MATH 104 QUIZ I SOLUTIONS

CLAY SHONKWILER

(1) Consider the region bounded by $y = \sqrt{x}$, $y = 0$ and $x = 4$. Set up integrals that will compute the volume of the solid of revolution generated by revolving this region about the y-axis using both the disk/washer method and the shell method (that is, set up two integrals, one computing the volume using the disk/washer method and one computing the volume using the shell method). Choose one of these integrals and evaluate it.

Answer: Using the washer method (with horizontal washers), the outer radius of each washer is simply 4, while the inner radius is given by $x = y^2$. As x ranges from 0 to 4, $y = \sqrt{x}$ ranges from 0 to 2, so the relevant integral is:

$$V = \int_0^2 \pi(4^2 - (y^2)^2)dy = \int_0^2 \pi(16 - y^4)dy.$$

On the other hand, using the shell method, the height of the shell is simply $y = \sqrt{x}$ and the radius is x. Hence, the relevant integral is

$$V = \int_0^4 2\pi x\sqrt{x}dx.$$

Each integral looks relatively easy to evaluate, but let's evaluate the shell integral. Then

$$\begin{aligned} V &= \int_0^4 2\pi x\sqrt{x}dx \\ &= \int_0^4 2\pi x^{3/2}dx \\ &= 2\pi\frac{2}{5}x^{5/2}\Big]_0^4 \\ &= \frac{4\pi}{5}\left[4^{5/2} - 0\right] \\ &= \frac{4\pi}{5}(32) \\ &= \frac{128\pi}{5}. \end{aligned}$$

(2) Find the length of the curve $y = x^{3/2}$ from $x = 0$ to $x = 4$.

Answer: First, we need to determine $\left(\frac{dy}{dx}\right)^2$. To that end, note that

$$\frac{dy}{dx} = \frac{3}{2}x^{1/2} = \frac{3}{2}\sqrt{x}.$$

Hence,

$$\left(\frac{dy}{dx}\right)^2 = \left(\frac{3}{2}\sqrt{x}\right)^2 = \frac{9}{4}x.$$

Therefore,

$$\begin{aligned} L &= \int_0^4 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \\ &= \int_0^4 \sqrt{1+\frac{9}{4}x}dx. \end{aligned}$$

Now, if we let $u = 1 + \frac{9}{4}x$, then $du = \frac{9}{4}dx$. Hence,

$$\begin{aligned} L &= \frac{4}{9}\int_0^4 \left(\sqrt{1+\frac{9}{4}x}\right)\frac{9}{4}dx \\ &= \frac{4}{9}\int_1^{10} \sqrt{u}du \\ &= \frac{4}{9}\frac{2}{3}u^{3/2}\Big]_1^{10} \\ &= \frac{8}{27}\left(10\sqrt{10}-1\right) \end{aligned}$$

DRL 3E3A, University of Pennsylvania
E-mail address: shonkwil@math.upenn.edu