Math 104 Summer Session II Final Examination

The exam consists of 15 questions. Each is worth 5 points. You should show all your work and try to keep it as neat as possible, as partial credit will be awarded for progress towards the correct solution. Correct answers presented with no justification will receive only minimal credit. Calculators are not allowed (or necessary) on the exam.

1. Solve the initial value problem

$$x\frac{dy}{dx} + \frac{y}{\ln x} = 2, \quad x > 1, \qquad y(e) = 3.$$

2. Consider the sequence $\{a_n\}$ where

$$a_n = \frac{e^n \cos n}{3^n}.$$

Determine whether the sequence converges or diverges. If it converges, find the limit of the sequence.

3. Find the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{x^n}{3^n}.$$

4. Evaluate the integral

$$\int 2x \csc^2 x \, dx.$$

5. Find the length of the curve $y = \frac{2}{3}x^{3/2}$ between $x = 0$ and $x = 3$.

6. Evaluate the integral

$$\int \frac{dx}{x^2 - 12x + 41}.$$

7. Evaluate the integral
$$\int \frac{3x - 4}{x^2 - 6x + 9} dx.$$

8. Does the series
$$\sum_{n=1}^{\infty} \frac{2n + 3}{8n^3 - 6n + 1}$$
converge or diverge?

9. Does the series

$$\sum_{n=1}^{\infty} \frac{n!}{3^{4n}}$$

converge or diverge?

10. Determine whether the series

$$\sum_{n=1}^{\infty} (-3)^n \frac{1}{2^n \pi^{n-1}}$$

converges or diverges. If it converges, find the sum of the series.

11. Consider a bacteria culture that starts with a single, isolated bacterium. Supposing that the rate of change in the population of the culture is proportional to its size and that there are 10 bacteria in the culture after 1 hour, how many bacteria should we expect in the culture after 3 hours? (Hint: the number you get should be quite simple)

12. Consider the region enclosed by $y = \ln x$, the x-axis and $x = e$. Find the volume of the solid obtained by rotating this region about the x-axis (Hint: there are two different methods for computing the volume of a solid of revolution, so if the first you try doesn't work, try the other).

13. Evaluate the integral

$$\int \frac{\sqrt{1-x^2}}{x^4}dx.$$

14. Does the series

$$\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{1+n^2}$$

converge or diverge?

15. Consider the sequence $\{a_n\}$ where

$$a_n = \frac{\ln \frac{1}{n}}{2\sqrt{n}}.$$

Does the sequence converge or diverge? If it converges, what is the limit of the sequence?