Books

- B. Borden, Radar Imaging of Airborne Targets, Institute of Physics, 1999.
- C. Elachi, Spaceborne Radar Remote Sensing: Applications and Techniques, IEEE Press, New York, 1987.
- W. C. Carrara, R. G. Goodman, R. M. Majewski, Spotlight Synthetic Aperture Radar: Signal Processing Algorithms, Artech House, Boston, 1995.
- G. Franceschetti and R. Lanari, Synthetic Aperture Radar Processing, CRC Press, New York, 1999.
- L.J. Cutrona, "Synthetic Aperture Radar", in Radar Handbook, second edition, ed. M. Skolnik, McGraw-Hill, New York, 1990.
- C.V. Jakowatz, D.E. Wahl, P.H. Eichel, D.C. Ghiglia, and P.A. Thompson, Spotlight-Mode Synthetic Aperture Radar: A Signal Processing Approach, Kluwer, Boston, 1996.
- I.G. Cumming and F.H. Wong, Digital Processing of SAR Data: Algorithms and Implementation, Artech House, 2005

Outline

1. introduction, history, frequency bands, dB, real-aperture imaging
2. radar systems: stepped-frequency systems, I/Q demodulation
3. 1D scattering by perfect conductor
4. receiver design, matched filtering
5. ambiguity function & its properties
6. range-doppler (unfocused) imaging
7. introduction to 3D scattering
8. ISAR
9. antenna theory
10. spotlight SAR
11. stripmap SAR
12. time permitting: deramp processing, Doppler from successive pulses

3D Mathematical Model

- We *should* use Maxwell's equations; but instead we use

$$\left(\nabla^2 - \frac{1}{c^2(\boldsymbol{x})}\partial_t^2\right)\mathcal{E}(t,\boldsymbol{x}) = \underbrace{j(t,\boldsymbol{x})}_{\text{source}}$$

- Scattering is due to a perturbation in the wave speed c:

$$\frac{1}{c^2(\boldsymbol{x})} = \frac{1}{c_0^2} - \underbrace{V(\boldsymbol{x})}$$

reflectivity function

- For a moving target, use $V(\boldsymbol{x}, t)$.

Basic facts about the wave equation

- fundamental solution g

$$\left(\nabla^2 - c_0^{-2}\partial_t^2\right) g(t,\boldsymbol{x}) = -\delta(t)\delta(\boldsymbol{x})$$

$$g(t,\boldsymbol{x}) = \frac{\delta(t-|\boldsymbol{x}|/c_0)}{4\pi|\boldsymbol{x}|} = \int \frac{e^{-i\omega(t-|\boldsymbol{x}|/c_0)}}{8\pi^2|\boldsymbol{x}|} d\omega$$

- $g(t,\boldsymbol{x}) =$ field at $(t,\boldsymbol{x})$ due to a source at the origin at time 0

- Solution of

$$\left(\nabla^2 - c_0^{-2}\partial_t^2\right) u(t,\boldsymbol{x}) = j(t,\boldsymbol{x}),$$

  is

$$u(t,\boldsymbol{x}) = -\int g(t-t',\boldsymbol{x}-\boldsymbol{y}) j(t',\boldsymbol{y}) dt' d\boldsymbol{y}$$

- frequency domain: $k = \omega/c_0$

$$(\nabla^2 + k^2)G = -\delta \qquad G(\omega,\boldsymbol{x}) = \frac{e^{ik|\boldsymbol{x}|}}{4\pi|\boldsymbol{x}|}$$

Introduction to scattering theory

$$\begin{aligned}\left(\nabla^2 - c^{-2}(\boldsymbol{x})\partial_t^2\right)\mathcal{E}(t,\boldsymbol{x}) &= j(t,\boldsymbol{x})\\ (\nabla^2 - c_0^{-2}\partial_t^2)\mathcal{E}^{in}(t,\boldsymbol{x}) &= j(t,\boldsymbol{x})\end{aligned}$$

write $\mathcal{E} = \mathcal{E}^{in} + \mathcal{E}^{sc}$, $\quad c^{-2}(\boldsymbol{x}) = c_0^{-2} - V(\boldsymbol{x})$, $\quad$ subtract:

$$\left(\nabla^2 - \partial_t^2\right)\mathcal{E}^{sc}(t,\boldsymbol{x}) = -V(\boldsymbol{x})\partial_t^2\mathcal{E}(t,\boldsymbol{x})$$

use fundamental solution $\Rightarrow$

$$\mathcal{E}^{sc}(t,\boldsymbol{x}) = \int g(t-\tau,\boldsymbol{x}-\boldsymbol{z})V(\boldsymbol{z})\partial_\tau^2\mathcal{E}(\tau,\boldsymbol{z})d\tau d\boldsymbol{z}.$$

Lippman-Schwinger integral equation

frequency domain Lippman-Schwinger equation:

$$E^{sc}(\omega,\boldsymbol{x}) = -\int G(\omega,\boldsymbol{x}-\boldsymbol{z})V(\boldsymbol{z})\omega^2 E(\omega,\boldsymbol{z})d\boldsymbol{z}$$

single-scattering or *Born* approximation

$$\mathcal{E}^{sc}(t,\boldsymbol{x}) \approx \mathcal{E}_B^{sc} := \int g(t-\tau,\boldsymbol{x}-\boldsymbol{z})V(\boldsymbol{z})\partial_\tau^2\mathcal{E}^{in}(\tau,\boldsymbol{z})d\tau d\boldsymbol{z}$$

useful: makes inverse problem linear

not necessarily a good approximation!

