$?

Answer: Using the third property of logarithms,

$$\ln \sqrt[5]{x+2} = \frac{1}{5} \ln(x+2).$$

Therefore,

$$f'(x) = \frac{d}{dx} \left(\frac{1}{5} \ln(x+2) \right) = \frac{1}{5} \frac{1}{x+2} = \frac{1}{5x+10}.$$

Plugging in $x = 3$, we see that

$$f'(3) = \frac{1}{5 \cdot 3 + 10} = \frac{1}{25}.$$

(4) Evaluate the definite integral

$$\int_e^6 \frac{dx}{x \ln x}.$$

Answer: Let $u = \ln x$. Then $du = \frac{1}{x} dx$. Also, $u = \ln 6$ when $x = 6$ and $u = 1$ when $x = e$. Therefore, we can re-write the integral as

$$\int_1^{\ln 6} \frac{du}{u} = \left[\ln |u| \right]_1^{\ln 6} = \ln \ln 6 - \ln 1 = \ln \ln 6.$$