Perimeters, Areas, and a Long Snowflake

Math 123

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1 Purpose

Suppose you are starting a vegetable garden. You want to put a fence around it but only have 40 feet of fence. You want the biggest garden you can make given your limited amount of fencing materials. Are there different ways to arrange the fence around the perimeter so that you get a bigger area inside the fence? This problem investigates the relationship between area and perimeter. You might think that if one shape has more area than another, then it is "bigger". In what ways is this true? Do bigger shapes have longer perimeters? On the other hand, are there shapes with the same perimeter and different areas? Today we will discover the answer to these questions and find out a few other interesting facts about the relationship between area and perimeter.

2 Materials

- Lots of square tiles
- Tracing paper
- Pencil and paper
- Triangular grid paper
- Graph paper

3 Vocabulary

- **area**
- **fractal**
- **perimeter**

4 What Perimeters Have the Same Area?

Count out 16 squares and assume that the length of the side of a square is 1. Arrange these squares into a larger square. What is the perimeter of the larger square?

Try to arrange these 16 squares into shapes that have perimeters of length
(1) 20
(2) 34
(3) 18
Sketch your results here:

Do these shapes all have the same are? (If so, what is it?)

4.1 What Areas Have the Same Perimeter?

Count out 9 triangles, and assume that the length of the side of a triangle is 1. Arrange them to form any shape you want.

1. What is the perimeter of the shape you made?

2. Take away 2 of the triangles. Can you arrange the remaining triangles into a shape that has the same perimeter? In other words, can you find a shape with the same perimeter but smaller area?

3. Can you arrange more than 9 triangles into a shape that has the same perimeter as your original large triangle?

5 The Koch Snowflake

So far, you have found that there are shapes that have the same area and different perimeter. You also found shapes that have the same perimeter and different area. How different can the area and perimeter be? You'll investivate this question further in this part of the lab. You'll need to ahve the page with the triangular grid and the page with the blank table handy. On the triangular grid, suppose the triangles have side length 1.

First Step: Draw an equilateral triangle on the triangular grid so that each side has length 9. In the table, fill in the number of sides, the length of a side, and the total perimeter. Also fill in the area of the triangle.

Second Step: You can make step 2 out of step 1 by dividing each side of the triangle into thirds and replacing the middle third by a triangle with side length 3. Each of the new segments has the same length.

1. How many sides are there in the new shape

2. What is the length of each side?

3. What is the perimeter of the new shape?

4. What is the area of the new shape?

Fill in the second row of the table with your answers.

Third Step: For step 3, divide each side into thirds. Replace the middle third of each side with a rectangle in the same way as in Step 2. Answer the questions in Step 2 again, and fill in the third row of the table with your answers.

More Steps: Repeat your actions in the third step on the shape you made in the third step. Answer the questions in Step 2 again and fill in the rows of the table with your answers (you'll probably want to stop drawing after the third or fourth step and try to find a pattern instead).

Make a Graph: Make a graph of the perimeter of the first eight shapes. Make another graph of the area of the first eight shapes. Describe what is happening to the perimeter and to the area.

If you continue this process infinitely many times, you get what is called a fractal.

# of steps	# of sides	Length of side	**Perimeter**	**Area**
1				
2				
3				
4				
5				
6				
7				
8				

6 Constructing Fractals Using the Computer

1. Go to the website http://compute2.shodor.org/snowflake
2. Change the number of iterations to 1. (This lets you see one step of the fractal construction at a time)
3. Click on "Grid Background" and "Snap Active Point to Grid" (this lets you space the points evenly)
4. Use the mouse to drage your "active point" to anywhere you want it.
5. Choose another active point and drag it anywhere you want.
6. Continue placing the points until you have a picture you want to make into a fractal.
7. Click on the button "up one" to see what the first "iteration" of the fractal looks like.
8. Continue clicking "up one" to see your fractal evolve.

6.1 Your Fractal.

1. Did your fractal start to look like a curve instead of a bunch of straight lines? If so, at what iteration?

2. Did your fractal ever intersect itself? (Why or why not?)

7 The Mandelbrot Set and some Fractals in Nature

- The Mandelbrot set is a fractal that models the "self-similarity" in the molecular behavior of blood vessels.
- Take a look at the webpage http://compute2.shodor.org/cgi-bin/mandy/cnew.pl
- Click on a part of the picture to zoom in (like on Google Maps or Mapquest) on that part.
- Notice that unlike Google Maps, the picture looks pretty much the same as the zoomed out version. This is "self-similarity". It means that the Mandelbrot set looks more or less the same at all scales.
- Check out the following websites to see examples of fractals in nature:

  classes.yale.edu/fractals/atma/FracNat/FracNat.html

  www.phys.uni.torun.pl/~duch/zdjecia/00Siberia/syb1.html

  www.fractal.org/Fractal-Research-and-Products/Dissecting-fractals.htm

  www.photo.net/photodb/photo?photo_id=1236856

  www.bendov.info/cours/sources/fracs/fraceng/sld008.htm