In the frequency domain,

$$E_B^{sc}(\omega,\boldsymbol{x}) = -\int \frac{e^{ik|\boldsymbol{x}-\boldsymbol{z}|}}{4\pi|\boldsymbol{x}-\boldsymbol{z}|}V(\boldsymbol{z})\omega^2 E^{in}(\omega,\boldsymbol{z})d\boldsymbol{z}$$

E^{in} is obtained by solving $(\nabla^2 + k^2)E^{in} = J$

For now, suppose $J(\omega, \boldsymbol{x}) = P(\omega)\delta(\boldsymbol{x} - \boldsymbol{x}^0)$ $\quad\Rightarrow$

$$E^{in}(\omega, \boldsymbol{x}) = -\int G(\omega, \boldsymbol{x} - \boldsymbol{y})P(\omega)\delta(\boldsymbol{y} - \boldsymbol{x}^0)dt'd\boldsymbol{y} = -P(\omega)\frac{e^{ik|\boldsymbol{x}-\boldsymbol{x}^0|}}{4\pi|\boldsymbol{x} - \boldsymbol{x}_0|}$$

Then the scattered field back at $\boldsymbol{x}_0$ is

$$E_B^{sc}(\omega, \boldsymbol{x}^0) = P(\omega)\,\omega^2 \int \frac{e^{2ik|\boldsymbol{x}^0-\boldsymbol{z}|}}{(4\pi)^2|\boldsymbol{x}^0 - \boldsymbol{z}|^2}V(\boldsymbol{z})d\boldsymbol{z}$$

In the time domain this is

$$\mathcal{E}_B^{sc}(t, \boldsymbol{x}^0) = \int \frac{e^{-i\omega(t-2|\boldsymbol{x}^0-\boldsymbol{z}|/c)}}{2\pi(4\pi|\boldsymbol{x}^0 - \boldsymbol{z}|)^2}k^2P(\omega)V(\boldsymbol{z})d\omega d\boldsymbol{z}$$

Note $1/R^2$ geometrical decay $\quad\Rightarrow\quad$ power decays like $1/R^4$

Matched filtering

$$
\begin{aligned}
\eta(t, \boldsymbol{x}^0) &\approx \int p^*(t' - t)\mathcal{E}_B^{sc}(t', \boldsymbol{x}^0)dt' \\
&= \int \left(\frac{1}{2\pi} \int e^{i\omega'(t'-t)} P(\omega')d\omega' \right) \\
&\quad \cdot \int \frac{e^{-i\omega(t'-2|\boldsymbol{x}^0-\boldsymbol{z}|/c)}}{2\pi(4\pi|\boldsymbol{x}^0 - \boldsymbol{z}|)^2} k^2 P(\omega)V(\boldsymbol{z})d\omega d\boldsymbol{z}\, dt' \\
&\vdots \quad \text{do } t' \text{ and } \omega' \text{ integrations} \\
&= \int \frac{e^{-i\omega(t-2|\boldsymbol{x}^0-\boldsymbol{z}|/c)}}{2\pi(4\pi|\boldsymbol{x}^0 - \boldsymbol{z}|)^2} k^2 |P(\omega)|^2 V(\boldsymbol{z})d\omega d\boldsymbol{z}
\end{aligned}
$$

Effect of matched filter is to replace $P(\omega)$ by $|P(\omega)|^2$.

Far-field expansion

$$|\boldsymbol{z}| << |\boldsymbol{x}^0| \qquad \Rightarrow \qquad |\boldsymbol{x}^0 - \boldsymbol{z}| = |\boldsymbol{x}^0| - \widehat{\boldsymbol{x}^0} \cdot \boldsymbol{z} + O(|\boldsymbol{x}^0|^{-1})$$

where $\widehat{\boldsymbol{x}} = \boldsymbol{x}/|\boldsymbol{x}|$

Proof:

$$|\boldsymbol{x}^0 - \boldsymbol{z}| = \sqrt{(\boldsymbol{x}^0 - \boldsymbol{z}) \cdot (\boldsymbol{x}^0 - \boldsymbol{z})} = \sqrt{|\boldsymbol{x}^0|^2 - 2\boldsymbol{x}^0 \cdot \boldsymbol{z} + |\boldsymbol{z}|^2}$$

