The Geometry of Random Polygons

Clayton Shonkwiler

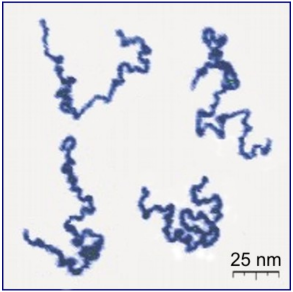
University of Georgia & Isaac Newton Institute

Geometry Seminar
University of Manchester
December 13, 2012

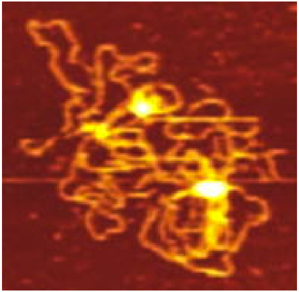
Random Polygons (and Polymer Physics)

Physics Question

What is the average shape of a polymer in solution?



Protonated P2VP
Roiter/Minko
Clarkson University



Plasmid DNA
Alonso-Sarduy, Dietler Lab
EPF Lausanne

Physics Question

What is the average shape of a polymer in solution?

Physics Answer

> *Modern polymer physics is based on the analogy between a polymer chain and a random walk.*
> *—Alexander Grosberg, NYU.*

Random Polygons (and Geometry and Topology)

Math Question

With respect to a given probability measure on closed polygons, what is the expected value of radius of gyration or total curvature?

Math Question

What fraction of 7-gons of length 2 are knotted?

Topology of cyclo-octane energy landscape
Martin, Thompson, Coutsias, Watson

Numerical Analysis Question

How can we construct random samples drawn from the space of closed space n-gons? More generally, how should we numerically integrate over the space of closed polygons?

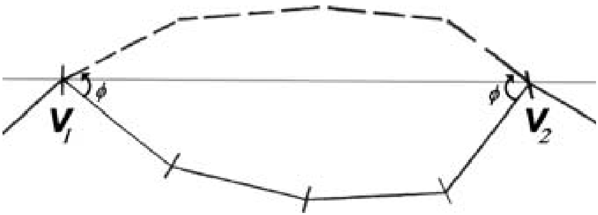


Illustration of crankshaft algorithm of Vologoskii et. al.
Benham/Mielke

Plan of Attack

Construct a mathematical structure underlying polygon spaces which makes these probability and numerical questions tractable.

- Construct a natural measure on framed polygons with edgelengths $r_1, \ldots, r_n$.

Plan of Attack

Construct a mathematical structure underlying polygon spaces which makes these probability and numerical questions tractable.

- Construct a natural measure on framed polygons with edgelengths $r_1, \ldots, r_n$.
- Assemble all these spaces with $r_1 + \cdots + r_n = 2$ into a simple space.

Plan of Attack

Construct a mathematical structure underlying polygon spaces which makes these probability and numerical questions tractable.

- Construct a natural measure on framed polygons with edgelengths $r_1, \dots, r_n$.
- Assemble all these spaces with $r_1 + \cdots + r_n = 2$ into a simple space.
- Develop sampling algorithm and make explicit computations on simple space.

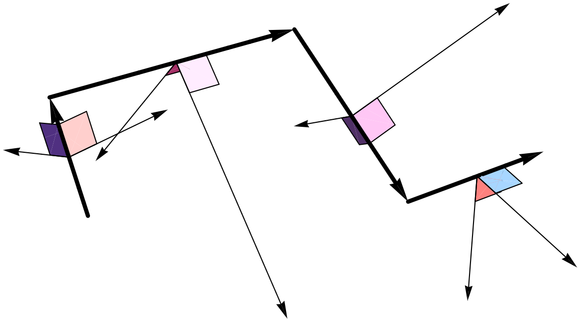
Plan of Attack

Construct a mathematical structure underlying polygon spaces which makes these probability and numerical questions tractable.

- Construct a natural measure on framed polygons with edgelengths $r_1, \ldots, r_n$.
- Assemble all these spaces with $r_1 + \cdots + r_n = 2$ into a simple space.
- Develop sampling algorithm and make explicit computations on simple space.
- (Future) Specialize back to fixed edgelength spaces.

Definition

Let $\mathrm{FArm}_3(n; r_1, \ldots, r_n)$ be the space of framed n-gons with edgelengths $r_1, \ldots, r_n$ (up to translation) in $\mathbb{R}^3$.



Quaternions: natural coordinates for frames

Definition

The quaternions $\mathbb{H}$ are the skew-algebra over $\mathbb{R}$ defined by adding $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ so that

$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = -1, \quad \mathbf{ijk} = -1$$

We can identify quaternions with frames in $\mathrm{SO}(3)$ via the Hopf map

$$\mathrm{Hopf}(q) = (\bar{q}\mathbf{i}q, \bar{q}\mathbf{j}q, \bar{q}\mathbf{k}q),$$

where the entries turn out to be purely imaginary quaternions, and hence vectors in $\mathbb{R}^3$.

Proposition

The unit quaternions (S^3) double-cover $\mathrm{SO}(3)$ *via the Hopf map.*

Question

Does the space of length r orthogonal frames for $\mathbb{R}^3$ have less volume than the space of length 1 orthogonal frames? If so, how much?

Question

Does the space of length r orthogonal frames for $\mathbb{R}^3$ have less volume than the space of length 1 orthogonal frames? If so, how much?

Lemma

The Hopf map takes quaternions of norm $\sqrt{r}$ to frames where each vector has norm r.

Question

Does the space of length r orthogonal frames for $\mathbb{R}^3$ have less volume than the space of length 1 orthogonal frames? If so, how much?

Lemma

The Hopf map takes quaternions of norm $\sqrt{r}$ to frames where each vector has norm r.

Definition

We take the measure on the space of frames of length r to be the pushforward under the Hopf map of the standard measure on the 3-sphere $S^3(\sqrt{r})$ of radius $\sqrt{r}$ inside $\mathbb{H}$.

Assembly of the measure on the space of framed arms

Definition

We take the measure on the space $\mathrm{FArm}_3(n; r_1, \ldots, r_n)$ to be the pushforward by the Hopf map

$$S^3(\sqrt{r_1}) \times \cdots \times S^3(\sqrt{r_n}) \stackrel{\text{Hopf}}{\to} r_1 \,\mathrm{SO}(3) \times \cdots \times r_n \,\mathrm{SO}(3).$$

Assembly of the measure on the space of framed arms

Definition

We take the measure on the space $\mathrm{FArm}_3(n; r_1, \ldots, r_n)$ to be the pushforward by the Hopf map

$$S^3(\sqrt{r_1}) \times \cdots \times S^3(\sqrt{r_n}) \stackrel{\text{Hopf}}{\to} r_1\, \mathrm{SO}(3) \times \cdots \times r_n\, \mathrm{SO}(3).$$

Definition

Let $\mathrm{FArm}_3(n)$ be the space of framed space polygons with total length 2.

$$\mathrm{FArm}_3(n) = \bigcup_{\sum r_i = 2} \mathrm{FArm}_3(n; r_1, \ldots, r_n).$$

Assembly of the measure on the space of framed arms

Definition

We take the measure on the space $\mathrm{FArm}_3(n; r_1, \ldots, r_n)$ to be the pushforward by the Hopf map

$$S^3(\sqrt{r_1}) \times \cdots \times S^3(\sqrt{r_n}) \overset{\text{Hopf}}{\to} r_1\,\mathrm{SO}(3) \times \cdots \times r_n\,\mathrm{SO}(3).$$

Definition

Let $\mathrm{FArm}_3(n)$ be the space of framed space polygons with total length 2.

$$\mathrm{FArm}_3(n) = \bigcup_{\sum r_i = 2} \mathrm{FArm}_3(n; r_1, \ldots, r_n).$$

Proposition

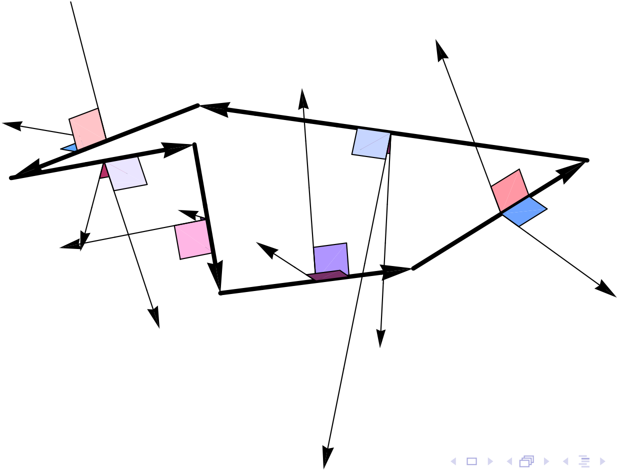
The space $\mathrm{FArm}_3(n)$ *is covered* 2^n *times by* $S^{4n-1} \subset \mathbb{H}^n$.

