Straight Shooters

September 21, 2005

1 Purpose

The purpose of this activity is to understand the way in which geometry influences the game of pool. We'll see how our knowledge of angles can improve how we play pool and we'll try it out on Geometer's Sketchpad.

2 Materials

- computer
- Geometer's Sketchpad
- paper
- pencil
- protractor

3 Vocabulary

- angle
- angle of incidence
- angle of refraction
- clockwise
- line segment
- reflection

4 Easy Shot

1. The easiest shots in pool look something like this:

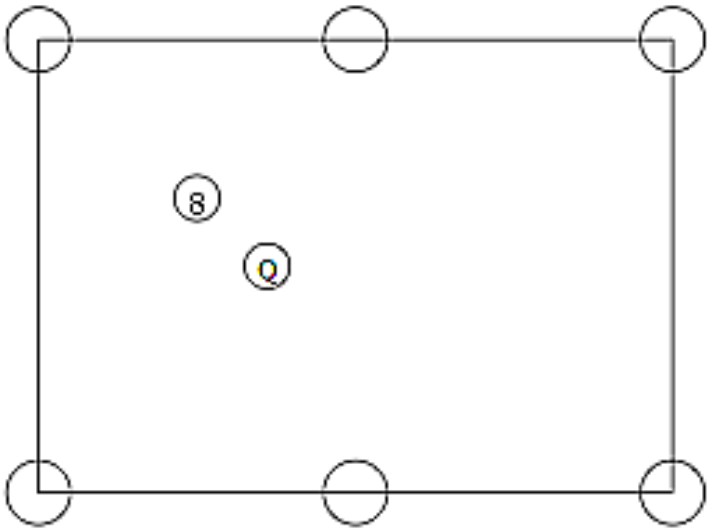

Figure 1: An easy shot

Here, the ball marked "Q" is the cue ball. Notice that the straight line passing through the cue ball and the top-left pocket also passes through the 8 ball, so to pocket the 8 ball, all you need to do is hit the cue ball directly at the pocket.

2. Draw a straight line from the top-left pocket through the cue ball and all the way past the edge of the table.

3. Label the pockets in alphabetical order starting with the top-left pocket and traveling clockwise.

4. Label the intersection of the line you drew with the edge of the table by I.

5. Use your protractor to find the measure of angle FAQ:

6. Without using your protractor, what is the measure of angle FIQ?

5 Reflected Shots

1. Of course, in a real game of pool, straight shots like what we dealt with above don't happen too often. For example, suppose you're playing 8 ball and are faced with the following situation:

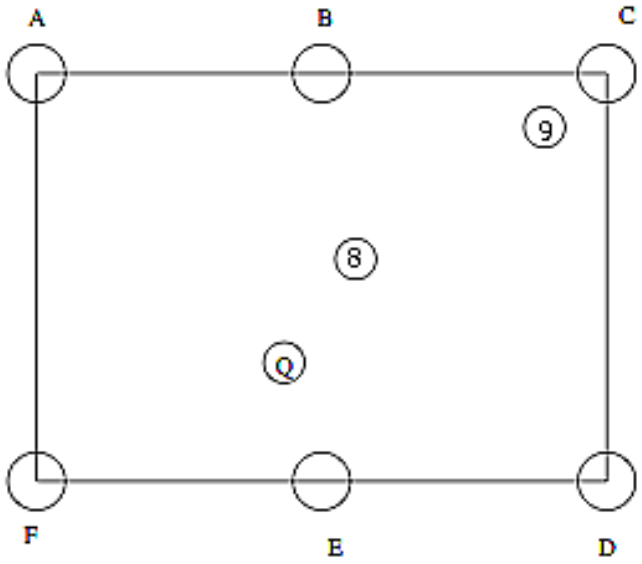

Figure 2: A harder shot

Your opponent's 9 ball is blocking your shot to the top-right corner; the only way you're going to be able to sink the 8 ball is by banking it off the top rail and into the bottom-right corner pocket.

2. How might we figure out in what direction to hit the 8 ball? Where should it bounce of the top rail in order to end up in the bottom-right pocket?

