Outline

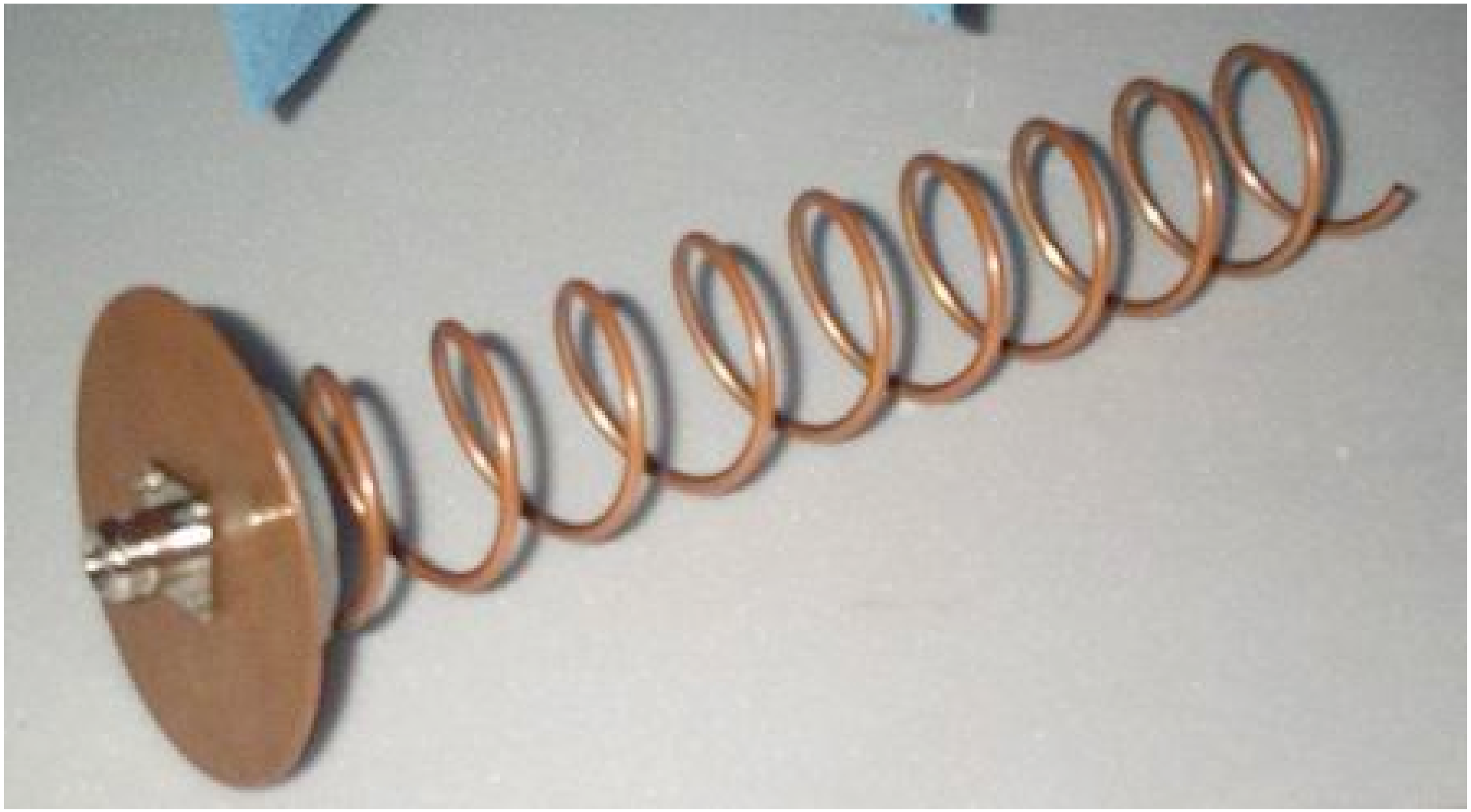
⋮

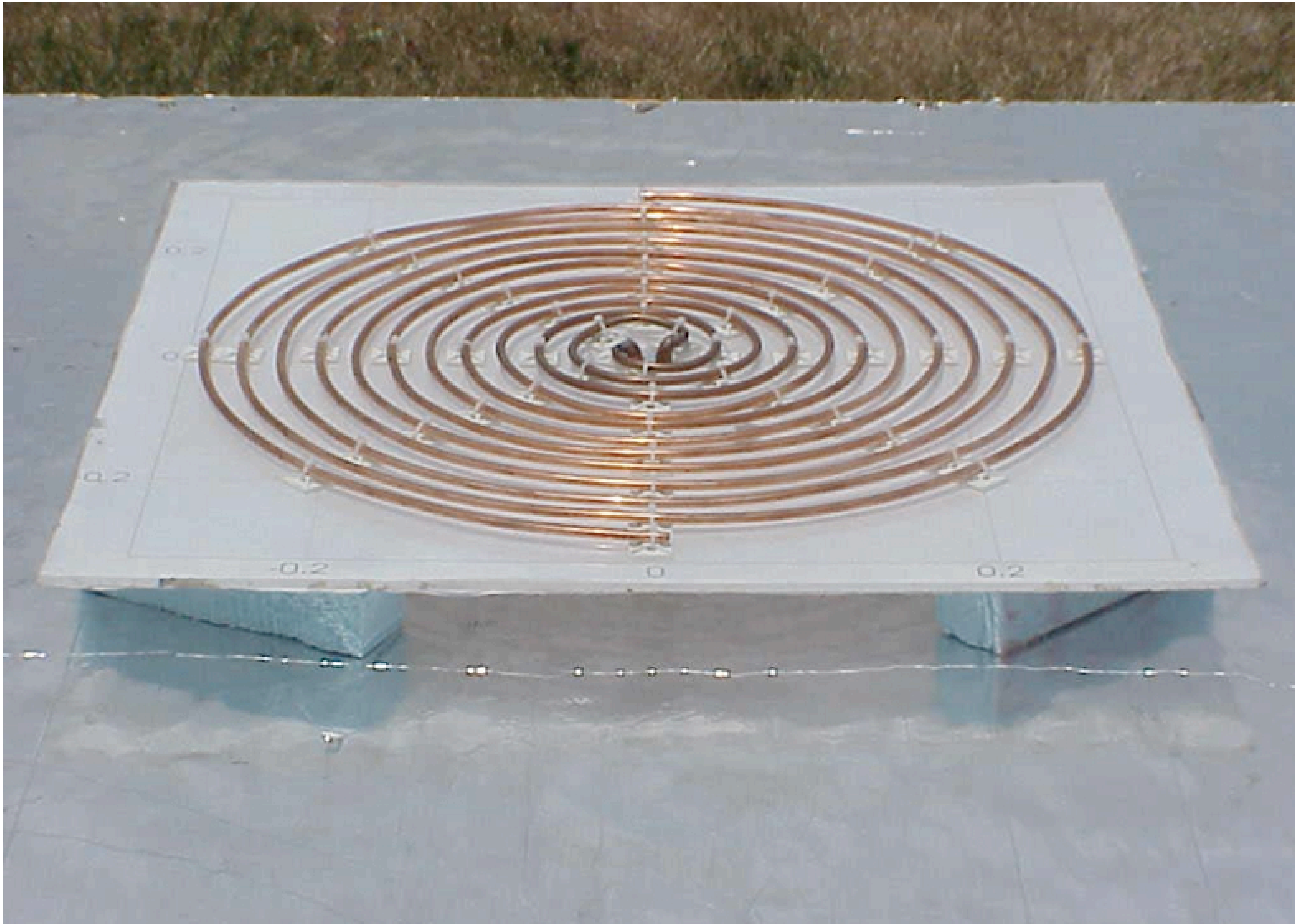
7. introduction to 3D scattering
8. ISAR
9. antenna theory
   (a) antenna examples
   (b) vector and scalar potentials
   (c) radiation in the far field
10. spotlight SAR
11. stripmap SAR