We can forget the framing to generate a natural measure on the *unframed* polygon spaces $\mathrm{Arm}_3(n; r_1, \ldots, r_n)$ and $\mathrm{Arm}_3(n)$. We call this the *symmetric* measure on $\mathrm{FArm}_3(n)$ and $\mathrm{Arm}_3(n)$ since it comes from the round sphere.

$$
\begin{tikzcd}
S^3(\sqrt{r_1}) \times \cdots S^3(\sqrt{r_n}) \subset \mathbb{H}^n \arrow[r, hook] \arrow[d, two heads, "\mathrm{Hopf}"'] & S^{4n-1}(\sqrt{2}) \subset \mathbb{H}^n \arrow[d, two heads, "\mathrm{Hopf}"'] \\
\mathrm{FArm}_3(n; r_1, \ldots, r_n) \arrow[r, hook] \arrow[d, two heads, "\pi"'] & \mathrm{FArm}_3(n) \arrow[d, two heads, "\pi"'] \\
\mathrm{Arm}_3(n; r_1, \ldots, r_n) \arrow[r, hook] & \mathrm{Arm}_3(n)
\end{tikzcd}
$$

Definition

Let $\mathrm{FPol}_3(n; r_1, \ldots, r_n) \subset \mathrm{FArm}_3(n; r_1, \ldots, r_n)$ be the space of **closed** framed n-gons with edgelengths $r_1, \ldots, r_n$ (up to translation) in $\mathbb{R}^3$.



From Arm Space to Closed Polygon Space

The quaternionic n-sphere S^{4n-1} is the join $S^{2n-1} \star S^{2n-1}$ of complex n-spheres:

$$(\vec{u}, \vec{v}, \theta) \mapsto \sqrt{2}(\cos\theta\vec{u} + \sin\theta\vec{v}\mathbf{j})$$

where $\vec{u}, \vec{v} \in \mathbb{C}^n$ lie in the unit sphere and $\theta \in [0, \pi/2]$. We focus on

$$S^{4n-1} \supset \{(\vec{u}, \vec{v}, \pi/4) \mid \langle \vec{u}, \vec{v} \rangle = 0\} = V_2(\mathbb{C}^n)$$

Knutson and Hausmann (1997) proved:

$$\begin{tikzcd}
V_2(\mathbb{C}^n) \subset \mathbb{H}^n \arrow[r, hook] \arrow[d, "\text{Hopf}"'] & S^{4n-1} \subset \mathbb{H}^n \arrow[d, "\text{Hopf}"'] \\
\mathrm{FPol}_3(n) \arrow[r, hook] & \mathrm{FArm}_3(n)
\end{tikzcd}$$

The proof is (a computation) worth doing!

Proof.

In complex form, the map $\mathrm{Hopf}_{\mathbf{i}}(q)$ can be written as

$$\mathrm{Hopf}(u + v\mathbf{j}) = \mathbf{i}(|u|^2 - |v|^2 + 2\bar{u}v\mathbf{j})$$

Thus the polygon closes $\iff$

$$\begin{aligned}\left|\sum \mathrm{Hopf}(q_i)\right|^2 &= \left|\sum 2|\cos\theta\, u_i|^2 - \sum 2|\sin\theta\, v_i|^2\right. \\ &\qquad \left. + 4\cos\theta\sin\theta \sum \overline{u_i} v_i \mathbf{j}\right|^2 \\ &= \left|2\cos^2\theta - 2\sin^2\theta\right|^2 + |4\cos\theta\sin\theta\,\langle u, v\rangle|^2 \\ &= 4\cos^2 2\theta + 4\sin^2 2\theta\, |\langle u, v\rangle|^2 = 0\end{aligned}$$

or $\iff$ $\theta = \pi/4$ and $\vec{u}$, $\vec{v}$ are orthogonal. $\square$

Proposition (with Cantarella, Deguchi)

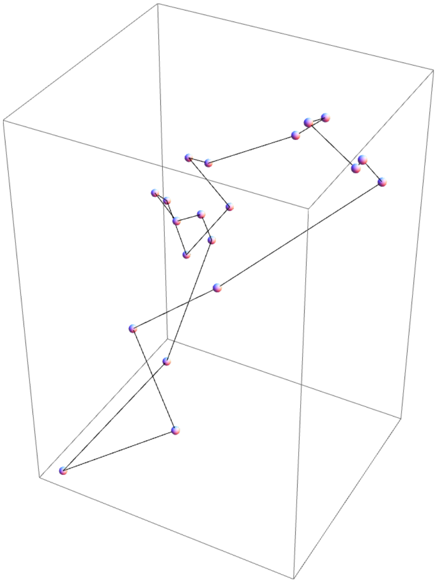
The natural (Haar) measure on $V_2(\mathbb{C}^n)$ is obtained by generating random complex n-vectors with independent Gaussian coordinates and applying (complex) Gram-Schmidt.

```
In[9]:= RandomComplexVector[n_] := Apply[Complex,
          Partition[#, 2] & /@ RandomVariate[NormalDistribution[], {1, 2 n}], {2}][[1]];

        ComplexDot[A_, B_] := Dot[A, Conjugate[B]];
        ComplexNormalize[A_] := (1 / Sqrt[Re[ComplexDot[A, A]]]) A;

        RandomComplexFrame[n_] := Module[{a, b, A, B},
            {a, b} = {RandomComplexVector[n], RandomComplexVector[n]};
            A = ComplexNormalize[a];
            B = ComplexNormalize[b - Conjugate[ComplexDot[A, b]] A];
           {A, B}
          ];
```

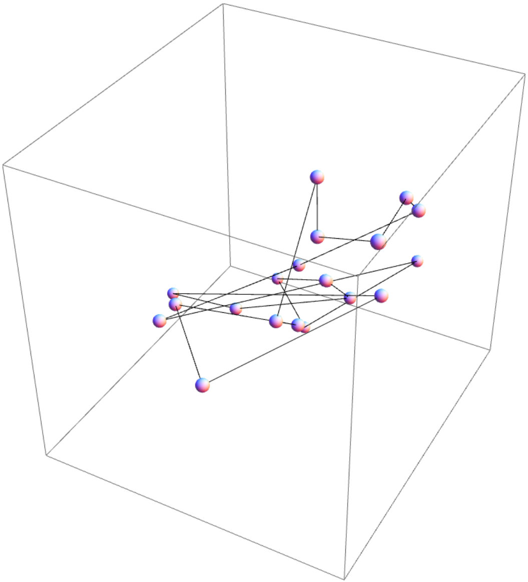
Now we need only apply the Hopf map to generate an edge set:

```
In[6]:= ToEdges[{A_, B_}] := {#[[2]], #[[3]], #[[4]]} & /@ (HopfMap /@ Transpose[{A, B}]);
```