3. We're trying to figure out the path the 8 ball should take, so we can safely ignore the cue ball and the 9 ball for now. Here's what we would like the 8 ball to do:

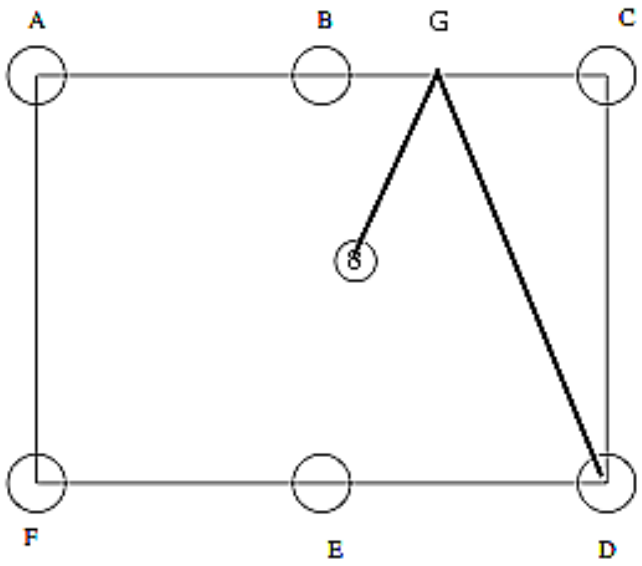

Figure 3: The path we want the 8 ball to take

4. Without using a protractor, what is the relationship between angle 8GB and angle DGC? Angle 8GB is called the **angle of incidence** and angle DGC is called the **angle of refraction**.

5. Our goal is to figure out where G is; let's use Geometer's Sketchpad!

6 Using Geometer's Sketchpad to model a pool shot

1. First, we need to construct our virtual pool table:

   (a) Make a line segment of suitable size. It is recommended, though not necessary, that the segment be perfectly vertical or horizontal.

   (b) Select one of the endpoints and the line segment, then use "construct — perpendicular line."

   (c) Then select the perpendicular line, click construct "point on perpendicular line."

(d) Move the point to a suitable place.

(e) Select the point and the first perpendicular line, then use the construct perpendicular line function again.

(f) Now go to the other endpoint of the original line segment. Construct a perpendicular line there.

(g) Construct the other corner by selecting the two intersecting lines and using "construct — point of intersection"

(h) Make the top side of the "table" into a segment by clicking on its endpoints and using "construct — segment"

(i) Then construct the midpoint by using "construct — midpoint"

(j) Do the same on the bottom side.

(k) Now label the "pockets" starting with the top left and going clockwise, A . . . F.

2. Now, put a point in an appropriate spot to use as your target ball, label it Y.

3. As with most problems, there's more than one way to figure out where to shoot Y. Before we do anything weird, let's just try shooting Y at different spots on the rail and seeing what happens.

   (a) Select the segment AC and use "construct — point on segment"; this will be the point we shoot at.

   (b) Construct the segment YG.

   (c) Select AC and G and use "construct — perpindicular line". Once it's constructed, this line should still be selected; use "transform — mark mirror". We don't want this line cluttering up our picture, so select it and use "display — hide perpendicular line" to hide it.

   (d) Now, select YG and G, then use "transform — reflect" to reflect them about the line we just hid.

   (e) Select G then the new point you just created and "construct — ray". Then select just the new point and "display — hide point" to hide it.

   (f) Move G (if necessary) until the ray you just made intersects the bottom rail. Select the ray and FD then "construct — intersection". The intersection point should still be selected at this point, so use "display — label intersection" to label it; call it Z.

   (g) Remember that G is the point on the top rail that you're shooting at; as you move G around, Z shows where your bank shot will end up.

   (h) Let's measure some angles: select A, G, Y **in that order**, then use "measure — angle" to measure the angle. Next, select C, G, Z and again use "measure — angle".

   (i) What can you say about angles AGY and CGZ? What are AGY and CGZ when you move G to a spot where Z matches up with D (that is, when the bank shot will go in the pocket)?

(j) What do you think of this method of solving the problem?

4. Let's try a different method. First, move G over close to A so it won't get in the way (don't put G on top of A, or you may not be able to select it again). Next, we want to reflect the pocket D across the AC rail.

   (a) Select AC, use "transform — mark mirror."

   (b) Select D and CD, then use "transform — reflect"

   (c) Look up, find the new point. It should be already selected. Use "display — label point" to label the point D. Note that D is the default name for this reflected point.

5. Now, we just shoot our ball at the "reflection" of D: construct the line segment between Y and D.

6. Select YD and AC. Click "construct — intersection."

7. Label the intersection K.

8. Now, we measure some angles.

   (a) Select the points A, K, Y in that order!!

   (b) Use "measure — angle." Note what happens if you click the points in the wrong order. This is the angle of incidence.

   (c) Now, measure the angle of refraction: construct the segment KD, and measure the angles DKY, DKC. How does the angle of refraction compare to the angle of incidence?

9. Try moving your ball around; what happens to the angles of incidence and refraction?

10. Do these two methods give you the same solution to the problem? How could you determine this?

11. Which method do you think is better? Why? Which is more interesting or fun? Which seems "better"?

12. Obviously, you can't refer to Geometer's Sketchpad every time you play pool, but how could you use what we did to help you win at pool?