Dipole antenna

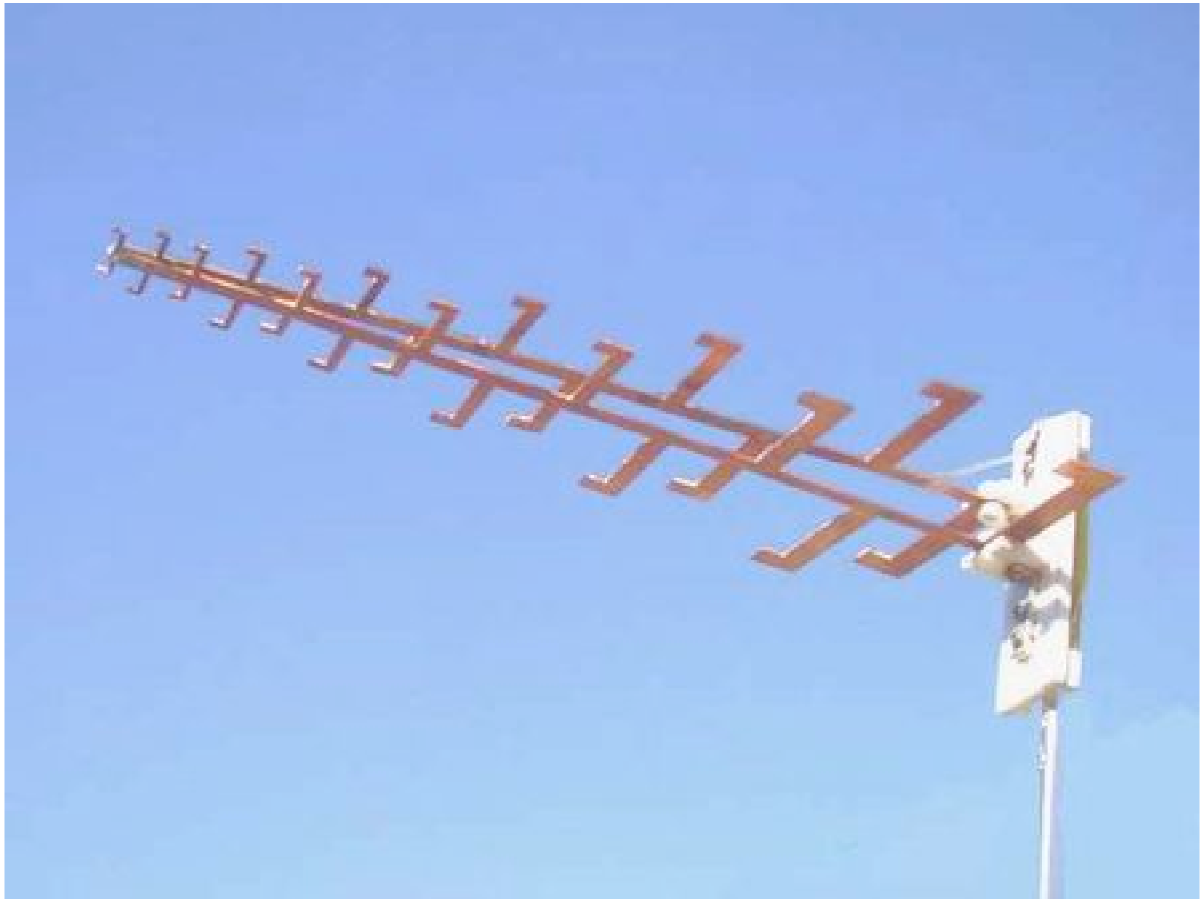
Conical monopole

Helical antenna

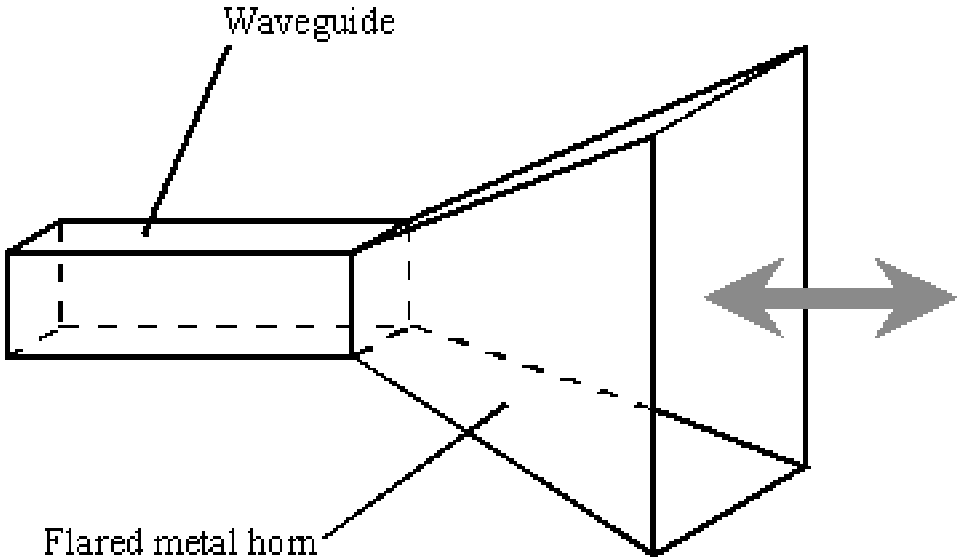


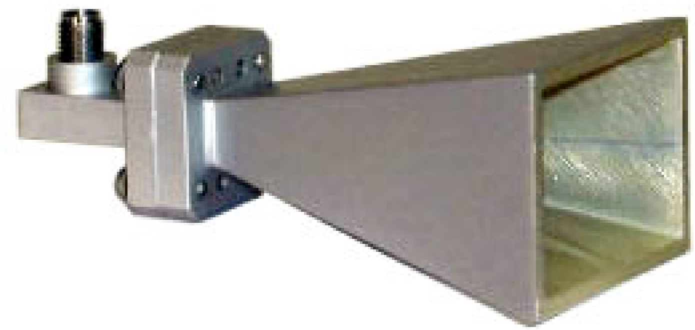

-0.2
0
0.2
0.2

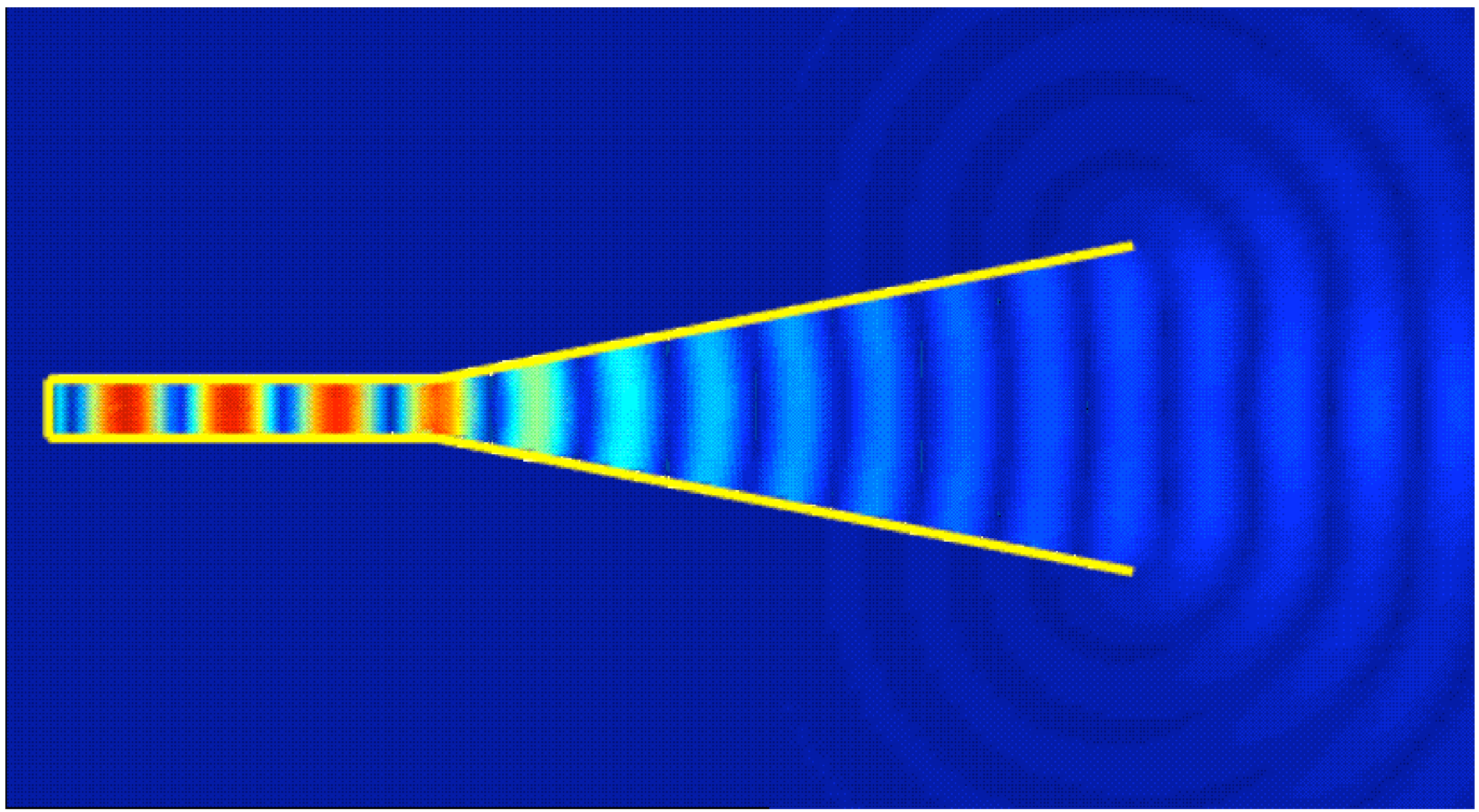
Log-periodic antenna