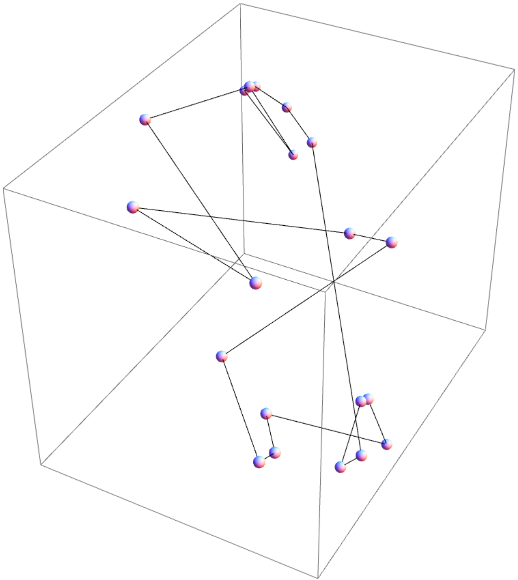
Examples of 20-gons



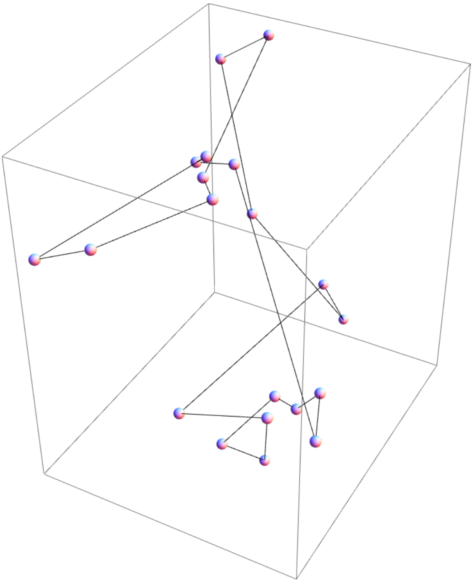
Examples of 20-gons



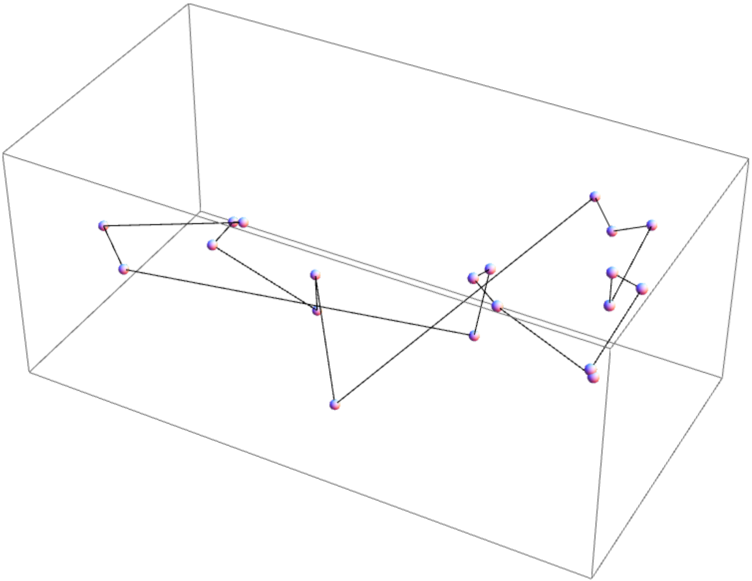
Examples of 20-gons



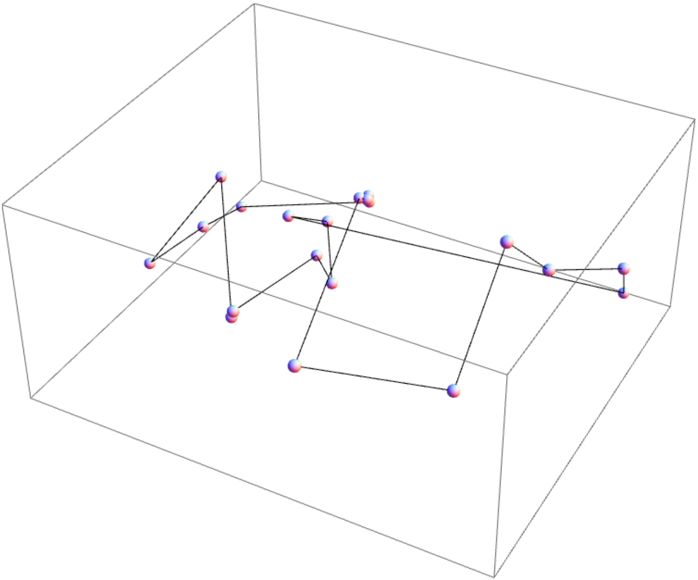
Examples of 20-gons

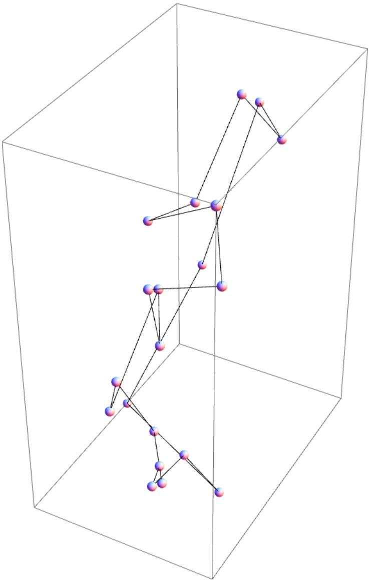


Examples of 20-gons

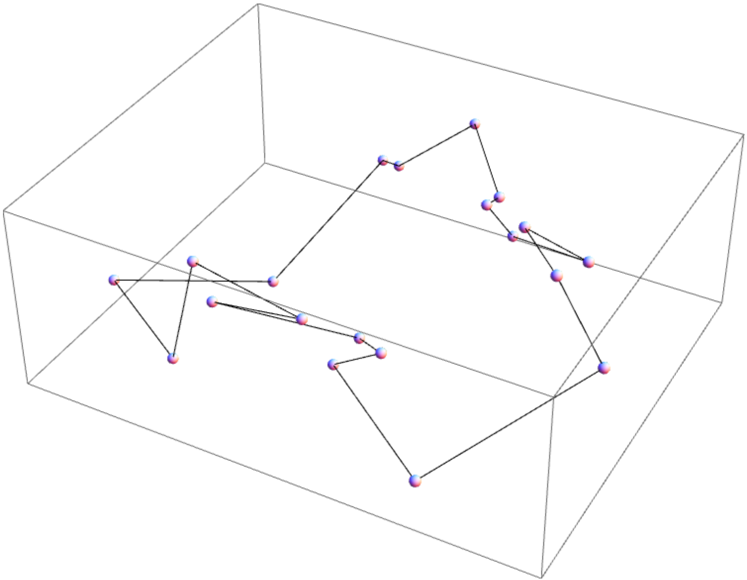


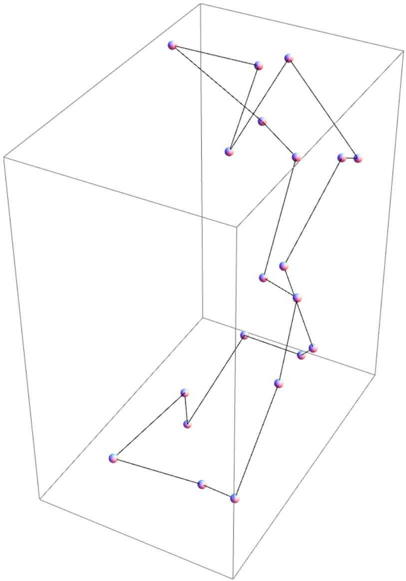
Examples of 20-gons



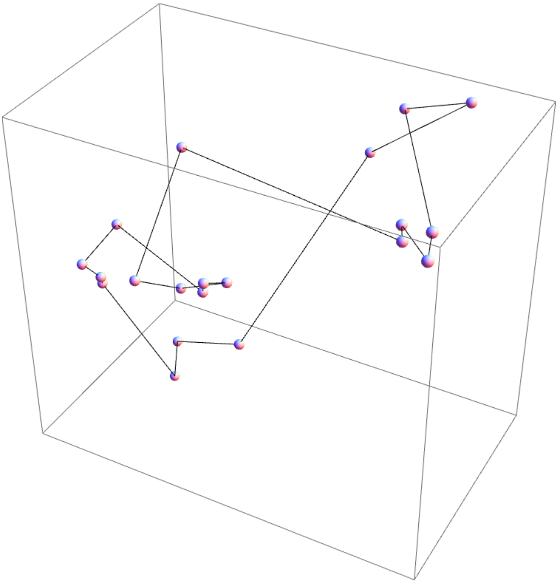


Examples of 20-gons

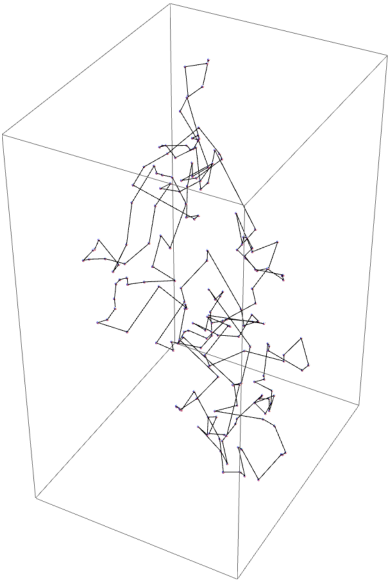




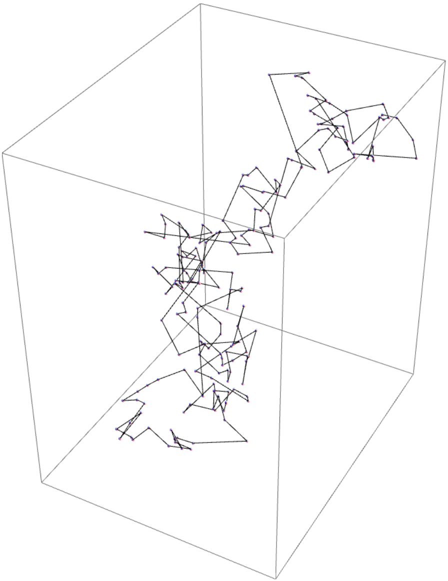
Examples of 20-gons



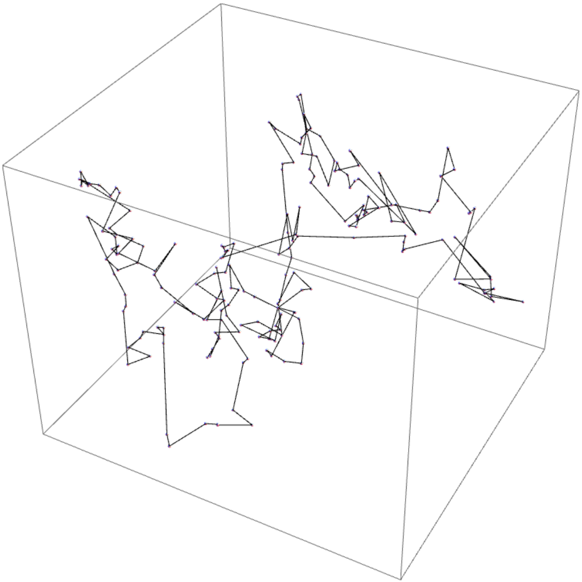
Examples of 200-gons



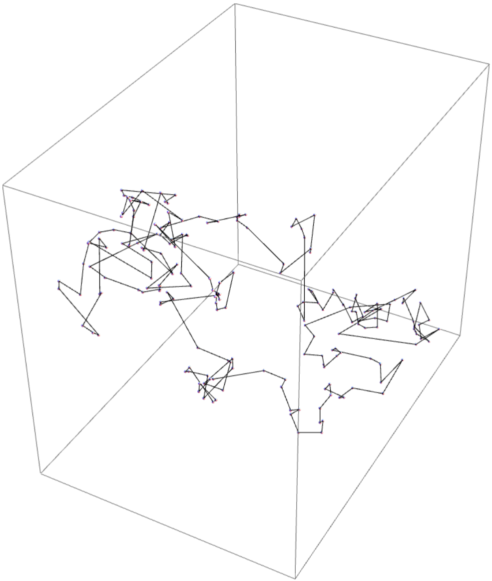
Examples of 200-gons



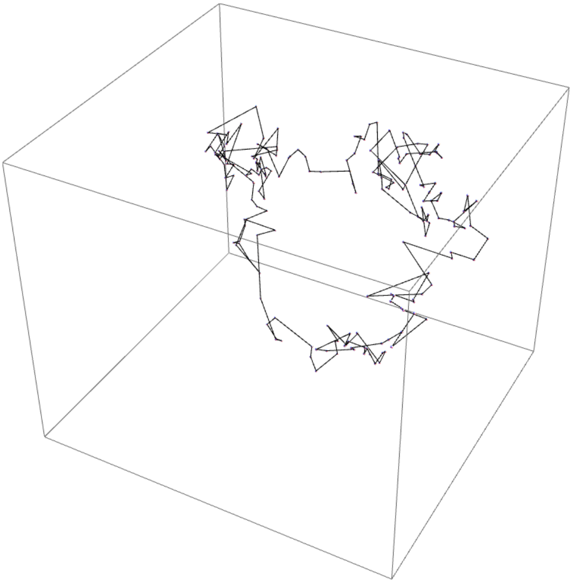
