MATH 104 QUIZ VII SOLUTIONS

CLAY SHONKWILER

In each problem, determine whether the sequence or series converges or diverges. If it converges, determine the limit or sum.

(1) The series

$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2-1}}.$$

Answer: Using the integral test

$$\begin{aligned}
\int_1^{\infty} \frac{dx}{x\sqrt{x^2-1}} &= \lim_{b\to\infty} \int_1^b \frac{dx}{x\sqrt{x^2-1}} \\
&= \lim_{b\to\infty} \left[\sec^{-1}|x|\right]_1^b \\
&= \lim_{b\to\infty} \left[\sec^{-1} b - \sec^{-1} 1\right] \\
&= \frac{\pi}{2} - 0 \\
&= \frac{\pi}{2}.
\end{aligned}$$

Since the integral converges, we know, by the Integral Test, that the series converges.

(2) The series

$$\sum_{n=1}^{\infty} \frac{1+\cos n}{n^2}.$$

Answer: Note that $1+\cos n \leq 2$. Hence,

$$\frac{1+\cos n}{n^2} \leq \frac{2}{n^2}.$$

Therefore, since the series

$$\sum_{n=1}^{\infty} \frac{2}{n^2}$$

converges (it is a p-series with $p = 2$), we know that the original series also converges.

(3) The series

$$\sum_{n=1}^{\infty} n! e^{-n}.$$

Answer: Using the Ratio Test,

$$\lim_{n\to\infty} \frac{(n+1)!e^{-(n+1)}}{n!e^{-n}} = \lim_{n\to\infty}(n+1)e^{-1} = \infty,$$

so the series diverges.

DRL 3E3A, University of Pennsylvania
E-mail address: `shonkwil@math.upenn.edu`