MATH 104 HW 1

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§2.4

4. Find dy/dx where $y = x^2 \cot x - \frac{1}{x^2}$.

Answer: Remember that the derivative of $\cot x$ is $-\csc^2 x$. We use the product rule:

$$\begin{aligned}\frac{dy}{dx} &= 2x\cot x + x^2(-\csc^2 x) - (-2)x^{-3} \\ &= 2x\cot x - x^2\csc^2 x + \frac{2}{x^3}\end{aligned}$$

5. Find dy/dx where $y = (\sec x + \tan x)(\sec x - \tan x)$.

Answer: Remember that the derivative of $\sec x$ is $\sec x \tan x$ and the derivative of $\tan x$ is $\sec^2 x$. Then, using the product rule,

$$\begin{aligned}\frac{dy}{dx} &= (\sec x\tan x + \sec^2 x)(\sec x - \tan x) + (\sec x + \tan x)(\sec x \tan x - \sec^2 x) \\ &= (\sec^2 x\tan x + \sec^3 x - \sec x\tan^2 x - \sec^2 x\tan x) \\ &\quad +(\sec^2 x\tan x + \sec x\tan^2 x - \sec^3 x - \sec^2 x\tan x) \\ &= 0\end{aligned}$$

22. Find dp/dq where $p = (1 + \csc q)\cos q$.

Answer: Recall that the derivative of $\csc q$ is $-\csc q\cot q$. Then, using the product rule,

$$\begin{aligned}\frac{dp}{dq} &= (-\csc q\cot q)\cos q + (1+\csc q)(-\sin q) \\ &= -\csc q\cot q\cos q - \sin q - \csc q\sin q \\ &= -\frac{1}{\sin q}\frac{\cos q}{\sin q}\cos q - \sin q - \frac{1}{\sin q}\sin q \\ &= -\frac{\cos^2 q}{\sin^2 q} - \sin q - 1 \\ &= -\cot^2 q - \sin q - 1.\end{aligned}$$

§2.6

20. Use implicit differentiation to find dy/dx where $x^3 + y^3 = 18xy$.

Answer: If we differentiate the left side in terms of x, then we get

$$3x^2 + 3y^2\frac{dy}{dx}.$$

On the other hand, if we differentiate the right side with respect to x, then, using the product rule, we see get

$$18(1)y + 18x(1)\frac{dy}{dx} = 18y + 18x\frac{dy}{dx}.$$

Hence, differentiating both sides simultaneously yields

$$3x^2 + 3y^2\frac{dy}{dx} = 18y + 18x\frac{dy}{dx}.$$

Solving for $\frac{dy}{dx}$, we see that

$$(3y^2 - 18x)\frac{dy}{dx} = 18y - 3x^2,$$

so

$$\frac{dy}{dx} = \frac{18y - 3x^2}{3y^2 - 18x}.$$

27. Find dy/dx where $x = \tan y$.

Answer: Differentiating both sides by x yields

$$1 = \sec^2 y\frac{dy}{dx}.$$

Hence,

$$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{\frac{1}{\cos^2 y}} = \cos^2 y.$$

§4.3

11. Evaluate the integral

$$\int \csc^2 2\theta \cot 2\theta d\theta$$

(a) using $u = \cot 2\theta$ and (b) using $u = \csc 2\theta$.

Answer: (a) If we let $u = \cot 2\theta$, then $du = -\csc^2 2\theta \cdot 2d\theta$ (since the derivative of 2θ is 2). Therefore,

$$\begin{aligned}
\int \csc^2 2\theta \cot 2\theta d\theta &= -\frac{1}{2}\int \cot 2\theta(-2\csc^2 2\theta)d\theta \\
&= -\frac{1}{2}\int u du \\
&= -\frac{1}{2}\left(\frac{u^2}{2} + C\right) \\
&= -\frac{u^2}{4} + C_1 \\
&= -\frac{\cot^2 2\theta}{4} + C_1.
\end{aligned}$$

(b) If $u = \csc 2\theta$, then $du = (-\csc 2\theta \cot 2\theta)(2)d\theta$. Therefore,

$$\begin{aligned}\int \csc^2 2\theta \cot 2\theta d\theta &= -\frac{1}{2}\int \csc 2\theta(-2\csc 2\theta \cot 2\theta)d\theta \\ &= -\frac{1}{2}\int u du \\ &= -\frac{u^2}{4} + C \\ &= -\frac{\csc^2 2\theta}{4} + C.\end{aligned}$$

20. Evaluate the integral

$$\int \frac{4y dy}{\sqrt{2y^2+1}}$$

Answer: Let $u = 2y^2 + 1$. Then $du = 4ydy$. Hence,

$$\begin{aligned}\int \frac{4ydy}{\sqrt{2y^2+1}} &= \int \frac{du}{\sqrt{u}} \\ &= \int u^{-1/2} du \\ &= 2u^{1/2} + C \\ &= 2\sqrt{2y^2+1} + C.\end{aligned}$$

§4.7

Don't worry about the problems I assigned from this section. I misunderstood what the questions were asking when I assigned them, and I'm not that enamored with what they are asking.

§4.8

3. Evaluate the integrals:

$$(a) \int_0^{\pi/4} \tan x \sec^2 x dx \qquad (b) \int_{-\pi/4}^{0} \tan x \sec^2 x dx$$

Answer: In both cases, we will let $u = \tan x$. Then $du = \sec^2 x$. In (a), $\tan(0) = 0$ and $\tan(\pi/4) = 1$, so the limits of integration change as follows:

$$\begin{aligned}\int_0^{\pi/4} \tan x \sec^2 x dx &= \int_0^1 u du \\ &= \left.\frac{u^2}{2}\right]_0^1 \\ &= \frac{1}{2} - 0 \\ &= \frac{1}{2}.\end{aligned}$$

In part (b), $\tan(-\pi/4) = -1$, so the integral becomes

$$\begin{aligned}\int_0^{\pi/4} \tan x \sec^2 x dx &= \int_{-1}^0 u du \\ &= \left.\frac{u^2}{2}\right]_{-1}^0 \\ &= 0 - \frac{1}{2} \\ &= -\frac{1}{2}.\end{aligned}$$

10. Evaluate the integrals

$$(a) \int_0^1 \frac{x^3}{\sqrt{x^4+9}} dx \qquad (b) \int_{-1}^0 \frac{x^3}{\sqrt{x^4+9}} dx.$$

Answer: In both cases, we let $u = x^4+9$, so $du = 4x^3 dx$. Since $(0)^4+9 = 9$ and $1^4 + 9 = 10$, the integral in (a) becomes

$$\begin{aligned}\int_0^1 \frac{x^3}{\sqrt{x^4+9}} dx &= \frac{1}{4}\int_9^{10} \frac{du}{\sqrt{u}} \\ &= \frac{1}{4}\left[2\sqrt{u}\right]_9^{10} \\ &= \frac{1}{4}\left[2\sqrt{10} - 6\right] \\ &= \frac{\sqrt{10}-3}{2}.\end{aligned}$$

In part (b), the integral is exactly the same, except evaluated from $(-1)^4 + 9 = 10$ to $0^4 + 9 = 9$, so we get the same answer with a negative sign out front.

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