Examples of 200-gons



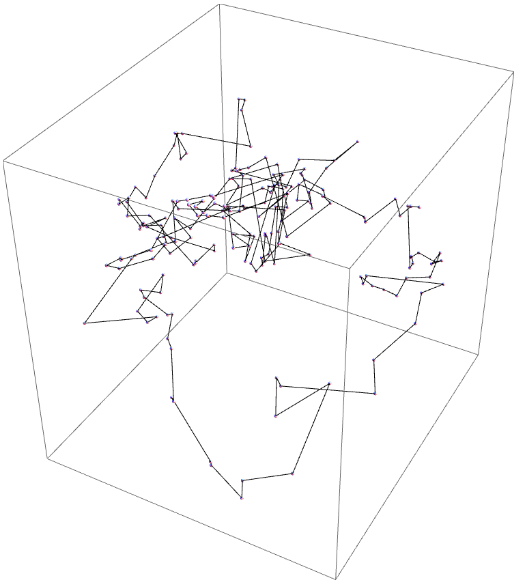
Examples of 200-gons



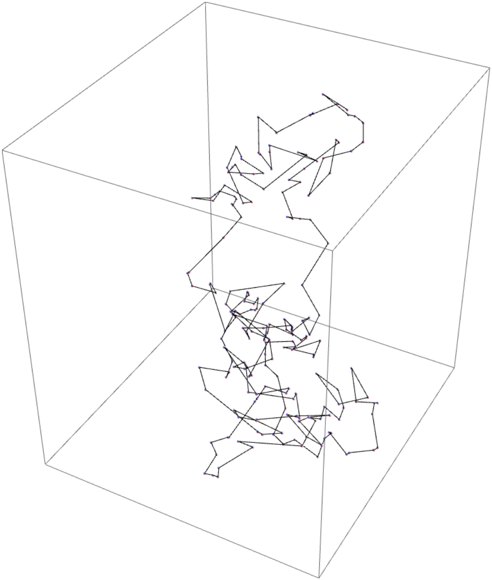
Examples of 200-gons



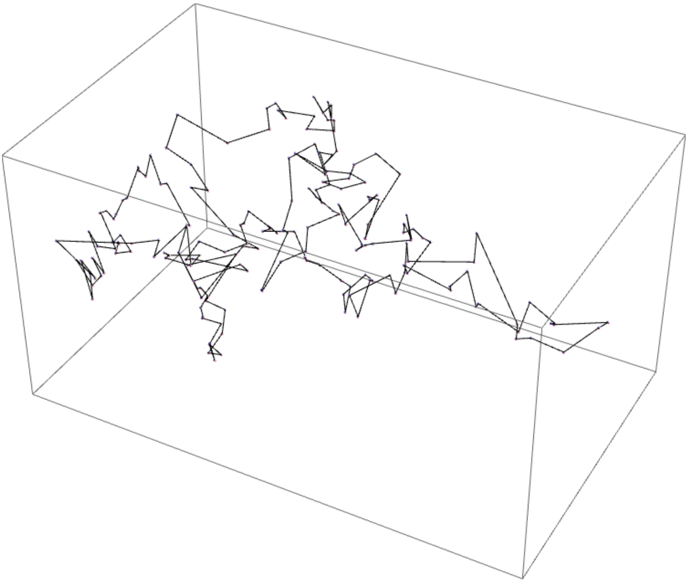
Examples of 200-gons



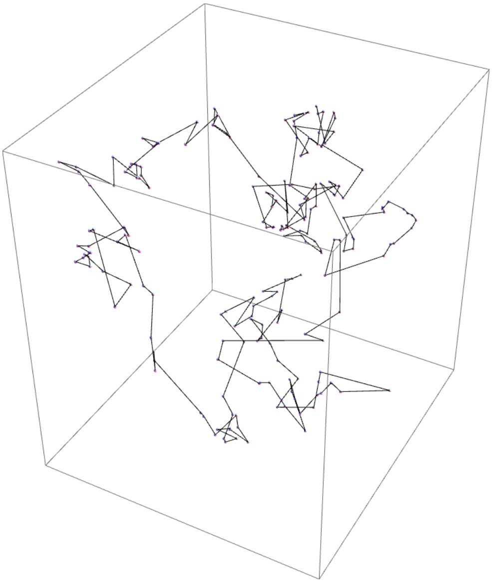
Examples of 200-gons



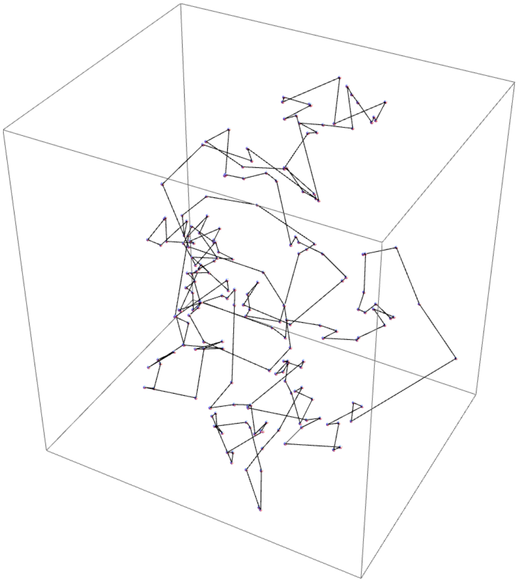
Examples of 200-gons



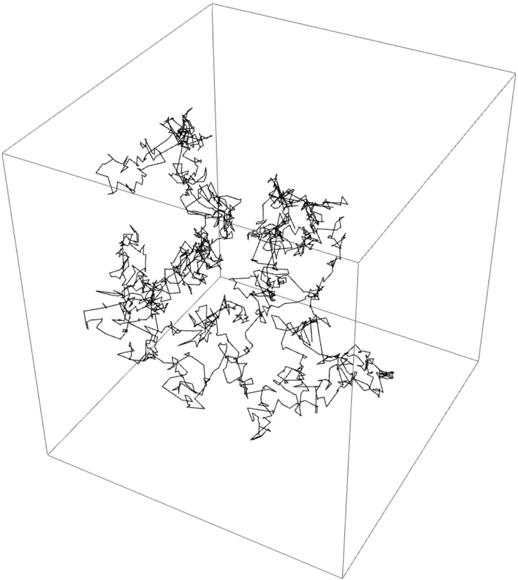
Examples of 200-gons



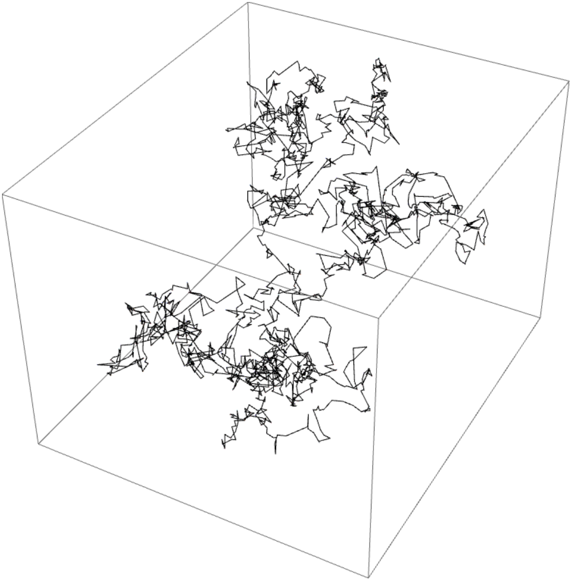
Examples of 200-gons

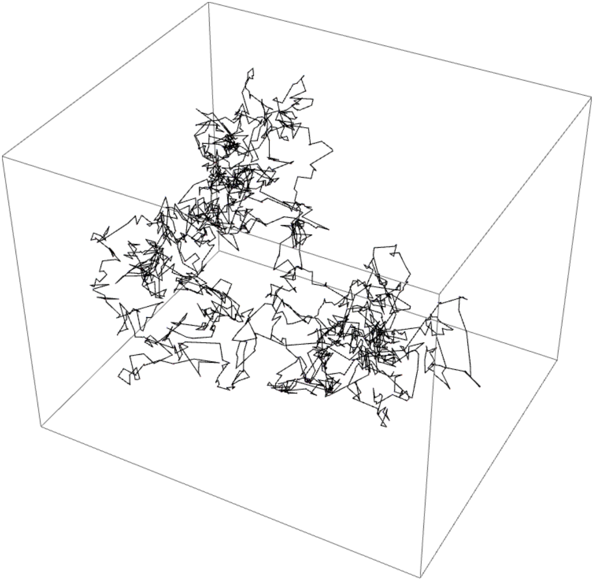


Examples of 2,000-gons

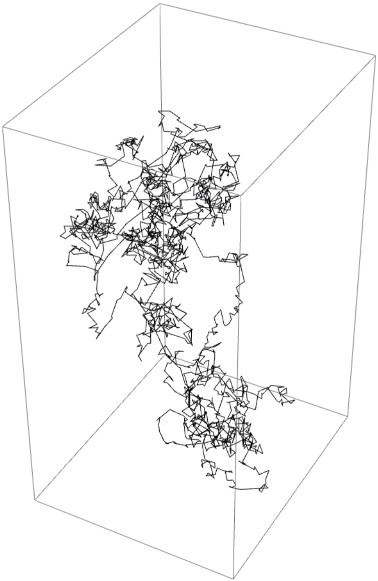


Examples of 2,000-gons

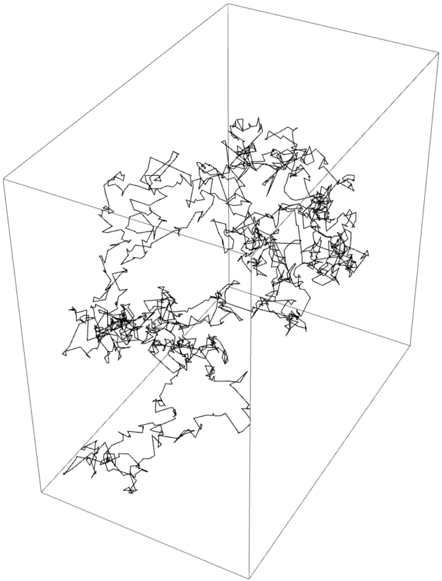




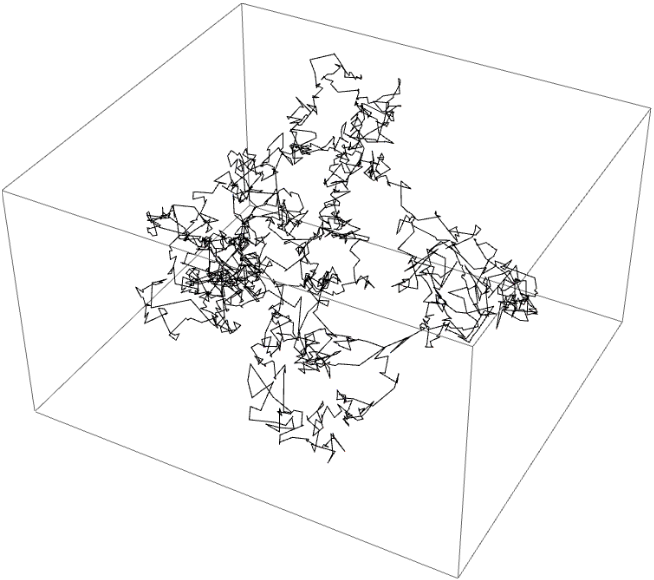
Examples of 2,000-gons



