PFP Math 104 Final Exam

1. Evaluate the definite integral

$$\int_1^3 \frac{x^4 + 1}{x^2} dx.$$

2. Let $f(x) = x^{\cos x}$. What is $f'(\pi/2)$?

3. Assuming $x > 3$, evaluate the integral

$$\int \frac{x}{x^2 - 6x + 9} \, dx.$$

4. Define the function $g(x)$ by the equation

$$g(x) = \int_1^{x^2} \frac{\sin t}{\sqrt{t}} \, dt.$$

What is the derivative of g?

5. Use right endpoints and 3 subintervals to estimate

$$\int_1^4 \frac{1}{x} dx,$$

then write this definite integral as a limit of Riemann sums.

6. Find the inverse of the function

$$f(x) = 1000(1 + 0.07)^x.$$

7. Suppose the velocity of a particle is given by

$$v(t) = 4(t-3)^3 + 7(t-3) - 4.$$

How far is the particle at time $t = 6$ from its starting point (at $t = 0$)?

8. Evaluate the definite integral

$$\int_0^1 \frac{dx}{\sqrt[3]{x}\left(x^{2/3} + 1\right)}.$$

9. What is

$$\frac{d}{dx}\left(e^{e^{e^x}}\right)?$$

10. Find an antiderivative of the function

$$f(x) = \frac{x^3}{\sqrt{x^2+1}}.$$

11. Evaluate the definite integral

$$\int_1^{100} \frac{1}{\ln 10} \frac{(\log_{10} x)^3}{x} \, dx.$$

12. Let

$$f(x) = \sqrt{x^3 + x^2 + x + 1},$$

which is well-defined for $x > -1$. What is $(f^{-1})'(2)$?