# MATH 104 QUIZ VI SOLUTIONS

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In each problem, determine whether the sequence or series converges or diverges. If it converges, determine the limit or sum.

(1) The sequence $\{a_n\}$ where

$$a_n = \frac{\ln n}{\cos^2 n}.$$

**Answer:** Since $|\cos n| \leq 1$ for all $n$, $\cos^2 n \leq 1$ for all $n$. Hence,

$$\frac{\ln n}{\cos^2 n} \geq \frac{\ln n}{1} = \ln n$$

for all $n$. As $n \to \infty$, $\ln n \to \infty$, so the sequence $\{\ln n\}$ diverges. Therefore, $\{a_n\}$ diverges as well.

(2) The sequence $\{a_n\}$ where

$$a_n = n \sin \frac{1}{n}.$$

**Answer:** Re-write $n \sin \frac{1}{n} = \frac{\sin \frac{1}{n}}{\frac{1}{n}}$. Then, by L'Hôpital's Rule,

$$\lim_{n\to\infty} n \sin \frac{1}{n} = \lim_{n\to\infty} \frac{\sin \frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{\cos \frac{1}{n} \cdot \frac{-1}{n^2}}{\frac{-1}{n^2}} = \lim_{n\to\infty} \cos \frac{1}{n} = 1.$$

Thus, the sequence $\{a_n\}$ converges to 1.

(3) The series

$$\sum_{n=1}^{\infty} e^{-2n}.$$

**Answer:** Note that

$$\sum_{n=1}^{\infty} e^{-2n} = \sum_{n=1}^{\infty} \frac{1}{e^{2n}} = \sum_{n=1}^{\infty} \left(\frac{1}{e^2}\right)^n = \sum_{n=1}^{\infty} \frac{1}{e^2} \left(\frac{1}{e^2}\right)^{n-1},$$

which is a geometric series with $a = r = \frac{1}{e^2}$. Hence, the series converges to

$$\frac{\frac{1}{e^2}}{1 - \frac{1}{e^2}} = \frac{\frac{1}{e^2}}{\frac{e^2-1}{e^2}} = \frac{1}{e^2 - 1}.$$

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