$$= |\boldsymbol{x}^0| \sqrt{1 - 2\frac{\boldsymbol{x}^0 \cdot \boldsymbol{z}}{|\boldsymbol{x}^0|^2} + \frac{|\boldsymbol{z}|^2}{|\boldsymbol{x}^0|^2}} \qquad \text{use } \sqrt{1-a} = 1 - \tfrac{a}{2} + \cdots$$

$$= |\boldsymbol{x}^0| \left(1 - \frac{\widehat{\boldsymbol{x}^0} \cdot \boldsymbol{z}}{|\boldsymbol{x}^0|} + O(|\boldsymbol{x}^0|^{-2})\right)$$

$$= |\boldsymbol{x}^0| - \widehat{\boldsymbol{x}^0} \cdot \boldsymbol{z} + O\left(\frac{|\boldsymbol{z}|}{|\boldsymbol{x}^0|}\right)$$

Note: Whether $|\boldsymbol{z}| << |\boldsymbol{x}^0|$ depends on location of origin of coordinates.

$$\frac{e^{ik|\boldsymbol{x}^0 - \boldsymbol{z}|}}{|\boldsymbol{x}^0 - \boldsymbol{z}|} = \frac{e^{ik|\boldsymbol{x}^0|}}{|\boldsymbol{x}^0|} e^{-ik\widehat{\boldsymbol{x}^0} \cdot \boldsymbol{z}} \left(1 + O\left(\frac{|\boldsymbol{z}|}{|\boldsymbol{x}^0|}\right)\right) \left(1 + O\left(\frac{k|\boldsymbol{z}|^2}{|\boldsymbol{x}^0|}\right)\right)$$

Far-field approximation to data

applies to smallish target

(Born-approximated) output of correlation receiver is:

$$\eta_B(t, \boldsymbol{x}^0) = \int \frac{e^{-i\omega(t-2|\boldsymbol{x}^0-\boldsymbol{z}|/c)}}{2\pi(4\pi|\boldsymbol{x}^0 - \boldsymbol{z}|)^2} k^2 |P(\omega)|^2 V(\boldsymbol{z}) d\omega d\boldsymbol{z}$$

put origin of coordinates in target, use far-field expansion $\Rightarrow$

$$\eta_B(t, \boldsymbol{x}^0) \approx \frac{1}{32\pi^2|\boldsymbol{x}^0|^2} \int e^{-i\omega(t-2|\boldsymbol{x}^0|/c+2\widehat{\boldsymbol{x}^0}\cdot\boldsymbol{z}/c)} k^2 |P(\omega)|^2 V(\boldsymbol{z}) d\omega d\boldsymbol{z}$$

in the frequency domain:

$$D_B(\omega, \boldsymbol{x}^0) \approx \frac{e^{2ik|\boldsymbol{x}^0|}}{32\pi^2|\boldsymbol{x}^0|^2} k^2 |P(\omega)|^2 \underbrace{\int e^{2ik\widehat{\boldsymbol{x}^0}\cdot\boldsymbol{z}} V(\boldsymbol{z}) d\boldsymbol{z}}_{\mathcal{F}[V](2k\widehat{\boldsymbol{x}^0})\ !}$$

