MATH 104 QUIZ III SOLUTIONS

CLAY SHONKWILER

(1) Let $y = \ln(\ln(\ln x))$. Find dy/dx.

    Answer: Using repeated applications of the chain rule:

$$\frac{dy}{dx} = \frac{1}{\ln(\ln x)} \cdot \frac{d}{dx}\left[\ln(\ln x)\right]$$

$$= \frac{1}{\ln(\ln x)} \cdot \left[\frac{1}{\ln x} \cdot \frac{d}{dx}(\ln x)\right]$$

$$= \frac{1}{\ln(\ln x)} \cdot \left[\frac{1}{\ln x} \cdot \frac{1}{x}\right]$$

$$= \frac{1}{x \ln x \ln(\ln x)}.$$

(2) Evaluate the integral

$$\int \frac{e^{1/x}}{4x^2} dx.$$

    Answer: Let $u = 1/x$. Then $du = \frac{-1}{x^2} dx$. Hence, we can re-write the integral as

$$\frac{-1}{4} \int e^{1/x}\left(\frac{-1}{x^2}\right) dx = \frac{-1}{4} \int e^u du = \frac{-1}{4} e^u + C = \frac{-1}{4} e^{1/x} + C.$$

(3) Let $y = x^{x^2}$. Find dy/dx.

    Answer: Recall that $y = x^{x^2} = e^{\ln(x^{x^2})} = e^{x^2 \ln x}$. Therefore,

$$\frac{dy}{dx} = e^{x^2 \ln x} \cdot \frac{d}{dx}\left[x^2 \ln x\right]$$

$$= x^{x^2}\left[x^2 \cdot \frac{1}{x} + 2x \ln x\right]$$

$$= x^{x^2}\left[x + 2x \ln x\right]$$

$$= x^{x^2+1}\left[1 + 2\ln x\right].$$

DRL 3E3A, University of Pennsylvania
E-mail address: shonkwil@math.upenn.edu