The Center of a Triangle

October 5, 2005

1 Purpose

In this lab we'll try to figure out what it means to talk about the "center" of a triangle and we'll investigate how to find various different centers of triangles. We'll also try to figure out how to apply this knowledge to real problems.

2 Materials

- cardboard
- compass
- computer (optional)
- paper
- pencil

3 Vocabulary

- centroid
- center of mass
- circumcenter
- incenter
- orthocenter

4 Centers of Triangles?

1. It might seem sort of weird to talk about the "center" of a triangle; after all, a triangle isn't symmetric like a circle or a square. What would you consider the "center" of a triangle? Can you think of more than one possible "center"? Will these different conceptions of the center describe the same point?

2. Can you come up with a strategy for finding the center(s) of triangles you described above?

5 One Possibility

1. Take a piece of cardboard and cut out a triangle. Be careful to make sure the sides are straight.
2. Here's a possible center of a triangle: the point on which it will balance. Try to balance your triangle on a pencil eraser. It may take a few attempts, but there should be a point where you can balance the triangle...mark this point.
3. The point you've marked is called the **centroid** of the triangle. Another name for the centroid is the **center of mass**, because, for many practical purposes (like balancing the triangle), we can pretend that all of the mass is concentrated at this point. All objects have a center of mass. In what contexts might it be useful to know an object's center of mass?
4. In general, finding the center of mass for an object is difficult (usually it requires calculus), but for the triangle it's easy. How might you locate the centroid of a triangle without trying to balance it? (Hint: for a flat triangle, mass is proportional to area. The center of mass is, as its name implies, in the very center of an object's mass, so the center of mass (centroid) of a triangle will be in the very center of the triangle's area)
5. Test your method of finding the centroid of a triangle on your triangle...did it work?

6 What about circles?

That might seem like an odd question, but we often think about centers in the context of circles; so what about different circles that might relate to triangles?

1. Can you construct a circle around your triangle that just barely touches the three corners of the triangle? Try it! Don't get discouraged if this turns out to be harder than it sounds; if your first attempt doesn't work, try again.

2. Such a circle is called a **circumscribed circle**. The center of a circumscribed circle is called the **circumcenter** of the triangle. Can you come up with a strategy for finding the circumcenter of your triangle that's more reliable than just guess-and-check? (Hint: the corners of the triangle are supposed to be on the edge of the circle, so they should all be exactly the same distance from the circumcenter)

3. Where might it be useful to find the circumcenter of a triangle in the real world? Think about some possible applications.

4. What about other circles? Can you construct a circle *inside* your triangle that just barely touches the edges of the triangle? Try it! Again, don't get discouraged if this turns out to be harder than it seems like it ought to be.

5. A circle like this is called an **inscribed circle** and its center is called the **incenter** of the triangle. Can you think of a good way to find the incenter of your triangle without having to draw any circles? (Hint: each of the edges of the triangle is supposed to just touch the edge of the inscribed circle, so the incenter should be exactly the same distance from all three edges)

6. When might it be useful to find to find the incenter of a real-world triangle? What professions might find this technique useful?

7 Other Centers

1. If we draw the altitudes from each side, the point where they intersect is called the **orthocenter** of a triangle. Try it!

2. Do the different centers of the triangle we've discussed coincide?

3. If they don't, can you think of certain special triangles where they *would* intersect? Can you think of triangles where some of the "centers" wouldn't even be in the interior of the triangle?

4. It turns out that the triangle centers we've discussed are special cases of what are called **Kimberling centers**, which are defined by special functions called **center functions** and special coordinates called **trilinear coordinates**. More than 3000 such centers have been compiled by Professor Clark Kimberling and are listed online at: `http://faculty.evansville.edu/ck6/tcenters/`

8 Sketchpad (Optional)

1. Make a triangle in Sketchpad (hint: place three points anywhere in the sketch, then construct the line segments connecting them to form a triangle).

2. Using the "Construct — Midpoint" command, construct the midpoint of each side of the triangle. Connect these to the appropriate vertices. Select two of these new line segments and construct their intersection. Label this point. This is the centroid of the triangle.

3. Use the "Construct — Midpoint" command and the "Construct — Perpendicular Line" command to construct the triangle's circumcenter. Label it.

4. Use the "Construct — Angle Bisector" command to help you construct the triangle's incenter. Label it.

5. Try moving the vertices of the triangle around and check out what happens to the different centers of the triangle (here's where it's useful to have labeled the centers so you don't get confused which is which). Can you confirm or deny your answers to question 7.3?