CHAPTER 5 REVIEW, PROBLEM 6

(a): Write $\int_1^5 (x+2x^5)dx$ as a limit of Riemann sums, taking the sample points to be the right endpoints.

(b): Use the Fundamental theorem to evaluate the integral.

Solution:

(a): If we let $f(x) = x+2x^5$, then we know that, as a limit of Riemann sums,

(1) $$\int_1^5 f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} \Delta x f(x_i)$$

where $x_0 = 1$, $x_n = 5$ and the interval $[1,5]$ is split up into n subintervals $[x_i, x_{i+1}]$ of equal width Δx.

Breaking the interval $[1,5]$ into n subintervals of equal width means that each such subinterval must be of width

$$\Delta x = \frac{5-1}{n} = \frac{4}{n}.$$

Now, the point x_1 is the right endpoint of the first subinterval, $[1, x_1]$, meaning that x_1 is at a distance $\Delta x = \frac{4}{n}$ from 1. Therefore,

$$x_1 = 1 + \Delta x = 1 + \frac{4}{n}.$$

Similarly, the point x_i is the right endpoint of the ith subinterval, so there are i subintervals of width $\Delta x = \frac{4}{n}$ between x_i and 1. Therefore,

$$x_i = 1 + i\Delta x = 1 + i\frac{4}{n}.$$

Therefore,

$$f(x_i) = f\left(1 + i\frac{4}{n}\right) = \left(1 + i\frac{4}{n}\right) + 2\left(1 + i\frac{4}{n}\right)^5.$$

Plugging Δx and $f(x_i)$ into (1), we see that

$$\int_1^5 (x+2x^5)dx = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{n}\left[1 + i\frac{4}{n} + 2\left(1 + i\frac{4}{n}\right)\right]$$

$$= \lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^{n} \left[1 + i\frac{4}{n} + 2\left(1 + i\frac{4}{n}\right)\right]$$

(b): Using the Fundamental Theorem of Calculus,

$$\begin{aligned}\int_1^5 (x + 2x^5)dx &= \left[\frac{x^2}{2} + 2\frac{x^6}{6}\right]_1^5 \\ &= \left(\frac{5^2}{2} + \frac{5^6}{3}\right) - \left(\frac{1^2}{2} + \frac{1^6}{3}\right) \\ &= \frac{25}{2} + \frac{15625}{3} - \frac{1}{2} - \frac{1}{3} \\ &= 5220\end{aligned}$$