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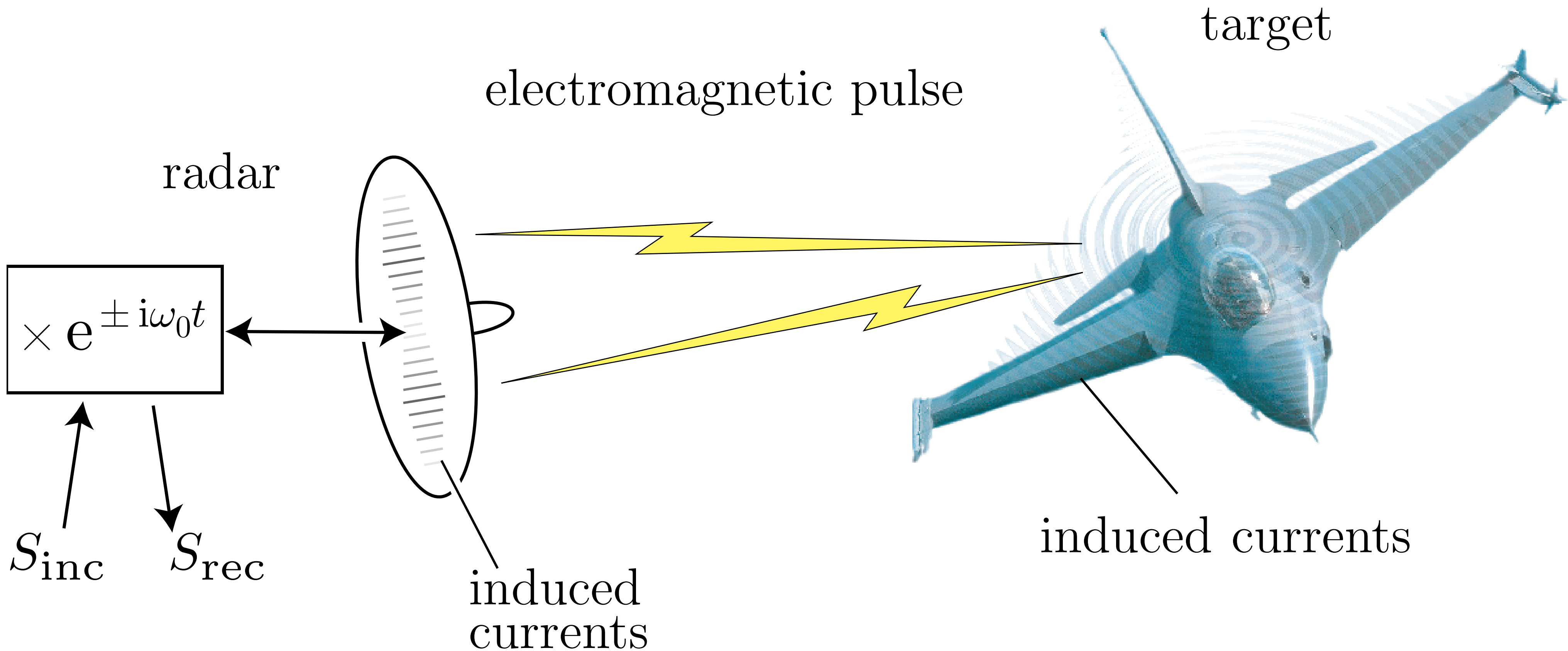
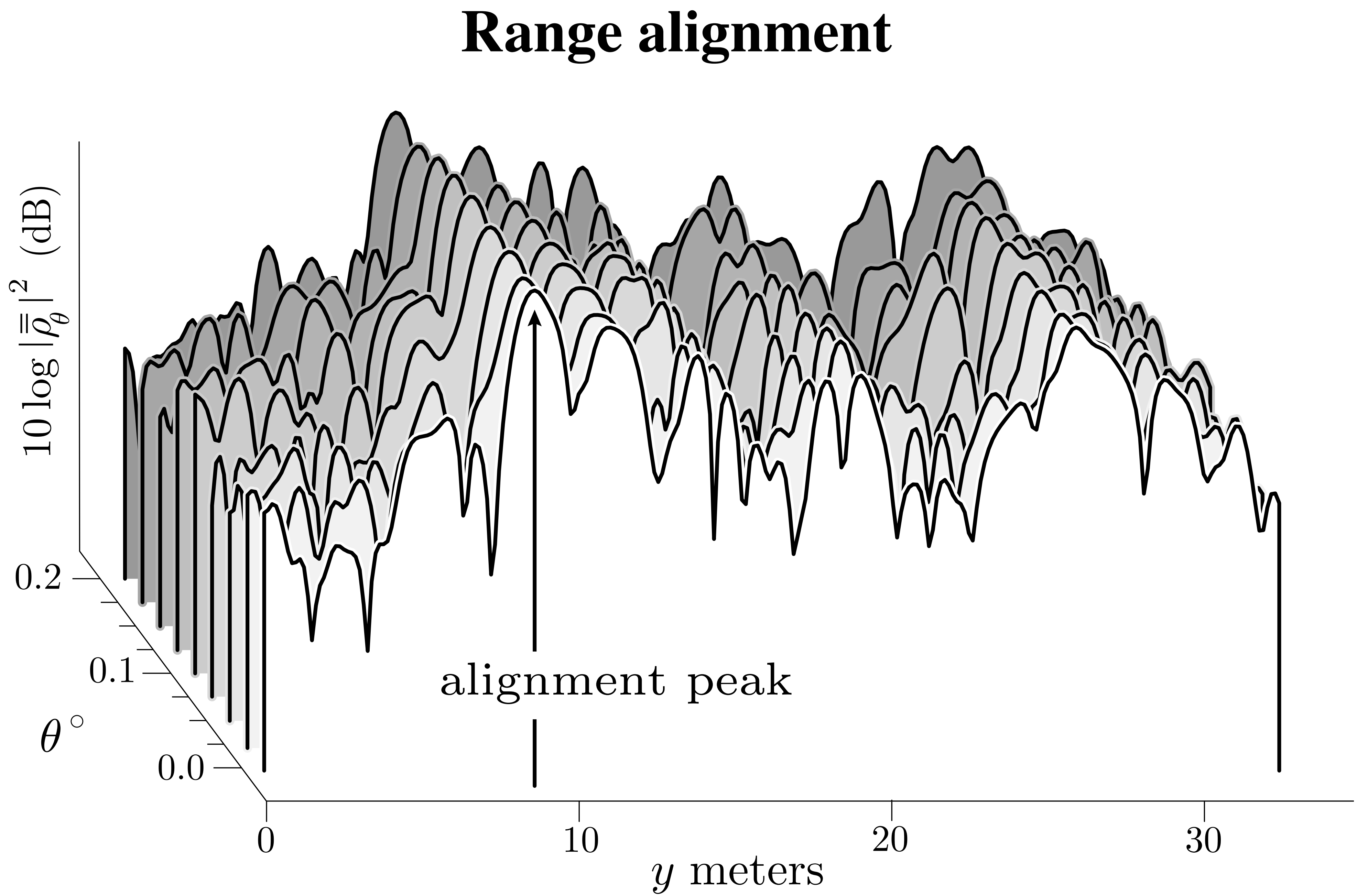
Airborne targets