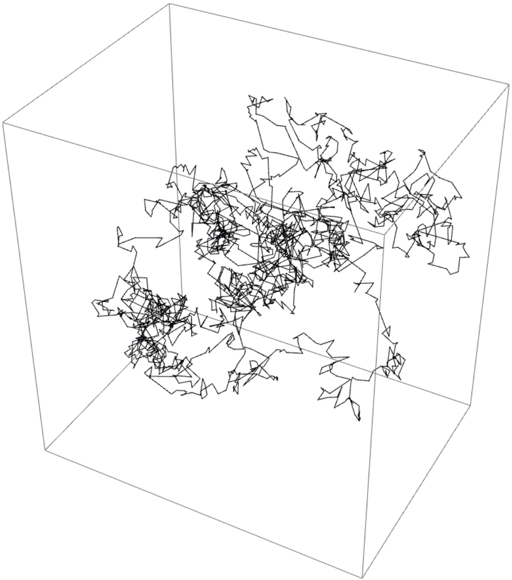
Examples of 2,000-gons



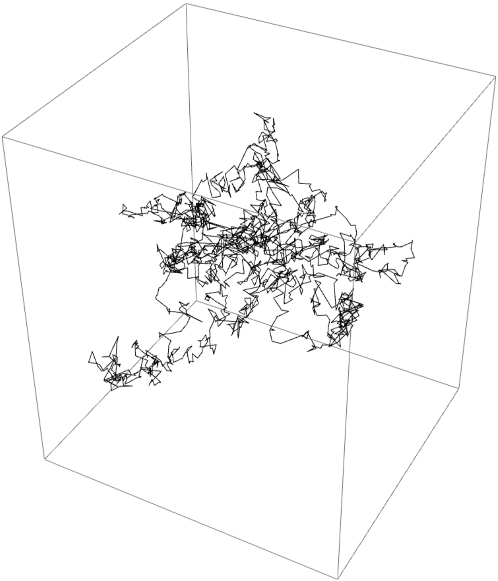
Examples of 2,000-gons



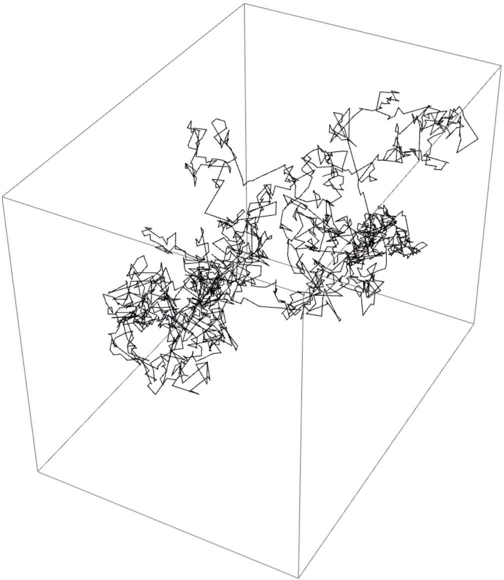
Examples of 2,000-gons



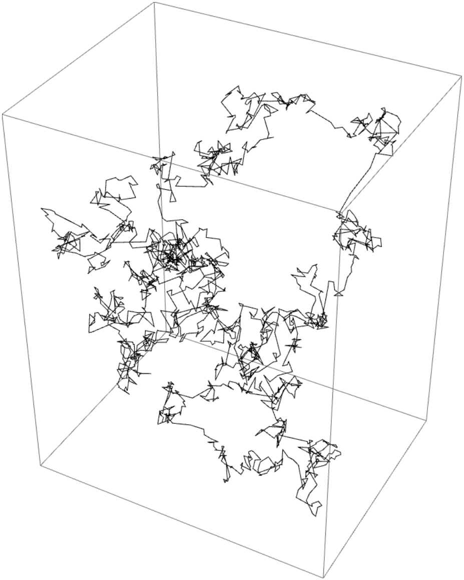
Examples of 2,000-gons



Examples of 2,000-gons

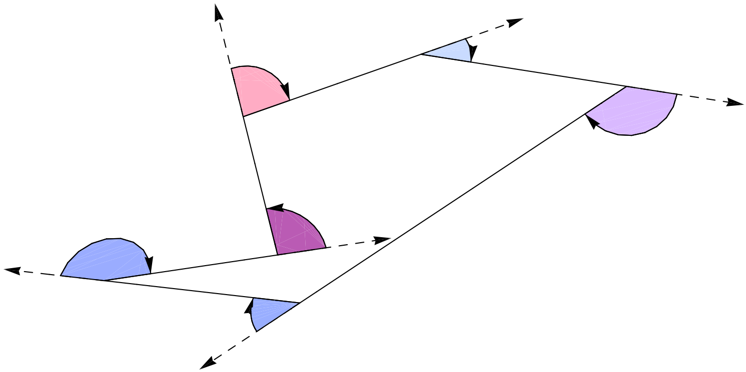


Examples of 2,000-gons



Definition

The total curvature κ of a space polygon is the sum of the turning angles $\theta_1, \ldots, \theta_n$.



Lemma

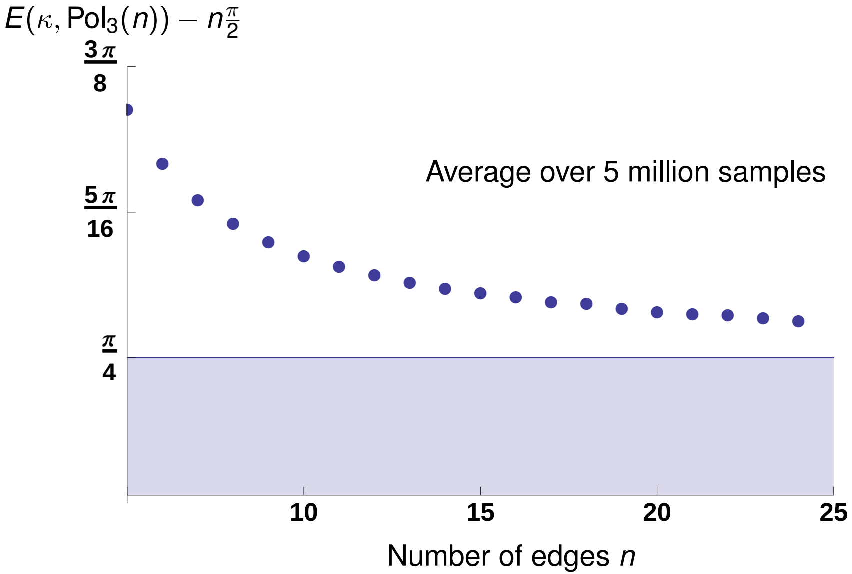
The expected total curvature of a polygon in $\mathrm{Arm}_3(n; r_1, \ldots, r_n)$ *is* $(n-1)\pi/2$.

In 2007, Plunkett et. al. sampled random equilateral closed polygons and noticed that

$$E(\kappa, \mathrm{Pol}_3(n; 1, \ldots, 1)) \to \frac{\pi}{2} n + \alpha$$

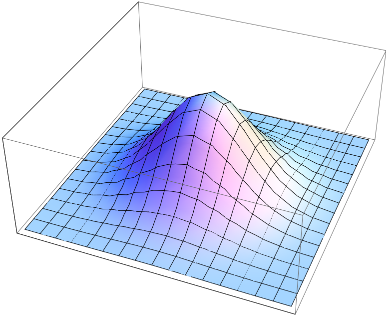
> We have observed that the normalized average total curvature and total torsion of the phantom polygons appears to be constant, approximately 1.2 and −1.2, respectively. A simple estimate derived from the inner product of the sum of the edge vectors[26] suggests an approximation of 1.0 for the excess total curvature. The case of the total torsion and the search for more accurate estimates remains an interesting research question.

Total curvature surplus in our measure on $\mathrm{Pol}_3(n)$



Definition

Let $\mathcal{A}_3(n)$ be the space of open n-gons in $\mathbb{R}^3$, and $\mathcal{P}_3(n) \subset \mathcal{A}_3(n)$ be the space of closed n-gons in $\mathbb{R}^3$.

