MATH 104 QUIZ I SOLUTIONS

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(1) Evaluate the integral

$$\int_0^{\pi/4} \frac{\cos\theta}{\sin^2\theta} d\theta$$

Answer: Let $u = \sin\theta$. Then $du = \cos\theta d\theta$. Now, $\sin 0 = 0$ and $\sin \pi/4 = \frac{\sqrt{2}}{2}$, so

$$\begin{aligned}\int_0^{\pi/4} \frac{\cos\theta}{\sin^2\theta} d\theta &= \int_0^{\sqrt{2}/2} \frac{du}{u^2} \\ &= -\frac{1}{u}\Big]_0^{\sqrt{2}/2} \\ &= -\frac{2}{\sqrt{2}} \\ &= -\sqrt{2}.\end{aligned}$$

(2) Find the area of the region enclosed by the curves $y = x^2 - 2x$ and $y = x$.

Answer: First, note that the curves intersect when

$$x^2 - 2x = x,$$

which is when $x^2 - 3x = 0$. Factoring, we see that $x(x-3) = 0$, so the points of intersection are at $x = 0$ and $x = 3$. The top curve is $y = x$, so the area of the region is given by

$$\begin{aligned}A &= \int_0^3 \left[x - (x^2 - 3x)\right] dx \\ &= \int_0^3 \left[4x - x^2\right] dx \\ &= 2x^2 - \frac{x^3}{3}\Big]_0^3 \\ &= (18 - 9) - 0 \\ &= 9.\end{aligned}$$

(3) Find the volume of the solid generated by rotating the curve $y = \sqrt{\sin x}$ about the x-axis.

Answer: We'll use the slicing method, which is exactly equivalent to the disk method in this situation. Since this is a surface of revolution, the cross-sections are disks, so we need only determine the radius of each cross-sectional disk. However, the radius is simply $y = \sqrt{\sin x}$, so we see that the area of each cross-sectional disk is given by $A(x) = \pi y^2 = \pi(\sin x)$. Now, the curve intersects the x-axis at $x = 0$ and $x = \pi$. Hence, the volume is given by

$$\begin{aligned} V &= \int_0^\pi \pi \sin x dx \\ &= -\pi \cos x]_0^\pi \\ &= -\pi(-1) - (-\pi(1)) \\ &= 2\pi. \end{aligned}$$

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