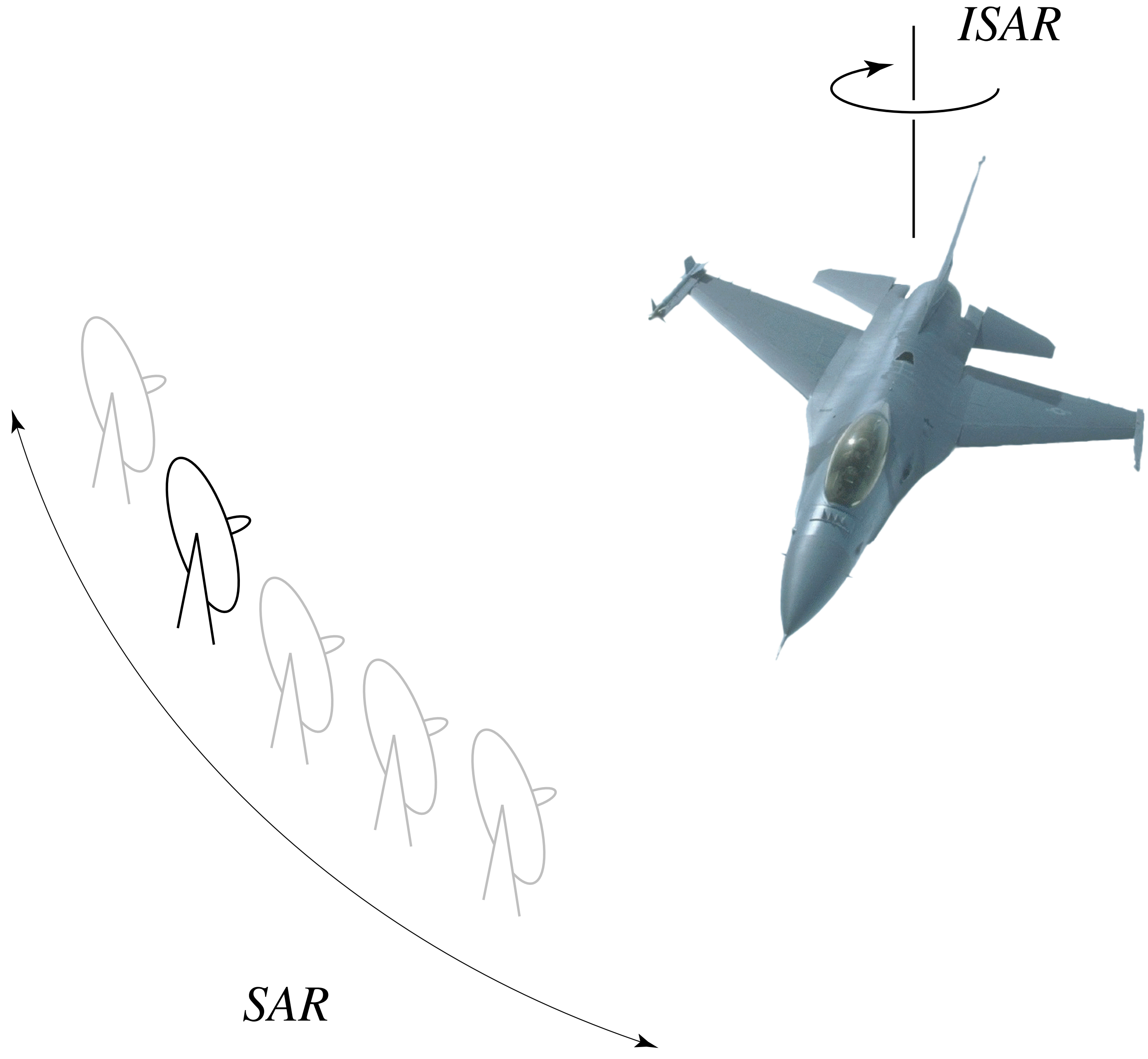


Inverse Synthetic Aperture Radar (ISAR)

- fixed radar, moving target (usually airplane or ship)
- transmit pulse train
- typical pulse length $\approx 10^{-4}$ sec,
  rotation rate $\Omega = 10^\circ/\text{sec} \approx 1/6$ R/sec, target radius $\approx 10\text{m} \Rightarrow$
  distance traveled during pulse $\approx 10^{-4}m \Rightarrow$ start-stop approximation
- for airborne target, measurements from nth pulse contain no reflections from earlier pulses
- take out translational motion via tracking and *range alignment*, leaving only rotational motion