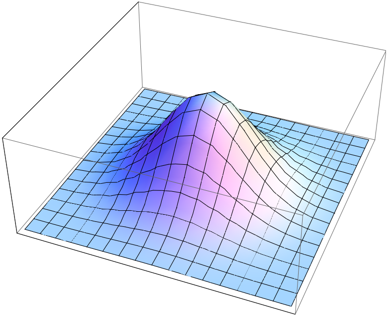


Definition

Let $\mathcal{A}_3(n)$ be the space of open n-gons in $\mathbb{R}^3$, and $\mathcal{P}_3(n) \subset \mathcal{A}_3(n)$ be the space of closed n-gons in $\mathbb{R}^3$.

Definition

The *Hopf-Gaussian measure* on $\mathcal{A}_3(n)$ is the pushforward of a multivariate Gaussian measure on $\mathbb{H}^n$ by the Hopf map. The Hopf-Gaussian measure on $\mathcal{P}_3(n)$ is the subspace measure.



Proposition (with Cantarella, Grosberg, Kusner)

For any scale invariant function F on space polygons,

$$E(F, \mathcal{A}_3(n)) = E(F, \mathrm{Arm}_3(n)), E(F, \mathcal{P}_3(n)) = E(F, \mathrm{Pol}_3(n)).$$

Proposition (with Cantarella, Grosberg, Kusner)

For any scale invariant function F on space polygons,

$$E(F, \mathcal{A}_3(n)) = E(F, \mathrm{Arm}_3(n)), E(F, \mathcal{P}_3(n)) = E(F, \mathrm{Pol}_3(n)).$$

Proof.

The idea is that the Gaussian measure on $\mathbb{H}^n$ is spherically symmetric, so the subspace measure on each S^{4n-1} is the standard one. Since the average over each sphere is the average over the whole space (by scale invariance), this is enough. $\square$

Proposition (with Cantarella, Grosberg, Kusner)

For any scale invariant function F on space polygons,

$$E(F, \mathcal{A}_3(n)) = E(F, \mathrm{Arm}_3(n)), E(F, \mathcal{P}_3(n)) = E(F, \mathrm{Pol}_3(n)).$$

Proof.

The idea is that the Gaussian measure on $\mathbb{H}^n$ is spherically symmetric, so the subspace measure on each S^{4n-1} is the standard one. Since the average over each sphere is the average over the whole space (by scale invariance), this is enough. □

Corollary

$$E(\kappa, \mathcal{P}_3(n)) = E(\kappa, \mathrm{Pol}_3(n)).$$

The pdf of the failure to close of an arm

If we write the Hopf map in coordinates, we see if $q = q_0 + q_1\mathbf{i} + q_2\mathbf{j} + q_3\mathbf{k}$,

$$\mathrm{Hopf}(q) = (q_0^2 + q_1^2 - q_2^2 - q_3^2, 2q_1q_2 - 2q_0q_3, 2q_0q_2 + 2q_1q_3).$$

The pdf of the failure to close of an arm

If we write the Hopf map in coordinates, we see if $q = q_0 + q_1\mathbf{i} + q_2\mathbf{j} + q_3\mathbf{k}$,

$$\mathrm{Hopf}(q) = (q_0^2 + q_1^2 - q_2^2 - q_3^2, 2q_1q_2 - 2q_0q_3, 2q_0q_2 + 2q_1q_3).$$

Proposition (with Cantarella, Grosberg, Kusner)

The first coordinate of the sum of k edges in $\mathcal{A}_n$ is distributed as a difference of $\chi^2(2k)$ variables. The pdf of this coordinate is

$$f(y) = \frac{|y|^{k-1/2}}{4^k\sqrt{\pi}\Gamma(k)} K_{k-1/2}\left({}^{|y|}/_2\right).$$

The pdf of the failure to close of an arm

If we write the Hopf map in coordinates, we see if $q = q_0 + q_1\mathbf{i} + q_2\mathbf{j} + q_3\mathbf{k}$,

$$\mathrm{Hopf}(q) = (q_0^2 + q_1^2 - q_2^2 - q_3^2, 2q_1q_2 - 2q_0q_3, 2q_0q_2 + 2q_1q_3).$$

Proposition (with Cantarella, Grosberg, Kusner)

The pdf $G_k(\vec{r})$ of the vector joining the ends of k edges from a Hopf-Gaussian arm is spherically symmetric in $\mathbb{R}^3$ and given by:

$$G_k(\vec{r}) = \frac{r^{k-3/2}}{2^{2k+2}\pi^{3/2}\Gamma(k)} K_{k-3/2}\left(r/2\right),$$

where $r = |\vec{r}|$.

Proposition (with Cantarella, Grosberg, Kusner)

The pdf of a pair of edges $\vec{e}_1$, $\vec{e}_2$ in $\mathcal{P}_3(n)$ is

$$P(\vec{e}_1, \vec{e}_2) = \frac{G_1(\vec{e}_1)G_1(\vec{e}_2)G_{n-2}(-\vec{e}_1 - \vec{e}_2)}{G_n(\vec{0})}.$$

Proposition (with Cantarella, Grosberg, Kusner)

The pdf of a pair of edges $\vec{e_1}, \vec{e_2}$ in $\mathcal{P}_3(n)$ is

$$P(\vec{e_1}, \vec{e_2}) = \frac{G_1(\vec{e_1}) G_1(\vec{e_2}) G_{n-2}(-\vec{e_1} - \vec{e_2})}{G_n(\vec{0})}.$$

Proposition (with Cantarella, Grosberg, Kusner)

If $x = (|\vec{e}_1| + |\vec{e}_2|)/2$, $y = (|\vec{e}_1| - |\vec{e}_2|)/2$ and $z = |\vec{e_1} + \vec{e_2}|$, this pairwise pdf is

$$P(\vec{e}_1, \vec{e}_2) = \frac{\Gamma(n)}{2\sqrt{\pi}\Gamma(2n-4)} e^{-x} z^{n-\frac{5}{2}} K_{n-\frac{7}{2}}\left(\frac{z}{2}\right)$$

The curvature surplus explained

Proposition (with Cantarella, Grosberg, Kusner)

The expected value of the turning angle θ for a single pair of edges in $\mathcal{P}_3(n)$ is given by the formula

$$E(\theta) = \frac{\pi}{2} + \frac{\pi}{4}\frac{2}{2n-3}$$

so

$$E(\kappa, \mathrm{Pol}_3(n)) = \frac{\pi}{2}n + \frac{\pi}{4}\frac{2n}{2n-3}.$$

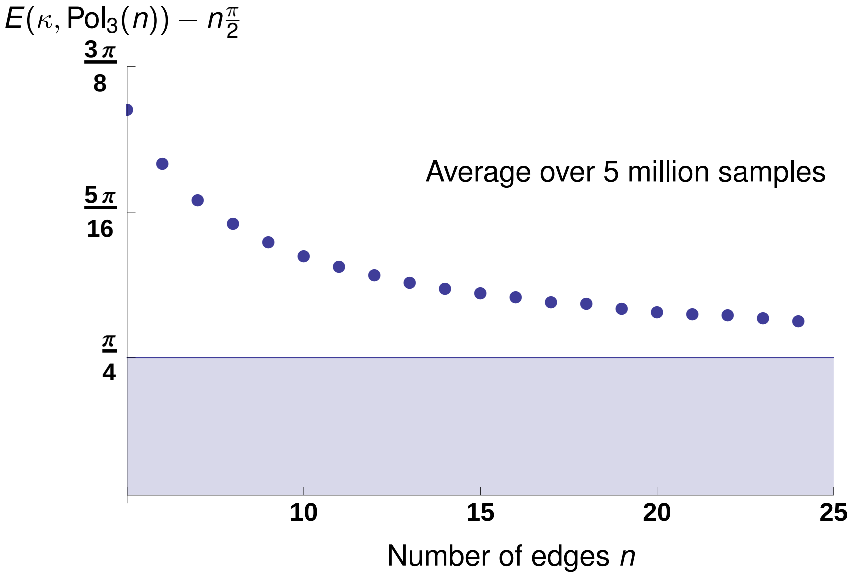
Proof.