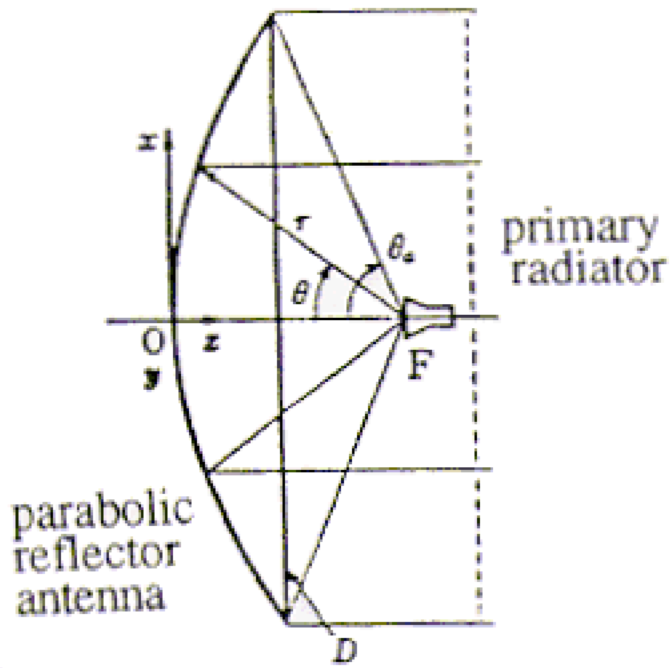
Horn antennas



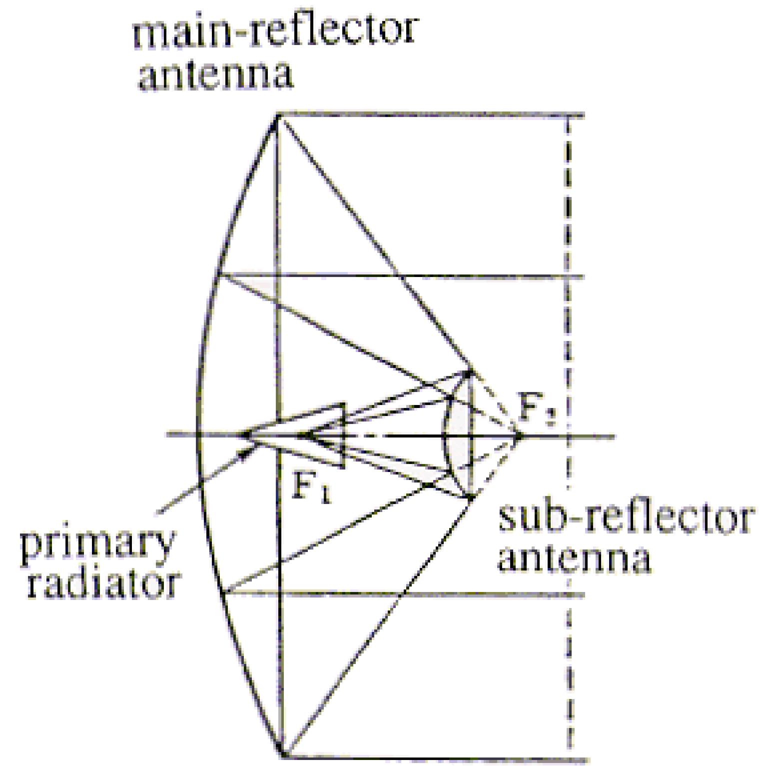




Corrugated horn



(a) Parabolic reflector antenna

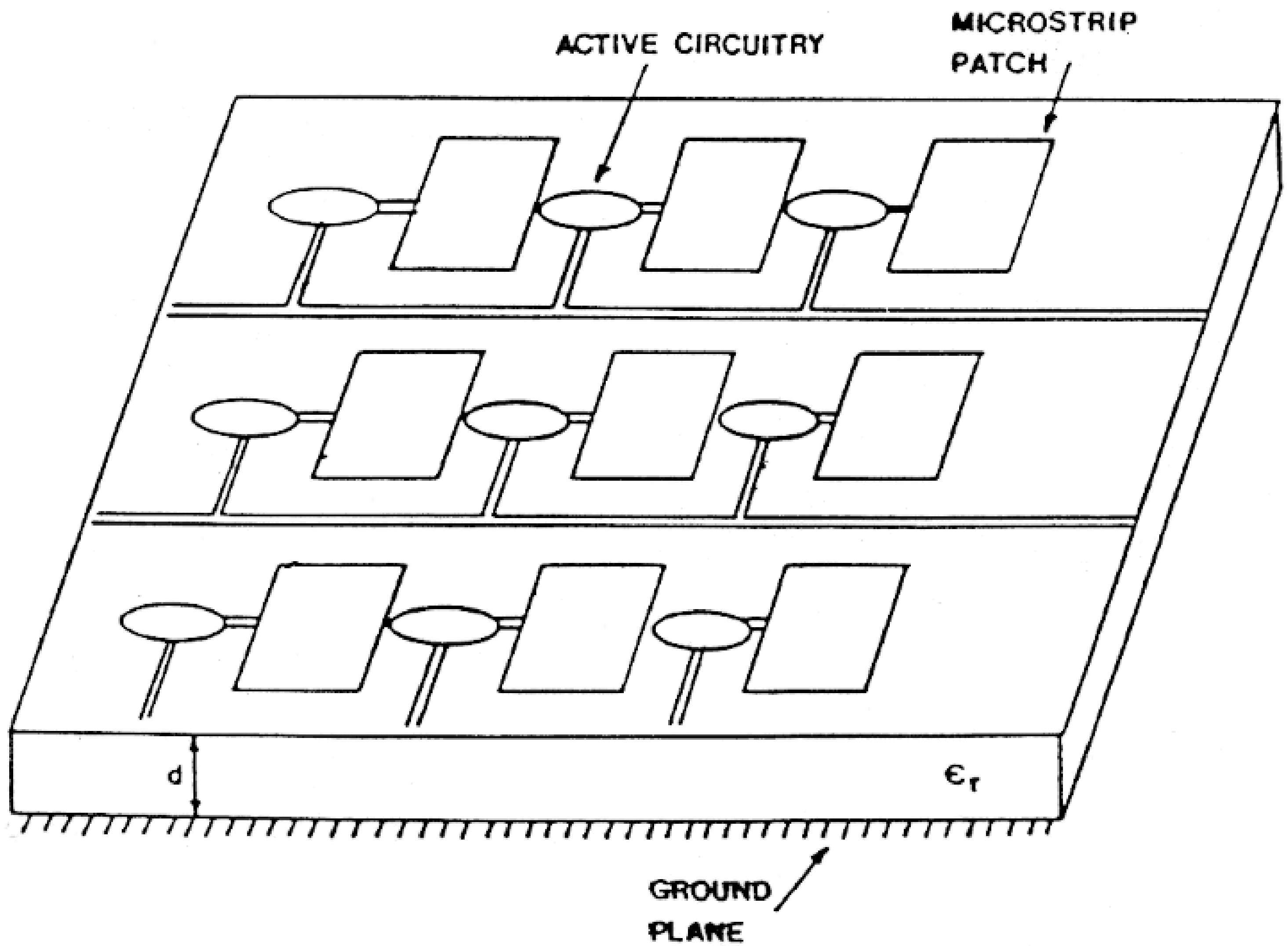


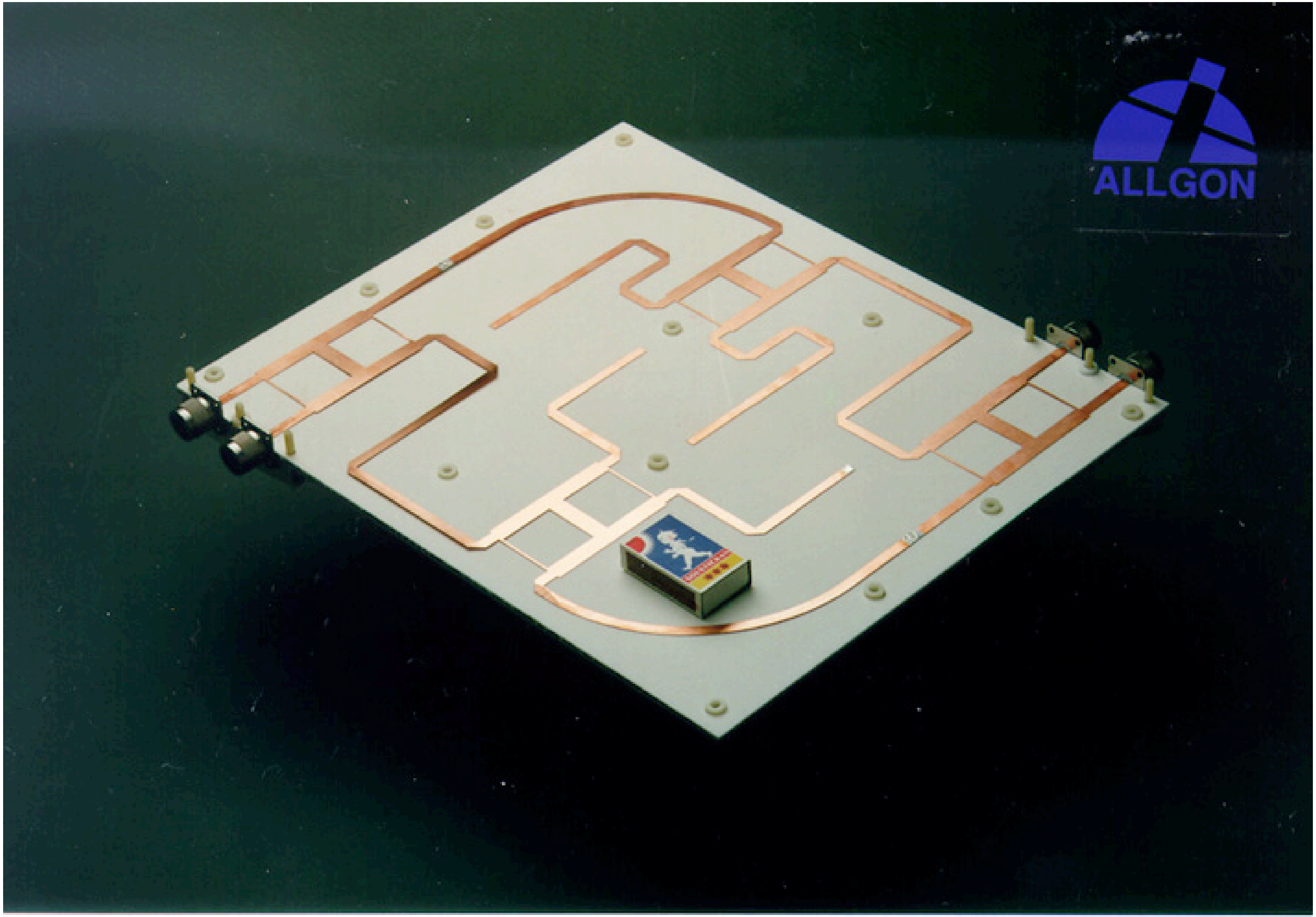
(b) Cassegrain antenna

Figure 3.6.2 Structure of reflector antennas

