Math 104 Summer Session II Sample Final Exam

The exam consists of 15 questions. Each is worth 5 points. You should show all your work and try to keep it as neat as possible, as partial credit will be awarded for progress towards the correct solution. Correct answers presented with no justification will receive only minimal credit. Calculators are not allowed (or necessary) on the exam.

1. Evaluate the integral
$$\int e^{-x/2} \cos x\, dx.$$

2. Consider the series
$$\sum_{n=1}^{\infty} \frac{12n}{6n^2 + \ln n}.$$
   Does this series converge or diverge?

3. Consider the sequence $\{a_n\}$ with
$$a_n = \frac{(-1)^{\frac{n(n+1)}{2}} 5^n}{\pi^{2n}}.$$
   Does this sequence converge or diverge? If it converges, what does it converge to?

4. Consider the curve $y = \sqrt{x}$. Find the surface area of the surface obtained by rotating the portion of the curve between $x = 3/4$ and $x = 15/4$ about the x-axis.

5. Suppose you invest \$1000 in an account earning 8% interest compounded continuously. After how many years will you quadruple your money (use the approximation $\ln 4 \approx 1.6$).

6. Consider the series
$$\sum_{n=1}^{\infty} \frac{4^n}{e^{2n+1}}.$$
   Does this series converge or diverge? If it converges, to what sum does it converge?

7. Does the series
$$\sum_{n=1}^{\infty} \frac{(2n)!}{2^n n!}$$
   converge or diverge?

8. Find the interval of convergence of the power series
$$\sum_{n=0}^{\infty} \frac{(2x)^n}{n^2}.$$
   For what values of x does the series converge absolutely? For what values does it converge uniformly?

9. Consider the region contained by $y = \frac{1}{1+x^2}$, the x- and y-axes and $x = 1$. What is the volume of the solid obtained by rotating this region about the y-axis?

10. Evaluate the integral

$$\int \frac{2dx}{(2x-2)\sqrt{4x^2-8x+1}}.$$

11. Evaluate the integral

$$\int \frac{2x+3}{x^2-7x+12}dx.$$

12. Solve the initial value problem

$$x\frac{dy}{dx}+y=\sin x+\cos x, \quad x>0, \qquad y(\pi)=\frac{2}{\pi}.$$

13. Evaluate the integral

$$\int \frac{x}{(x^2+3)^{3/2}}dx.$$

14. Consider the sequence $\{a_n\}$ where

$$a_n = \frac{\sqrt[3]{n}}{\ln(n+1)}.$$

Does the sequence converge or diverge? If it converges, what does it converge to?

15. Does the series

$$\sum_{n=1}^{\infty} \frac{1}{n(\ln n)^2}$$

converge or diverge?