Range alignment
$10 \log |\bar{\bar{\rho}}_\theta|^2$ (dB)
0.2
0.1
0.0
θ°
alignment peak
0
10
20
30
y meters

ISAR vs SAR



Mathematics of ISAR

at start of nth pulse, $V(\boldsymbol{x}) = q(\mathcal{O}(\theta_n)\boldsymbol{x})$ where $\mathcal{O}$ is an orthogonal matrix

For example: if radar is in plane $\perp$ to axis of rotation ("turntable geometry"),

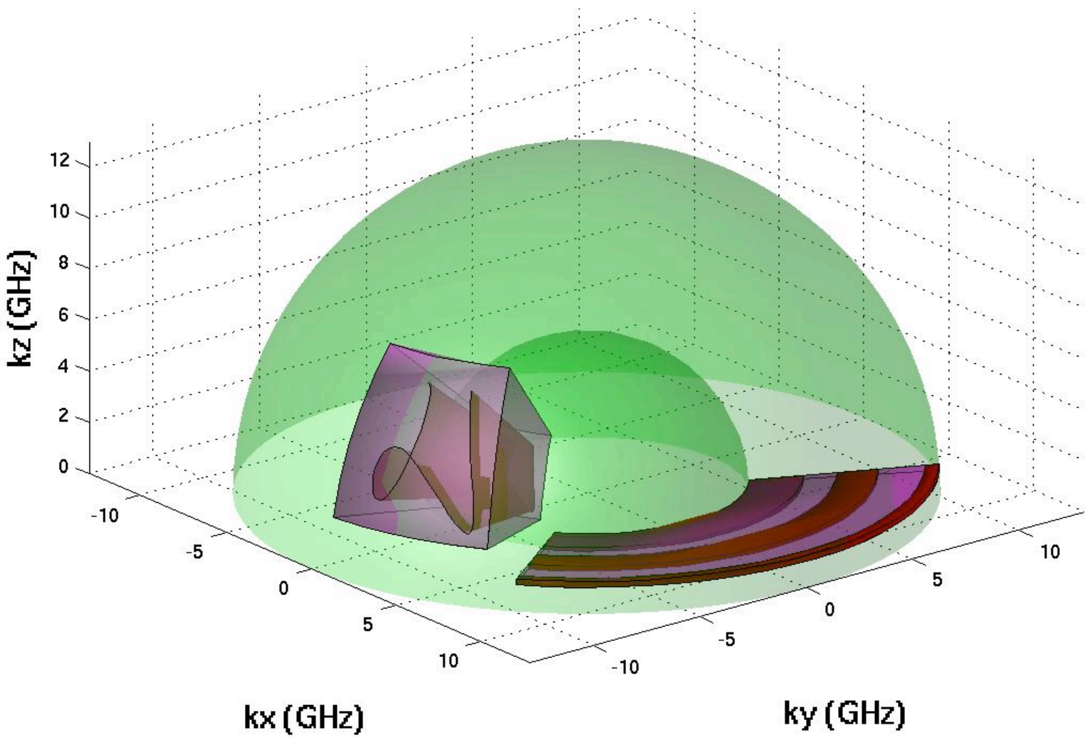
$$\mathcal{O}(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

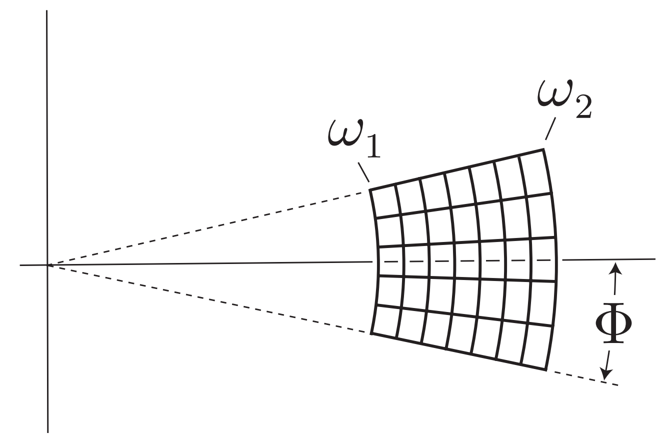
$$D_B(\omega, \theta_n) \approx \frac{e^{2ik|\boldsymbol{x}^0|}}{32\pi^2|\boldsymbol{x}^0|^2} k^2 |P(\omega)|^2 \int e^{2ik\widehat{\boldsymbol{x}^0}\cdot\boldsymbol{z}} q(\underbrace{\mathcal{O}(\theta_n)\boldsymbol{z}}_{\boldsymbol{y}}) d\boldsymbol{z}$$

$$(\text{ use } \boldsymbol{x}^0 \cdot \mathcal{O}^{-1}(\theta_n)\boldsymbol{y} = \mathcal{O}(\theta_n)\boldsymbol{x}^0 \cdot \boldsymbol{y})$$

$$= \frac{e^{2ik|\boldsymbol{x}^0|}}{32\pi^2|\boldsymbol{x}^0|^2} k^2 |P(\omega)|^2 \underbrace{\int e^{2ik\ \mathcal{O}(\theta_n)\widehat{\boldsymbol{x}^0}\cdot\boldsymbol{y}} V(\ \boldsymbol{y}) d\boldsymbol{y}}_{\mathcal{F}[q](2k\mathcal{O}(\theta_n)\widehat{\boldsymbol{x}^0})\ !}$$