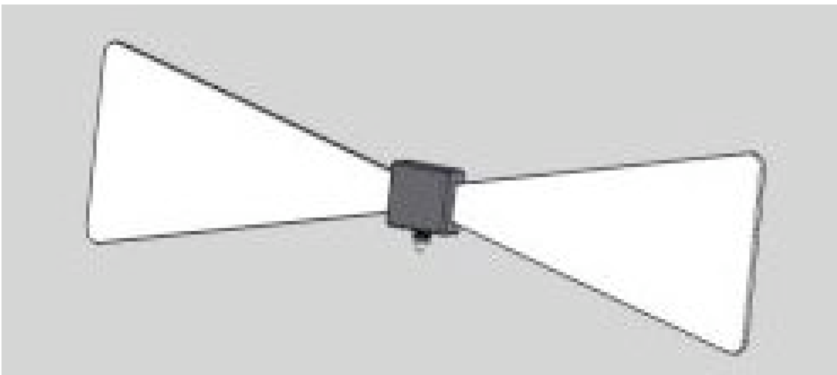

4.6 meter antenna

Microstrip antennas



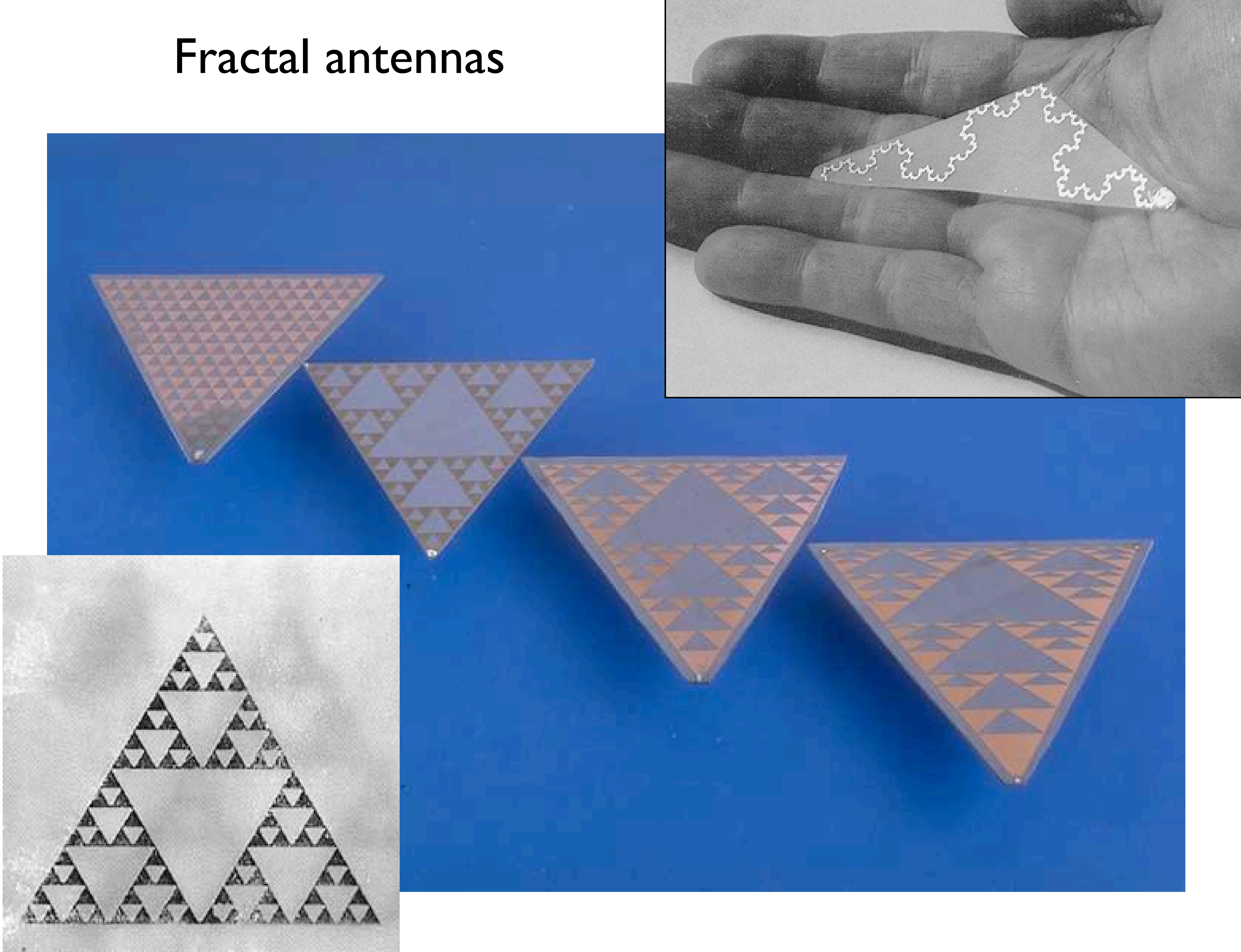

ALLGON

Bowtie antenna

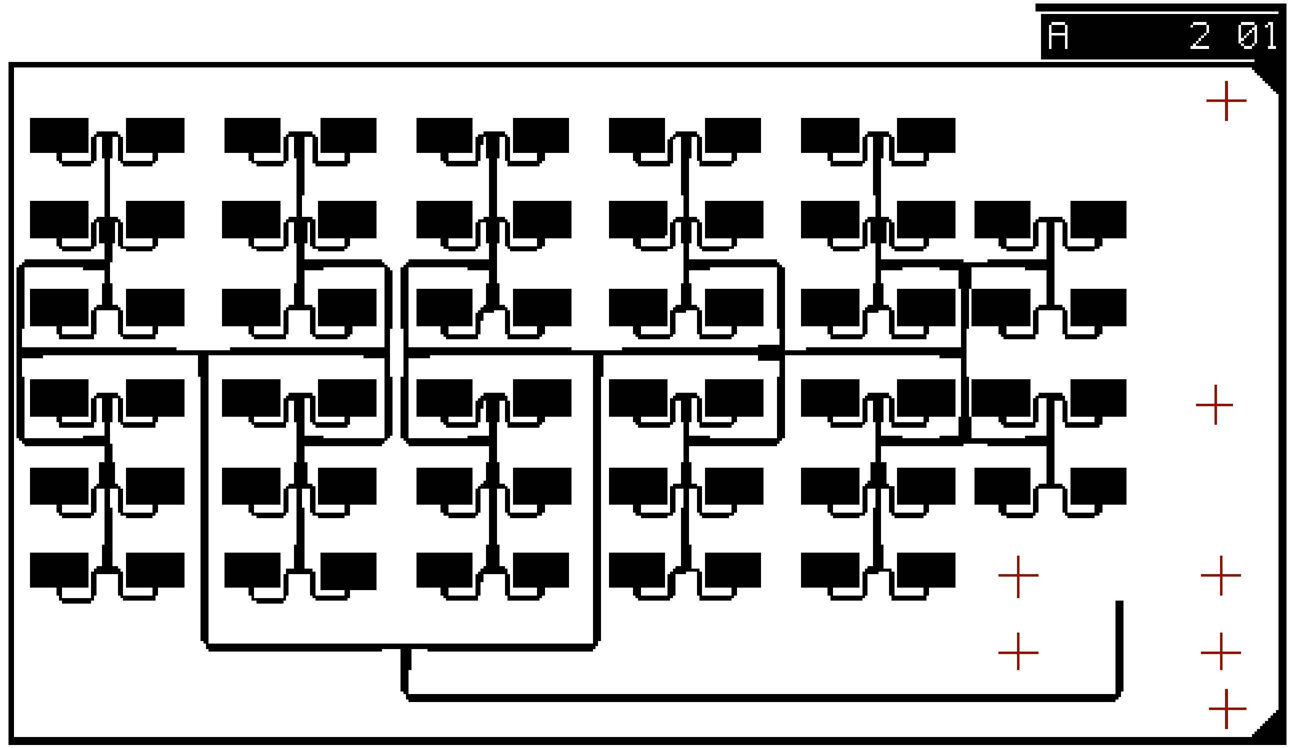


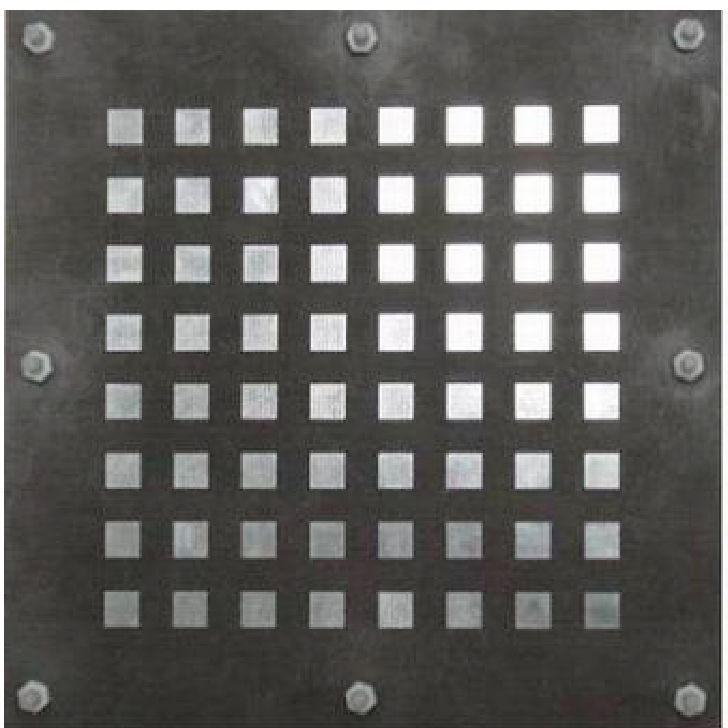
Toothed bowtie

