

MATH 104 QUIZ IV SOLUTIONS

CLAY SHONKWILER

(1) Evaluate the integral

$$\int \ln x dx.$$

Answer: To integrate by parts, let $u = \ln x$ and $dv = dx$. Then $du = \frac{1}{x}dx$ and $v = x$. Hence,

$$\int \ln x dx = x \ln x - \int \frac{x}{x} dx = x \ln x - \int dx = x \ln x - x + C.$$

(2) Evaluate the integral

$$\int \frac{2x - 6}{x^2 - 6x - 7} dx.$$

Answer: Note that we can factor $x^2 - 6x - 7 = (x - 7)(x + 1)$, so we let $\frac{2x-6}{x^2-6x-7} = \frac{A}{x-7} + \frac{B}{x+1}$ and solve for A and B :

$$2x - 6 = A(x + 1) + B(x - 7).$$

Letting $x = -1$, we see that

$$2(-1) - 6 = A(0) + B(-8) = -8B,$$

so $B = 1$. Letting $x = 7$, we see that

$$2(7) - 6 = A(8) + B(0) = 8A,$$

so $A = 1$. Hence,

$$\int \frac{2x - 6}{x^2 - 6x - 7} dx = \int \left[\frac{1}{x - 7} + \frac{1}{x + 1} \right] dx = \ln |x - 7| + \ln |x + 1| + C.$$

(3) Evaluate the integral

$$\int \frac{dx}{(4 - x^2)^{3/2}}.$$

Answer: Let $x = 2 \sin \theta$. Then $dx = 2 \cos \theta d\theta$, so we can re-write the integral as

$$\begin{aligned} \int \frac{2 \cos \theta d\theta}{(4 - 4 \sin^2 \theta)^{3/2}} &= 2 \int \frac{\cos \theta d\theta}{(4 \cos^2 \theta)^{3/2}} \\ &= 2 \int \frac{\cos \theta d\theta}{8 \cos^3 \theta} \\ &= \frac{1}{4} \int \frac{1}{\cos^2 \theta} d\theta \\ &= \frac{1}{4} \int \sec^2 \theta d\theta \\ &= \frac{1}{4} \tan \theta + C. \end{aligned}$$

Now, since $\theta = \sin^{-1} \frac{x}{2}$, $\tan \theta = \frac{x}{\sqrt{4-x^2}}$. Therefore,

$$\int \frac{dx}{(4-x^2)^{3/2}} = \frac{1}{4} \frac{x}{\sqrt{4-x^2}} + C = \frac{x}{4\sqrt{4-x^2}} + C.$$

DRL 3E3A, UNIVERSITY OF PENNSYLVANIA
E-mail address: shonkwil@math.upenn.edu