Region in Fourier space where we have data



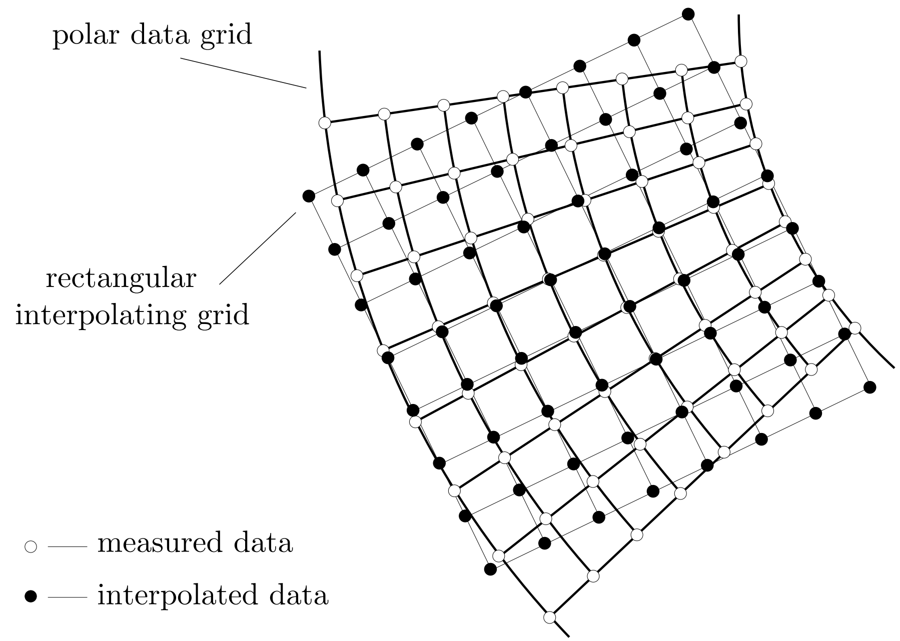


The backhoe data dome

Ω (turntable geometry)

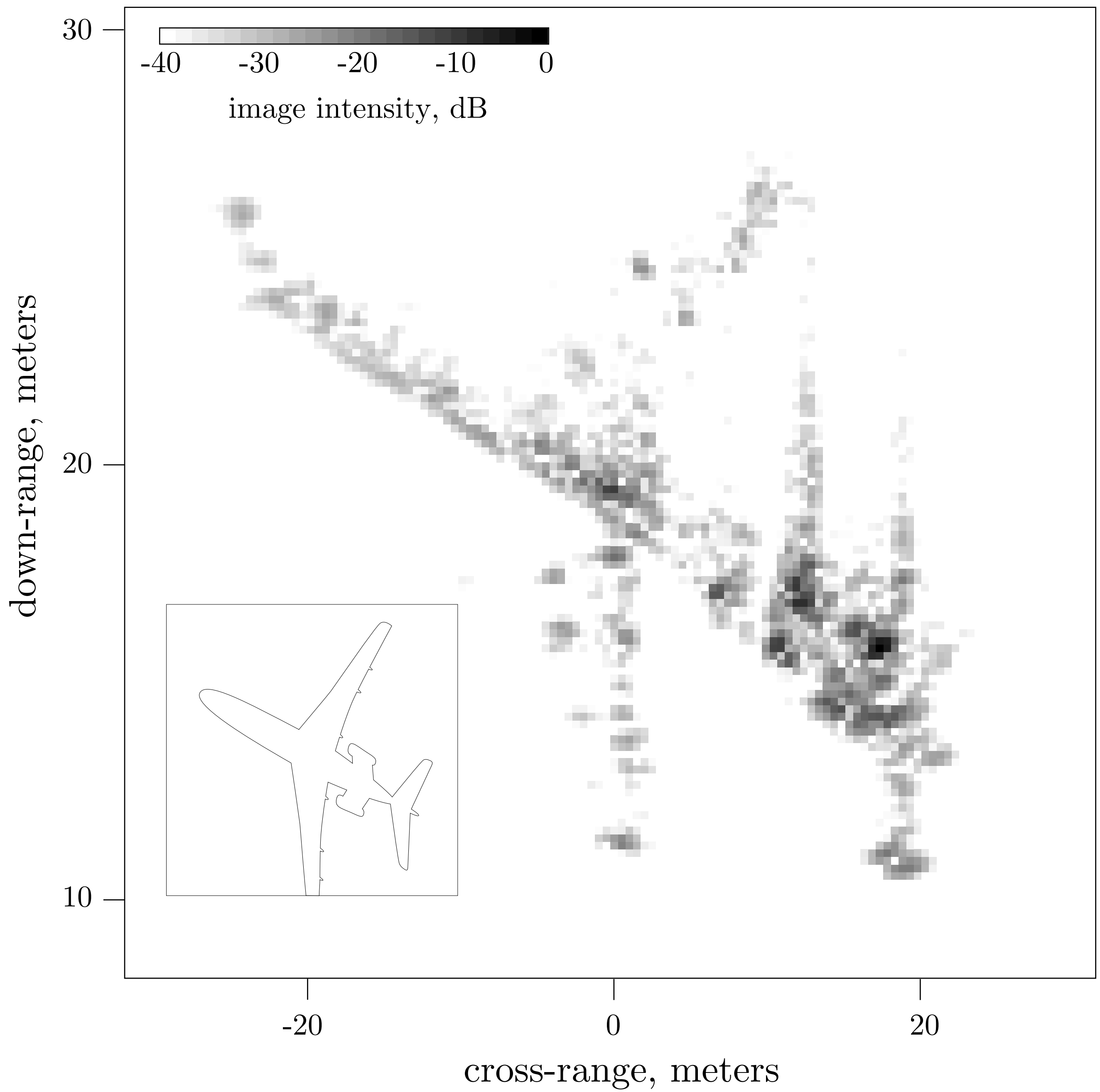
Polar Format Algorithm (PFA)