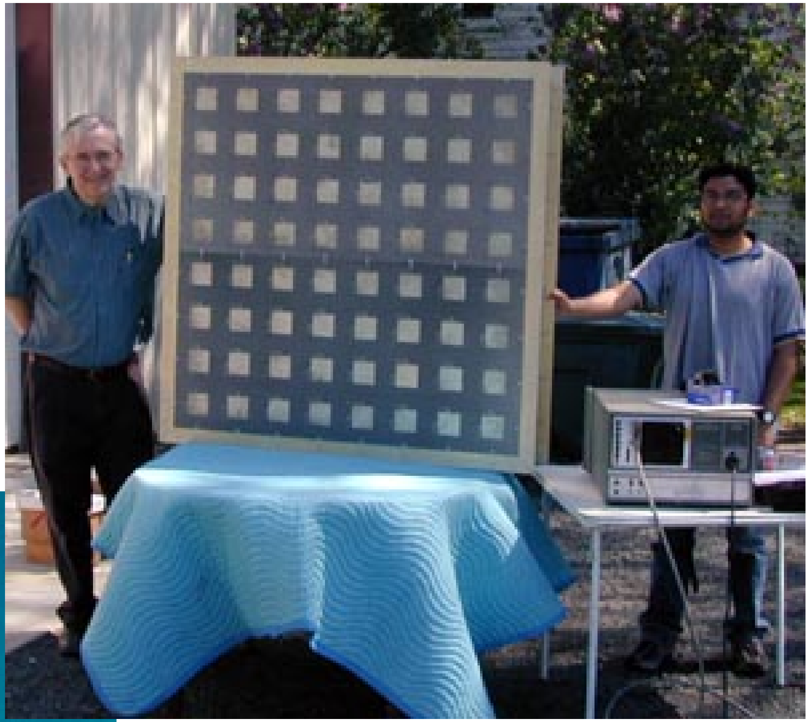
Fractal antennas

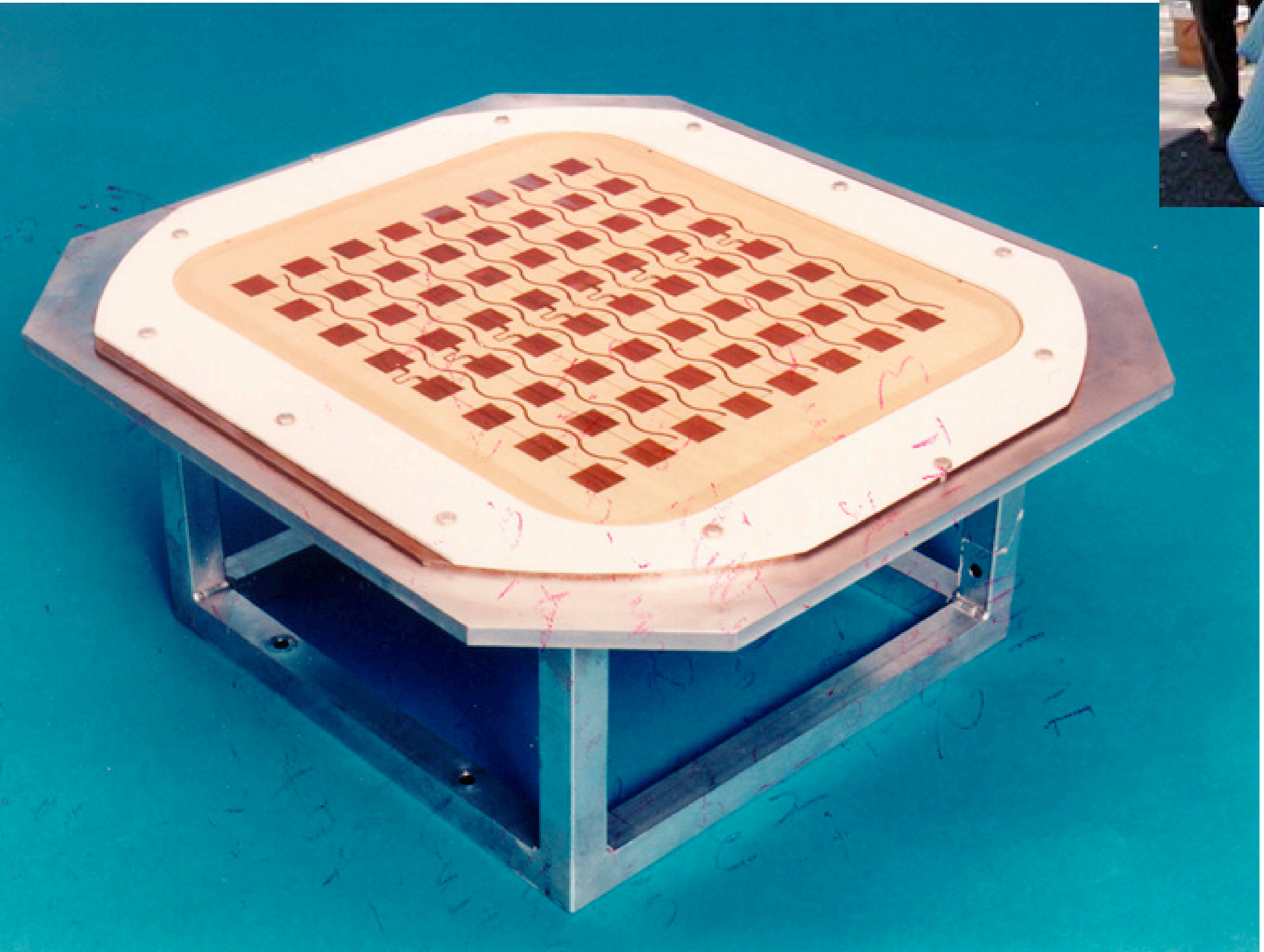


Array antennas

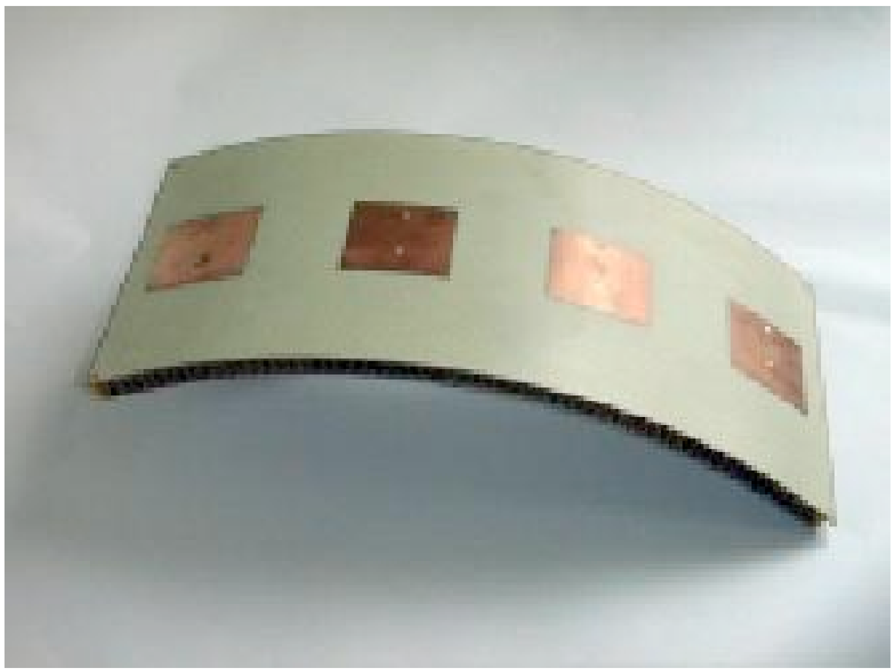


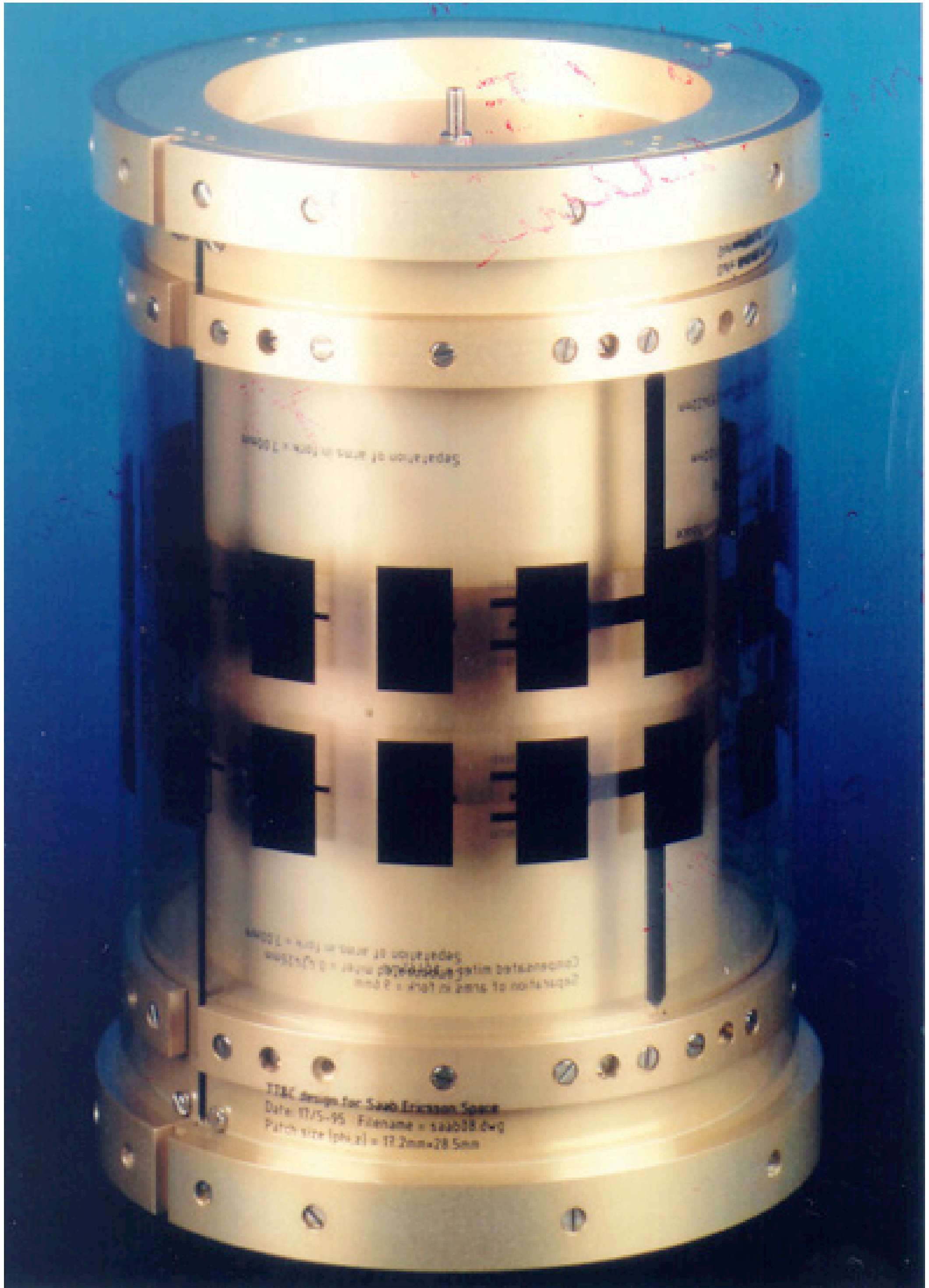




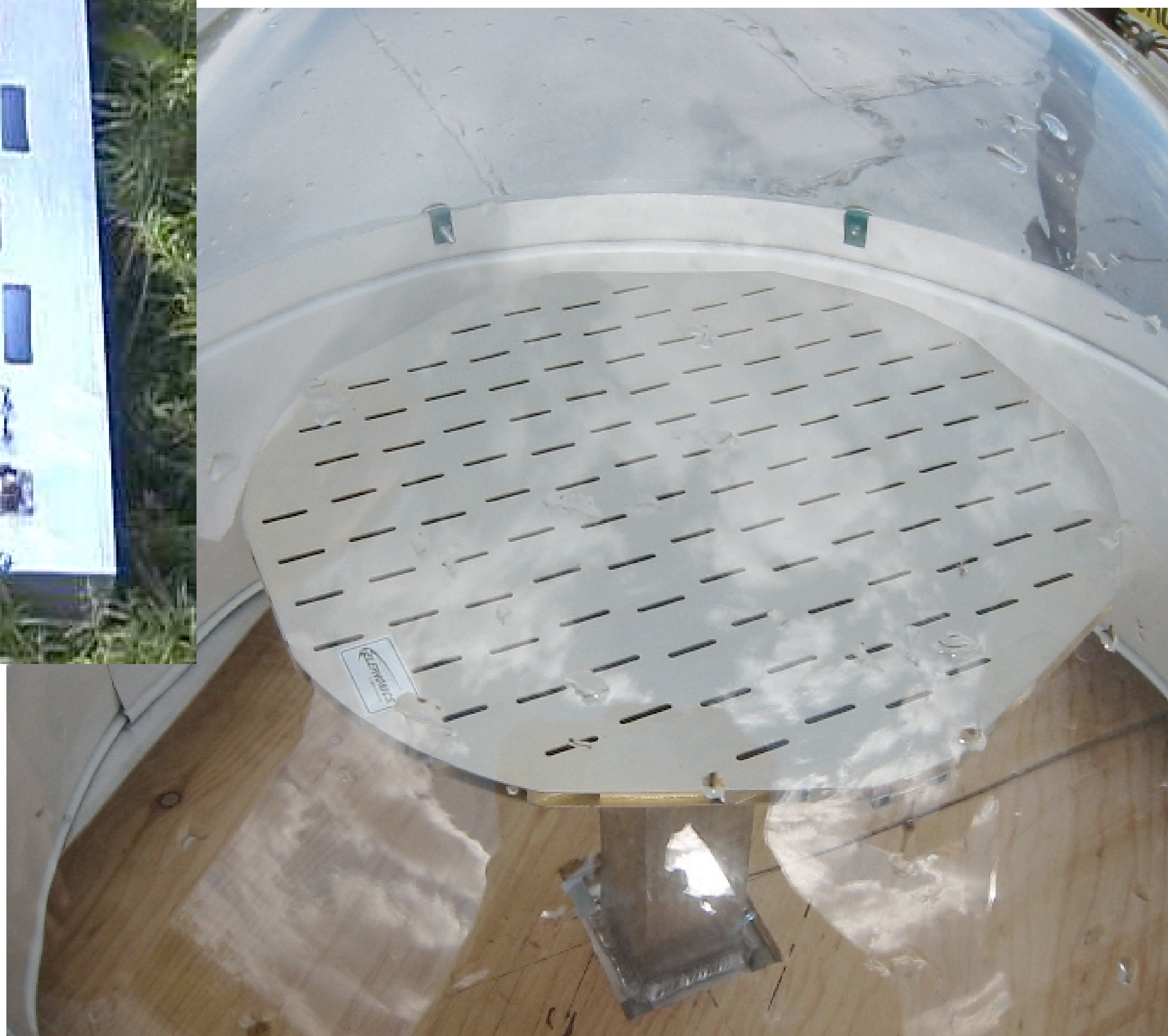


Conformal antennas (usually microstrip)

