Pick's Theorem

Math 123

February 2006

1 Purpose

The purpose of this activity is to gain a better understanding of area and to learn how to interpret data. We will experiment with different polygons and find their areas. We will explore a data set to form and test a conjecture about the relationship between area and grid points.

2 Materials

- pencil & paper
- geo-board
- rubber bands

3 Vocabulary

- **conjecture**
- **data**
- **interior**
- **perimeter**
- **polygon**

4 No Interior Points

1. Using a rubber band, make a shape on your geoboard by hooking the rubber band around several different pegs.
2. Rearrange your rubber band until you have a shape that has no pegs in the interior.

3. Count the number of pegs on the perimeter (boundary) of the shape and record this number in the table below.

4. Find the area of this shape. (We consider one unit of length to be the distance between two pegs that are horizontally side by side.)

5. Next create another shape (shape 2) with zero pegs in the interior on your geo-board with the rubber band.

6. Do steps 3-4, using this new shape instead of your first shape.

7. Now create 2 more shapes (again with no pegs on the interior) on the geo-board, and do steps 3-4 using these shapes.

Shape	Interior Points I(P)	Perimeter Points B(P)	Total Area A(P)
Shape 1			
Shape 2			
Shape 3			
Shape 4			
Shape 5			
Shape 6			
Shape 7			
Shape 8			
Shape 9			
Shape 10			
Shape 11			
Shape 12			
Shape 13			
Shape 14			
Shape 15			
Shape 16			
Shape 17			
Shape 18			
Shape 19			
Shape 20			

5 Form a Conjecture

1. From your data so far, do you notice any relationships between A(P) and B(P)?

2. If so, can you write the relationship as a precise formula?

6 One Interior Point

1. Now create a new polygon with exactly one interior peg.
2. Count the number of pegs on the perimeter and the number of pegs in the interior. Put this data into the table above.
3. Calculate the area of this polygon and put this information into the table above.
4. Repeat steps 1-3 three more times.

7 Elaborate on the Conjecture

1. What relationsip do you see between A(P), B(P), and I(P)?

2. Hint: Can you multiply B(P) by a number, multiply I(P) by a number, and then add them together to get A(P)?

3. Write your answer as a formula for A(P) in terms of B(P) and I(P).

8 Generalize

1. You have tried zero interior pegs and one interior peg. What situation should you investigate next? Do it!

2. Continue investigating more and more interior pegs until you find a pattern.

3. Write a formula for A(P) in terms of B(P) and I(P).

9 Test Your Conjecture

1. Use your geo-board to create any polygon you want, as long as it is different from the polygons you previously created.

2. Count the number of boundary points and interior points.

   Interior Points:

   Boundary Points:

3. Using I(P) and B(P), what does your formula tell you the area shoud be?

4. Calculate the area of your polygon without using the formula.

5. Was your prediction correct?

6. Do you believe that you have the right formula?

10 Proof

- What if I told you I could create a shape that doesn't fit this formula. Would you believe me?

- Sometimes in geometry formulas and relationships seem true based on examples but actually are only true in certain cases. Later on we will talk about how to "prove" that something is always true. (This particular formula is one that you can prove is always true.)