Write

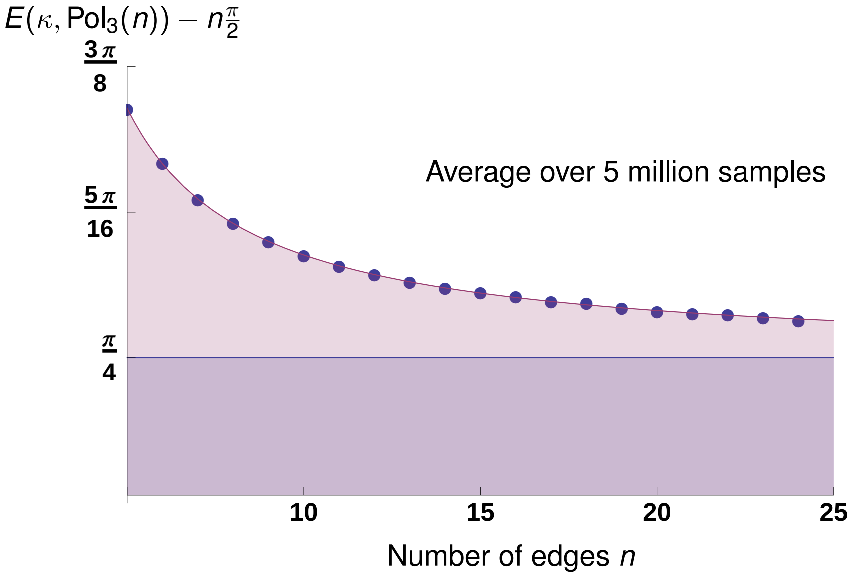
$$\theta(x, y, z) = \arccos \frac{z^2 - 2(x^2 + y^2)}{2x^2 - 2y^2}$$

and integrate this against the pairwise pdf for edges in $\mathcal{P}_3$. □

Checking against the numerical data



Checking against the numerical data



Proposition (with Cantarella, Grosberg, Kusner)

At least $1/3$ *of* $\mathrm{Pol}_3(6)$ *and* $1/11$ *of* $\mathrm{Pol}_3(7)$ *consists of unknots.*

A consequence: topology at last!

Proposition (with Cantarella, Grosberg, Kusner)

At least $1/3$ *of* $\mathrm{Pol}_3(6)$ *and* $1/11$ *of* $\mathrm{Pol}_3(7)$ *consists of unknots.*

Proof.

Let x be the fraction of polygons in $\mathrm{Pol}_3(n)$ with total curvature greater than 4π (by the Fáry-Milnor theorem, these are the only polygons which may be knotted). The expected value of total curvature then satisfies

$$E(\kappa; \mathrm{Pol}_3(n)) > 4\pi x + 2\pi(1 - x).$$

Solving for x and using our total curvature expectation, we see that

$$x < \frac{(n-2)(n-3)}{2(2n-3)}.$$

□

New Results: Expected Total Curvature for Equilateral Random Polygons

A similar analysis can be used to get results about equilateral polygons. Here we don't have a closed form (yet), but we can express the results as a sum.

Theorem (with Cantarella)

The (exact) expectation for the total curvature of an equilateral random n-gon (in the standard measure) is

n	$E(\kappa, \mathrm{ePol}_3(n))$	(decimal)
4	8	8.
6	$6\pi - 8$	10.8496
8	$\frac{15}{4}\pi + \frac{32}{15}$	13.9143
10	$\frac{11}{2}\pi - \frac{64}{245}$	17.0175
12	$\frac{331{,}545}{51{,}776}\pi + \frac{512}{28{,}315}$	20.1351

New Results: Expected Total Curvature for Equilateral Random Polygons

A similar analysis can be used to get results about equilateral polygons. Here we don't have a closed form (yet), but we can express the results as a sum.

Theorem (with Cantarella)

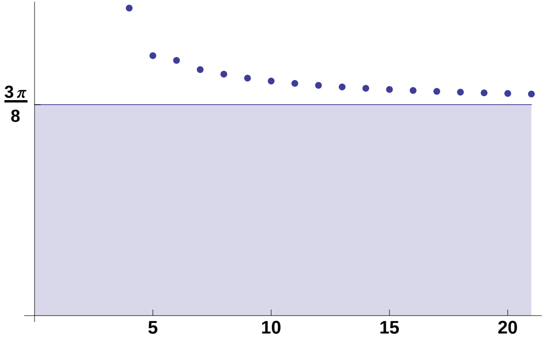
The (exact) expectation for the total curvature of an equilateral random n-gon (in the standard measure) is

n	$E(\kappa, \mathrm{ePol}_3(n))$	*(decimal)*
5	$-2\pi + 9\sqrt{3}$	*9.30527*
7	$\frac{316}{33}\pi - \frac{225}{22}\sqrt{3}$	*12.369*
9	$\frac{766}{289}\pi + \frac{11907}{2890}\sqrt{3}$	*15.463*
11	$\frac{90712}{14219}\pi - \frac{1686177}{1990660}\sqrt{3}$	*18.5751*
13	$\frac{23336570}{3407523}\pi + \frac{2381643}{22716820}\sqrt{3}$	*21.6969*

Theorem (Grosberg)

We have

$$E(\kappa, \mathrm{ePol}_3(n)) - \frac{\pi}{2} n \to \frac{3}{8}\pi \text{ as } n \to \infty.$$

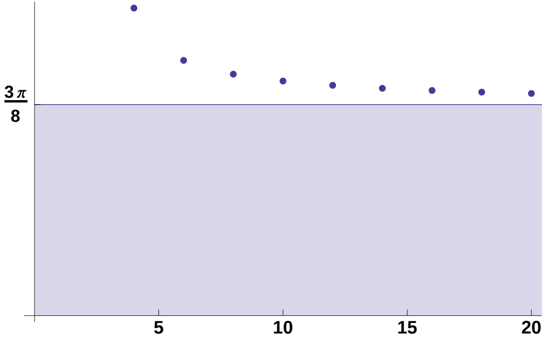


Expectations for all n

Theorem (Grosberg)

We have

$$E(\kappa, \mathrm{ePol}_3(n)) - \frac{\pi}{2} n \to \frac{3}{8}\pi \text{ as } n \to \infty.$$

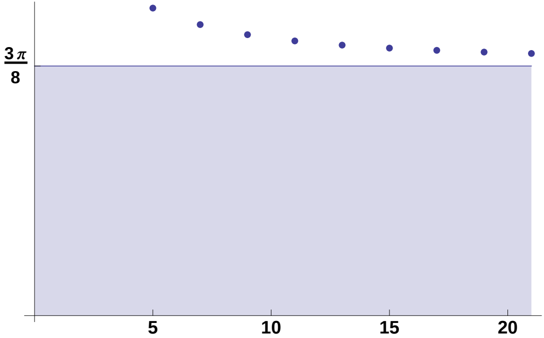


Expectations for even n

Theorem (Grosberg)

We have

$$E(\kappa, \mathrm{ePol}_3(n)) - \frac{\pi}{2} n \to \frac{3}{8}\pi \text{ as } n \to \infty.$$



Expectations for odd n

- There's a version of all this for *curves*. What can we derive about curve theory from this point of view? For plane curves, this is the Michor-Shah-Mumford-Younes metric on closed plane curves.

- What does Brownian motion on the Stiefel manifold look like? Is this a model for evolution of polygons?

- Geodesics in the Stiefel manifold are "optimal" reconfigurations of framed curves or polygons. Given a framed n-gon, there are 2^n lifts to "spin-framed" n-gons. So given n-gons P_a and P_b, which pair of lifts is closest?

- *Probability Theory of Random Polygons from the Quaternionic Viewpoint*
  Jason Cantarella, Tetsuo Deguchi, and Clayton Shonkwiler
  arXiv:1206.3161
  To appear in *Communications on Pure and Applied Mathematics*.

- *The Expected Total Curvature of Random Polygons*
  Jason Cantarella, Alexander Y. Grosberg, Robert Kusner, and Clayton Shonkwiler
  arXiv:1210.6537.

Thank you for inviting me!