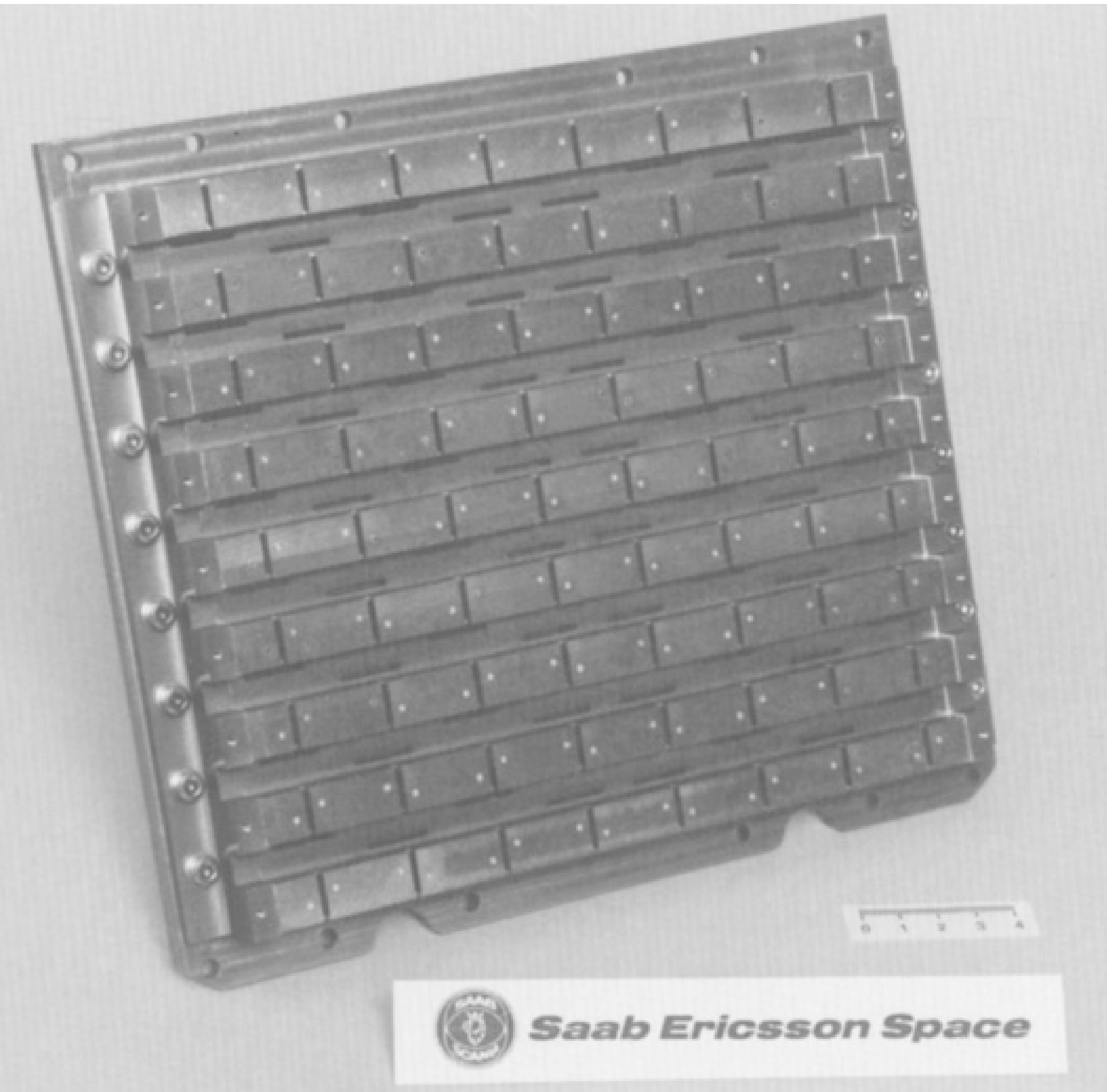


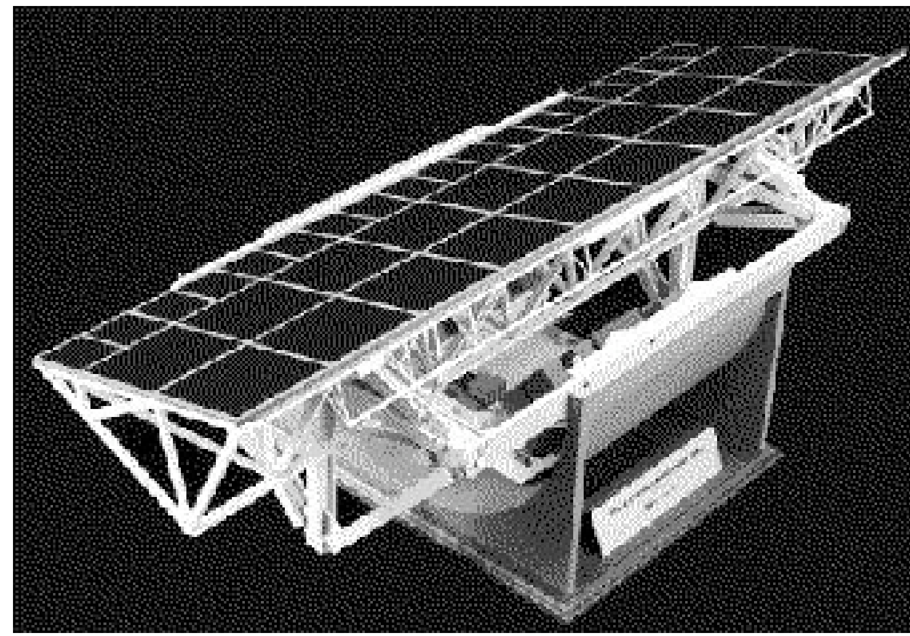

TT&C design for Saab Ericsson Space
Date: 17/5-95 Filename = saab08.dwg
Patch size (phi,z) = 17.2mm×28.5mm

Slotted waveguides



Slotted waveguide arrays





Shuttle imaging radar antenna

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Vector and scalar electromagnetic potentials

$$\nabla \times \boldsymbol{\mathcal{E}} = -\partial_t \boldsymbol{\mathcal{B}} \tag{1}$$

$$\nabla \times \boldsymbol{\mathcal{H}} = \boldsymbol{\mathcal{J}} + \partial_t \boldsymbol{\mathcal{D}} \tag{2}$$

$$\nabla \cdot \boldsymbol{\mathcal{D}} = \rho \qquad \nabla \cdot \boldsymbol{\mathcal{B}} = 0 \quad \Rightarrow \quad \boldsymbol{\mathcal{B}} = \nabla \times \underset{\substack{\uparrow \\ \text{vector potential}}}{\boldsymbol{\mathcal{A}}} \tag{3}$$

$$(1) \Rightarrow \quad \nabla \times \boldsymbol{\mathcal{E}} + \partial_t \nabla \times \boldsymbol{\mathcal{A}} = \mathbf{0} \quad \Rightarrow \quad \nabla \times (\boldsymbol{\mathcal{E}} + \partial_t \boldsymbol{\mathcal{A}}) = \mathbf{0}$$

$$\Rightarrow \boldsymbol{\mathcal{E}} + \partial_t \boldsymbol{\mathcal{A}} = -\nabla \underset{\substack{\uparrow \\ \text{scalar potential}}}{\phi}$$

plug into (2), use $\boldsymbol{\mathcal{B}} = \mu_0 \boldsymbol{\mathcal{H}}$, $\boldsymbol{\mathcal{D}} = \epsilon_0 \boldsymbol{\mathcal{E}}$ $\quad \Rightarrow$

$$\frac{1}{\mu_0} \underbrace{\nabla \times (\nabla \times \boldsymbol{\mathcal{A}})}_{\nabla(\nabla \cdot \boldsymbol{\mathcal{A}}) - \nabla^2 \boldsymbol{\mathcal{A}}} = \boldsymbol{\mathcal{J}} - \partial_t \epsilon_0 (\nabla \phi + \partial_t \boldsymbol{\mathcal{A}}) \quad \Rightarrow$$

$$\nabla^2 \boldsymbol{\mathcal{A}} - \mu_0 \epsilon_0 \partial_t^2 \boldsymbol{\mathcal{A}} + \left[\nabla(\nabla \cdot \boldsymbol{\mathcal{A}}) + \mu_0 \epsilon_0 \partial_t \nabla \phi \right] = -\mu_0 \boldsymbol{\mathcal{J}}$$

$\boldsymbol{\mathcal{A}}$ and Φ are not unique!

We get the same $\boldsymbol{\mathcal{B}}, \boldsymbol{\mathcal{E}}, \boldsymbol{\mathcal{H}}, \boldsymbol{\mathcal{D}}$ if $\begin{cases} \boldsymbol{\mathcal{A}} \to \boldsymbol{\mathcal{A}} + \nabla \Psi \\ \phi \to \phi - \partial_t \Psi \end{cases}$ *gauge invariance*

Two commonly used gauges:

$$\begin{cases} \nabla \cdot \boldsymbol{\mathcal{A}} + \mu_0 \epsilon_0 \partial_t \phi = 0 & \text{Lorentz gauge} \\ \nabla \cdot \boldsymbol{\mathcal{A}} = 0 & \text{Coulomb gauge} \end{cases}$$

We use the Lorentz gauge to obtain

$$\boxed{\nabla^2 \boldsymbol{\mathcal{A}} - \mu_0 \epsilon_0 \partial_t^2 \boldsymbol{\mathcal{A}} = -\mu_0 \boldsymbol{\mathcal{J}}}$$

in frequency domain: $\boxed{\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu_0 \boldsymbol{J}}$

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Strategy (frequency domain)

1. model antenna as current density $\boldsymbol{J}$

2. solve $\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu_0 \boldsymbol{J}$ for $\boldsymbol{A}$:

$$\boldsymbol{A}(\boldsymbol{x}) = \int \frac{e^{ik|\boldsymbol{x}-\boldsymbol{y}|}}{4\pi|\boldsymbol{x}-\boldsymbol{y}|}\mu_0 \boldsymbol{J}(\boldsymbol{y})d\boldsymbol{y} \approx \frac{e^{ik|\boldsymbol{x}|}}{4\pi|\boldsymbol{x}|}\mu_0 \underbrace{\int e^{-ik\hat{\boldsymbol{x}}\cdot\boldsymbol{y}}\boldsymbol{J}(\boldsymbol{y})d\boldsymbol{y}}_{\boldsymbol{F}(k\hat{\boldsymbol{x}})=\mathcal{F}[\boldsymbol{J}](k\hat{\boldsymbol{x}})}$$

3. recall that $\boldsymbol{E} = i\omega\boldsymbol{A} - \nabla\Phi$ and the Lorentz gauge condition is $i\omega\Phi = c_0^2 \nabla\cdot\boldsymbol{A} \qquad \Rightarrow$

$$\boldsymbol{E} = i\omega\left(\boldsymbol{A} - \frac{c_0^2\nabla(\nabla\cdot\boldsymbol{A})}{(i\omega)^2}\right) = i\omega\left[\boldsymbol{A} + k^{-2}\nabla(\nabla\cdot\boldsymbol{A})\right]$$

4. compute $\boldsymbol{E}$ using far-field approximation
   use expressions for ∇ and $\nabla\cdot$ in spherical coordinates

$$\boldsymbol{A}(\boldsymbol{x}) \approx \frac{e^{ik|\boldsymbol{x}|}}{4\pi|\boldsymbol{x}|}\mu_0 \boldsymbol{F}(k\hat{\boldsymbol{x}})$$

$$\begin{aligned}
\boldsymbol{E} &= i\omega\left[\boldsymbol{A} + \frac{1}{k^2}\nabla(\nabla\cdot\boldsymbol{A})\right] \\
&= i\omega\mu_0 \frac{e^{ik|\boldsymbol{x}|}}{4\pi|\boldsymbol{x}|}\left[\boldsymbol{F} + \frac{1}{k^2}(ik\hat{\boldsymbol{x}}(ik\hat{\boldsymbol{x}}\cdot\boldsymbol{F}))\right] + \mathcal{O}\left(\frac{1}{(k|\boldsymbol{x}|)^2}\right)
\end{aligned}$$

$$\boxed{\boldsymbol{E}(\boldsymbol{x}) = i\omega\mu_0 \frac{e^{ik|\boldsymbol{x}|}}{4\pi|\boldsymbol{x}|}\left[\boldsymbol{F} - \hat{\boldsymbol{x}}(\hat{\boldsymbol{x}}\cdot\boldsymbol{F})\right] + \mathcal{O}\left(\frac{1}{(k|\boldsymbol{x}|)^2}\right)}$$

$$\boldsymbol{A}\times(\boldsymbol{B}\times\boldsymbol{C}) = \boldsymbol{B}(\boldsymbol{A}\cdot\boldsymbol{C}) - \boldsymbol{C}(\boldsymbol{A}\cdot\boldsymbol{B}) \Rightarrow$$

$$\boxed{\boldsymbol{E}(\boldsymbol{x}) = -i\omega\mu_0 \frac{e^{ik|\boldsymbol{x}|}}{4\pi|\boldsymbol{x}|}\left[\hat{\boldsymbol{x}}\times(\hat{\boldsymbol{x}}\times\boldsymbol{F})\right] + \mathcal{O}\left(\frac{1}{(k|\boldsymbol{x}|)^2}\right)} \quad \text{Note } \boldsymbol{E}\perp\hat{\boldsymbol{x}}$$

Example: uniform current density on a line

$$\begin{aligned} \boldsymbol{F}(k\hat{\boldsymbol{x}}) &= \int_{-a}^{a} e^{-ik\hat{\boldsymbol{x}}\cdot(s\hat{\boldsymbol{e}})} I\hat{\boldsymbol{e}} ds = I\hat{\boldsymbol{e}} \frac{e^{-ika\hat{\boldsymbol{x}}\cdot\hat{\boldsymbol{e}}} - e^{ika\hat{\boldsymbol{x}}\cdot\hat{\boldsymbol{e}}}}{ik\hat{\boldsymbol{x}}\cdot\hat{\boldsymbol{e}}} \\ &= -I\hat{\boldsymbol{e}} \frac{2i\sin(ka\hat{\boldsymbol{x}}\cdot\hat{\boldsymbol{e}})}{ik\hat{\boldsymbol{x}}\cdot\hat{\boldsymbol{e}}} = -2aI\hat{\boldsymbol{e}}\,\mathrm{sinc}(ka\hat{\boldsymbol{x}}\cdot\hat{\boldsymbol{e}}) \end{aligned}$$

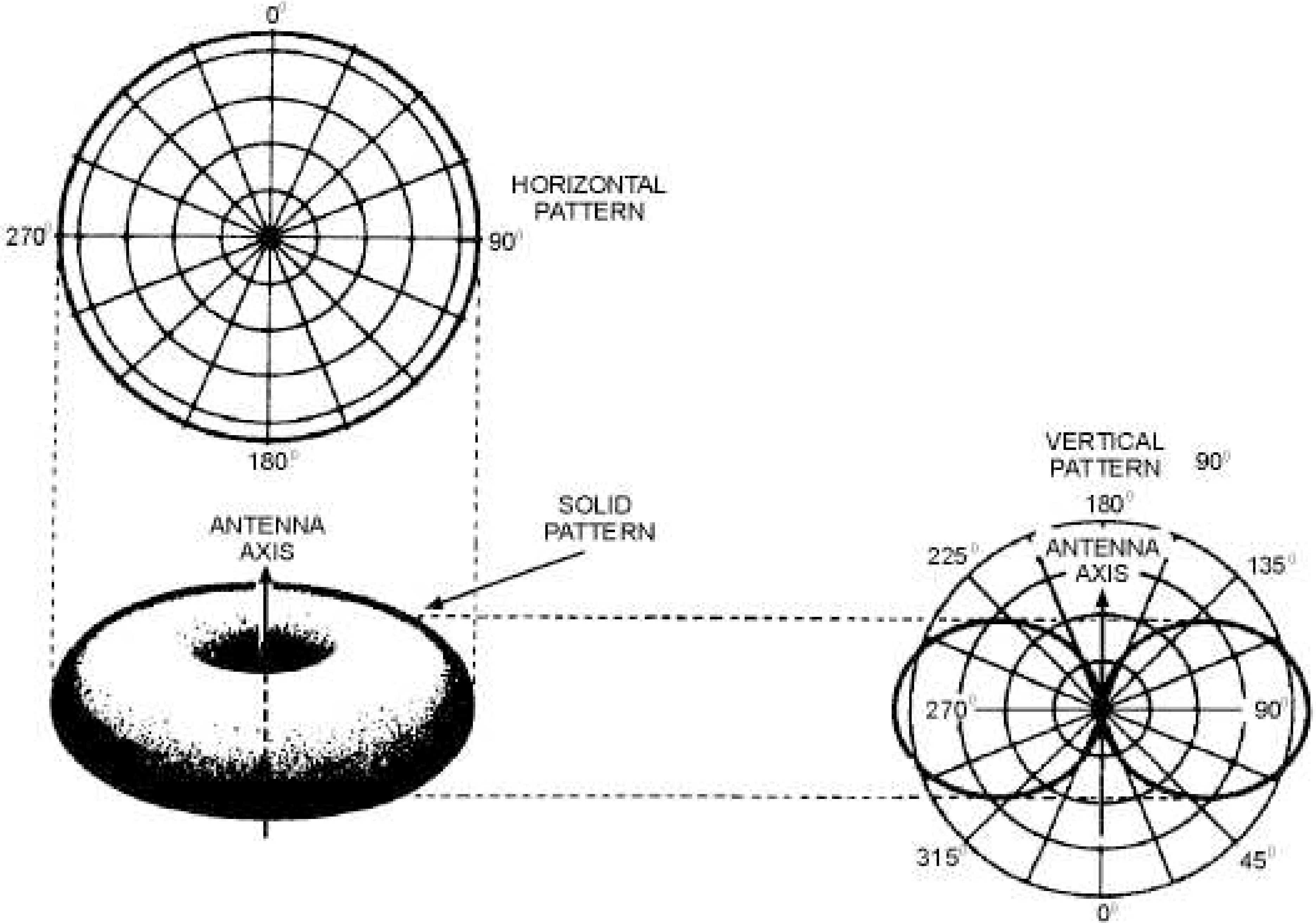
width of main lobe is obtained from $ka\,\underbrace{\hat{\boldsymbol{x}}\cdot\hat{\boldsymbol{e}}}_{\sin\theta} = \pi$

(θ measured from normal)

If $\sin\theta \approx \theta$, then beamwidth is

$$\approx \frac{2\pi}{ka} = \frac{2\pi}{(2\pi/\lambda)a} = \frac{\lambda}{a} = \frac{2}{\text{number of wavelengths that fit on antenna}}$$

$$\left(k = \frac{\omega}{c} \quad \text{and} \quad \nu = \frac{c}{\lambda} \quad \Rightarrow \quad k = \frac{2\pi}{\lambda} \right)$$



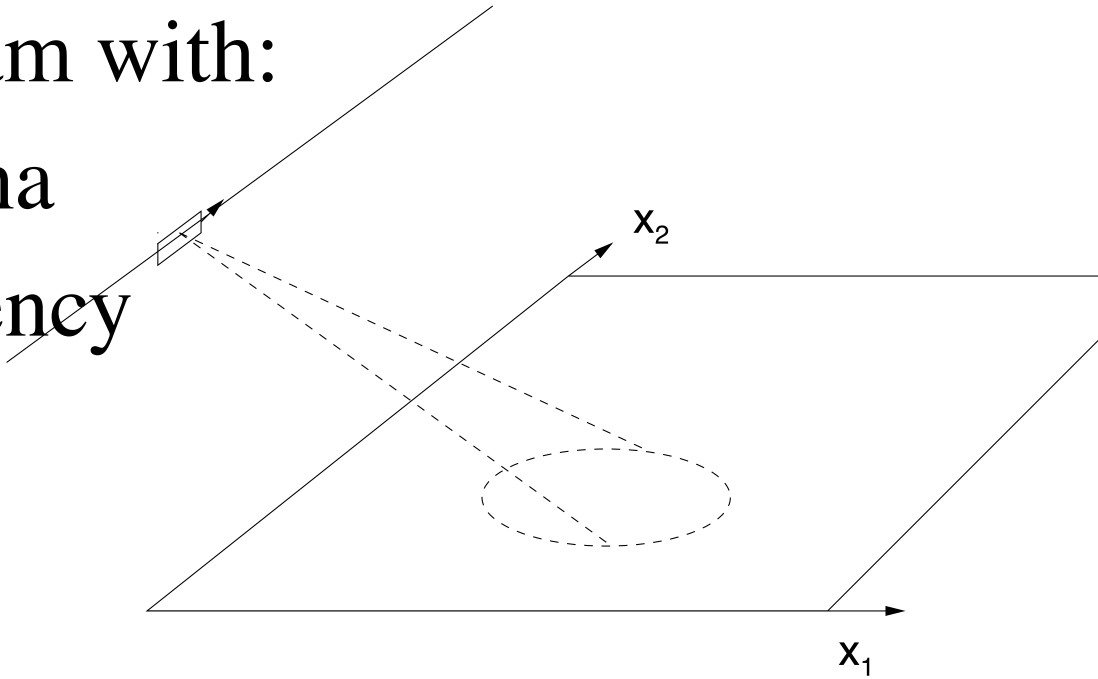
0°
HORIZONTAL PATTERN
270°
90°
180°
SOLID PATTERN
ANTENNA AXIS
VERTICAL PATTERN
90°
180°
225°
135°
ANTENNA AXIS
270°
90°
315°
45°
0°

Example: uniform current density on a rectangle

$$
\begin{aligned}
\boldsymbol{F}(k\hat{\boldsymbol{x}}) &= \int_{-a}^{a}\int_{-b}^{b} e^{-ik\hat{\boldsymbol{x}}\cdot(s_1\hat{\boldsymbol{e}_1}+s_2\hat{\boldsymbol{e}_2})}\boldsymbol{I}\; ds_1 ds_2 \\
&= \boldsymbol{I}\left(2b\,\mathrm{sinc}(kb\hat{\boldsymbol{x}}\cdot\hat{\boldsymbol{e}_1})\right)\left(2a\,\mathrm{sinc}(ka\hat{\boldsymbol{x}}\cdot\hat{\boldsymbol{e}_2})\right)
\end{aligned}
$$

Key features of antenna beam

- Main lobe of beam is perpendicular to antenna
- Antenna has length $L = 2a \Rightarrow$
  Angular width of main lobe $\approx \frac{4\pi}{kL} = \frac{2\lambda}{L}$
  $\Rightarrow$ width of antenna footprint $= \frac{4\pi}{kL} R = \frac{2\lambda}{L} R.$
- Get a more focused beam with:
  — bigger antenna
  — higher frequency



Phased arrays

Take $\boldsymbol{J}(k, \boldsymbol{y}) = \boldsymbol{I} e^{ik\hat{\boldsymbol{\mu}}\cdot\boldsymbol{y}}$ for $\boldsymbol{y} = s_1\hat{\boldsymbol{e}_1} + s_2\hat{\boldsymbol{e}_2}$ (phase depends on position):

$$\boldsymbol{F}(k\hat{\boldsymbol{x}}) = \int_{-a}^{a}\int_{-b}^{b} e^{-ik(\hat{\boldsymbol{x}}-\hat{\boldsymbol{\mu}})\cdot(s_1\hat{\boldsymbol{e}_1}+s_2\hat{\boldsymbol{e}_2})}\boldsymbol{I}\; ds_1 ds_2$$

same as above, but beam points in direction $\hat{\boldsymbol{x}} = \hat{\boldsymbol{\mu}}$.

Electronically Steered Array (ESA)

Real-Aperture Imaging

- Use big antenna or high frequencies to form small spot on ground
- Scan the spot over the ground
- For each spot position on a map, assign a color corresponding to the amplitude of the received energy

Synthetic-Aperture imaging

- Use a smaller antenna
- Illuminate same region from different antenna locations $\gamma(s)$
- Use mathematics to form high-resolution image

An array of point antennas

$$\boldsymbol{F}(k\hat{\boldsymbol{x}}) = \boldsymbol{I}\sum_{j=0}^{N-1}\int e^{-ik\hat{\boldsymbol{x}}\cdot\boldsymbol{y}}\delta(\boldsymbol{y}-dj\hat{\boldsymbol{e}})d\boldsymbol{y} = \boldsymbol{I}\sum_{j=0}^{N-1} e^{-ik\hat{\boldsymbol{x}}\cdot\hat{\boldsymbol{e}}dj}$$

$$= \frac{1-e^{ikdN\hat{\boldsymbol{x}}\cdot\hat{\boldsymbol{e}}}}{1-e^{ikd\hat{\boldsymbol{x}}\cdot\hat{\boldsymbol{e}}}} = \frac{e^{ikdN\hat{\boldsymbol{x}}\cdot\hat{\boldsymbol{e}}/2}}{e^{ikd\hat{\boldsymbol{x}}\cdot\hat{\boldsymbol{e}}/2}}\underbrace{\left[\frac{\sin(kd\hat{\boldsymbol{x}}\cdot\hat{\boldsymbol{e}}N/2)}{\sin(kd\hat{\boldsymbol{x}}\cdot\hat{\boldsymbol{e}}/2}\right]}_{\text{array factor}}$$

periodic with period $kd\hat{\boldsymbol{x}}\cdot\hat{\boldsymbol{e}} = 2\pi \Rightarrow$ *grating lobes* (extra main lobes) when kd is sufficiently big

(an issue for sparse apertures)

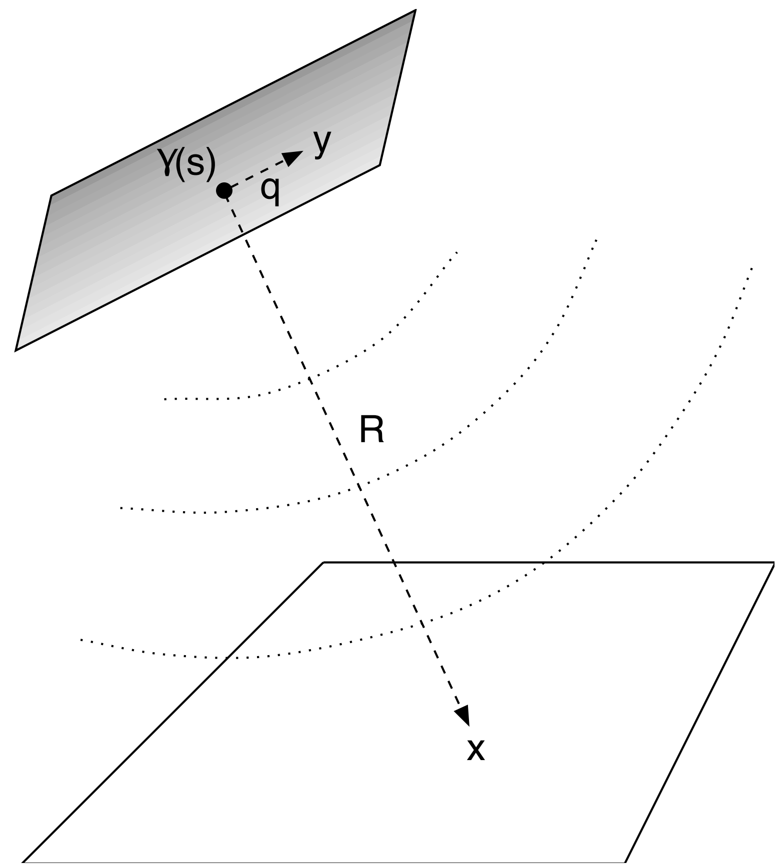
Far-field expansion when antenna is at $\gamma(s)$

Center of antenna is at $\boldsymbol{\gamma}(s)$

Point on antenna is $\boldsymbol{y} = \boldsymbol{\gamma}(s) + \boldsymbol{q}$

$\boldsymbol{x}$ is on the ground

$$|\boldsymbol{x} - \boldsymbol{y}| = |\underbrace{\boldsymbol{x} - \boldsymbol{\gamma}(s)}_{\boldsymbol{R}_{s,\boldsymbol{x}}} + \underbrace{\boldsymbol{\gamma}(s) - \boldsymbol{y}}_{-\boldsymbol{q}}| = |\boldsymbol{R}_{s,\boldsymbol{x}} - \boldsymbol{q}|$$



$$|\boldsymbol{q}| << |\boldsymbol{R}_{s,\boldsymbol{x}}| \Rightarrow$$

$$|\boldsymbol{R}_{s,\boldsymbol{x}} - \boldsymbol{q}| = |\boldsymbol{R}_{s,\boldsymbol{x}}| - \widehat{\boldsymbol{R}_{s,\boldsymbol{x}}} \cdot \boldsymbol{q} + O(|\boldsymbol{R}_{s,\boldsymbol{x}}|^{-1})$$

Field from antenna at $\boldsymbol{\gamma}(s)$

$$\hat{\boldsymbol{x}} \to \widehat{\boldsymbol{R}_{s,\boldsymbol{x}}} \qquad \Rightarrow \qquad \boldsymbol{F}(k\hat{\boldsymbol{x}}) \to \boldsymbol{F}(k\widehat{\boldsymbol{R}_{s,\boldsymbol{x}}}) = \int e^{ik\widehat{\boldsymbol{R}_{s,\boldsymbol{x}}}\cdot\boldsymbol{q}} \boldsymbol{J}(\omega, s, \boldsymbol{q}) d\boldsymbol{q}$$

$$\boldsymbol{E}(\omega, \boldsymbol{x}) \approx -\frac{i\omega\mu_0}{4\pi} \frac{e^{ik|\boldsymbol{R}_{s,\boldsymbol{x}}|}}{|\boldsymbol{R}_{s,\boldsymbol{x}}|} \left[\boldsymbol{F} - \widehat{\boldsymbol{R}_{s,\boldsymbol{x}}}\left(\widehat{\boldsymbol{R}_{s,\boldsymbol{x}}} \cdot \boldsymbol{F}\right)\right]$$

Vector model	Scalar model
$\nabla^2 \boldsymbol{\mathcal{A}} - \mu_0\epsilon_0\partial_t^2\boldsymbol{\mathcal{A}} = -\mu_0\boldsymbol{\mathcal{J}}$	$\nabla^2 \mathcal{E} - \mu_0\epsilon_0\partial_t^2\mathcal{E} = \mu_0\partial_t\mathcal{J} =: j$
$\nabla^2 \boldsymbol{A} + k^2\boldsymbol{A} = -\mu_0\boldsymbol{J}$	$\nabla^2 E + k^2 E = -i\omega\mu_0\mathrm{J}$
$\boldsymbol{A}(\omega,\boldsymbol{x}) = \int \frac{e^{ik\lvert\boldsymbol{x}-\boldsymbol{y}\rvert}}{4\pi\lvert\boldsymbol{x}-\boldsymbol{y}\rvert}\mu_0\boldsymbol{J}(\omega,\boldsymbol{y})d\boldsymbol{y}$	$E(\omega,\boldsymbol{x}) = \int \frac{e^{ik\lvert\boldsymbol{x}-\boldsymbol{y}\rvert}}{4\pi\lvert\boldsymbol{x}-\boldsymbol{y}\rvert}(i\omega\mu_0)J(\omega,\boldsymbol{y})d\boldsymbol{y}$
$\boldsymbol{E} = i\omega\left[\boldsymbol{A} + k^{-2}\nabla(\nabla\cdot\boldsymbol{A})\right]$	
$\boldsymbol{E}(\boldsymbol{x}) \approx i\omega\mu_0\frac{e^{ik\lvert\boldsymbol{x}\rvert}}{4\pi\lvert\boldsymbol{x}\rvert}\left[\boldsymbol{F} - \hat{\boldsymbol{x}}(\hat{\boldsymbol{x}}\cdot\boldsymbol{F})\right]$	$E(\boldsymbol{x}) \approx i\omega\mu_0\frac{e^{ik\lvert\boldsymbol{x}\rvert}}{4\pi\lvert\boldsymbol{x}\rvert}F$
$\boldsymbol{F}(k\hat{\boldsymbol{x}}) = \int e^{-ik\hat{\boldsymbol{x}}\cdot\boldsymbol{y}}\boldsymbol{J}(\omega,\boldsymbol{y})d\boldsymbol{y}$	$F(k\hat{\boldsymbol{x}}) = \int e^{-ik\hat{\boldsymbol{x}}\cdot\boldsymbol{y}}J(\omega,\boldsymbol{y})d\boldsymbol{y}$

Antenna reception

$$s_{rec}(t) = \int \int_{\text{antenna}} \mathcal{E}^{sc}(t - t', \boldsymbol{x}^0 - \boldsymbol{y}) \underbrace{\mathcal{W}(t', \boldsymbol{y})} \, d\boldsymbol{y} dt'$$

weighting factor depending on antenna shape

in the frequency domain:

$$\begin{aligned} S_{rec}(\omega) &= \int_{\text{antenna}} E^{sc}(\omega, \boldsymbol{x}^0 - \boldsymbol{y}) W(\omega, \boldsymbol{y}) d\boldsymbol{y} \\ &= \int \int \frac{e^{ik|\boldsymbol{x}^0 - \boldsymbol{y} - \boldsymbol{z}|}}{4\pi|\boldsymbol{x}^0 - \boldsymbol{y} - \boldsymbol{z}|} V(\boldsymbol{z}) E(\omega, \boldsymbol{z}) d\boldsymbol{z} W(\omega, \boldsymbol{y}) d\boldsymbol{y} \end{aligned}$$

use far-field expansion for $|\boldsymbol{x}^0 - \boldsymbol{z}| >> |\boldsymbol{y}|$:

$$S_{rec}(\omega) \approx \int \frac{e^{ik|\boldsymbol{x}^0 - \boldsymbol{z}|}}{4\pi|\boldsymbol{x}^0 - \boldsymbol{z}|} V(\boldsymbol{z}) E(\omega, \boldsymbol{z}) d\boldsymbol{z} \underbrace{\int e^{-ik\widehat{\boldsymbol{x}^0 - \boldsymbol{z}} \cdot \boldsymbol{y}} W(\omega, \boldsymbol{y}) d\boldsymbol{y}}_{\mathcal{F}[W](\omega, \widehat{\boldsymbol{x}^0 - \boldsymbol{z}})}$$

Antenna research questions

- How can antennas be designed to achieve certain desired far-field patterns?
  Antennas must be possible to manufacture and feed.
  Ideally antennas should be small.

- What can be done with array antennas in which each element is activated with a different waveform? What can be done with distributed networks of antennas?

- Is it possible to send a message only to a desired location, and send different messages in other directions? (Applications to secure wireless communications)