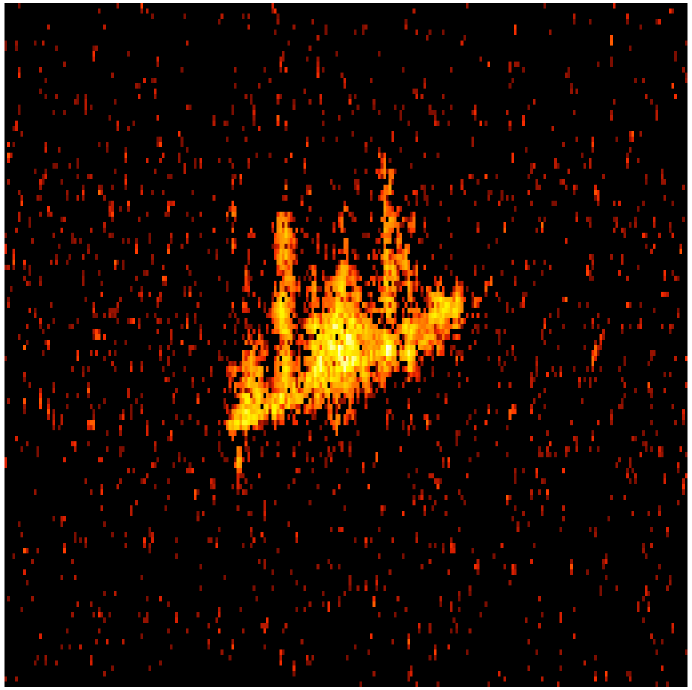
applies to turntable geometry, frequency-domain data

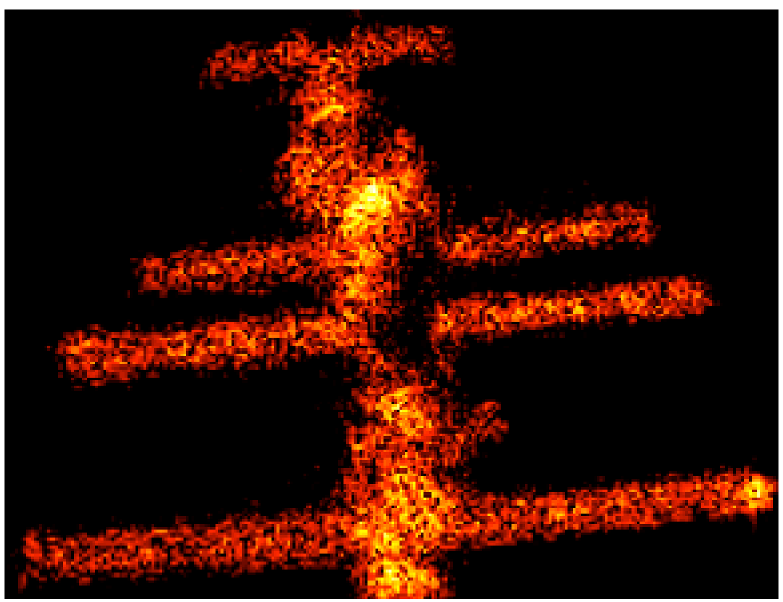


1. interpolate to rectangular grid
2. use 2D Fast Fourier Transform

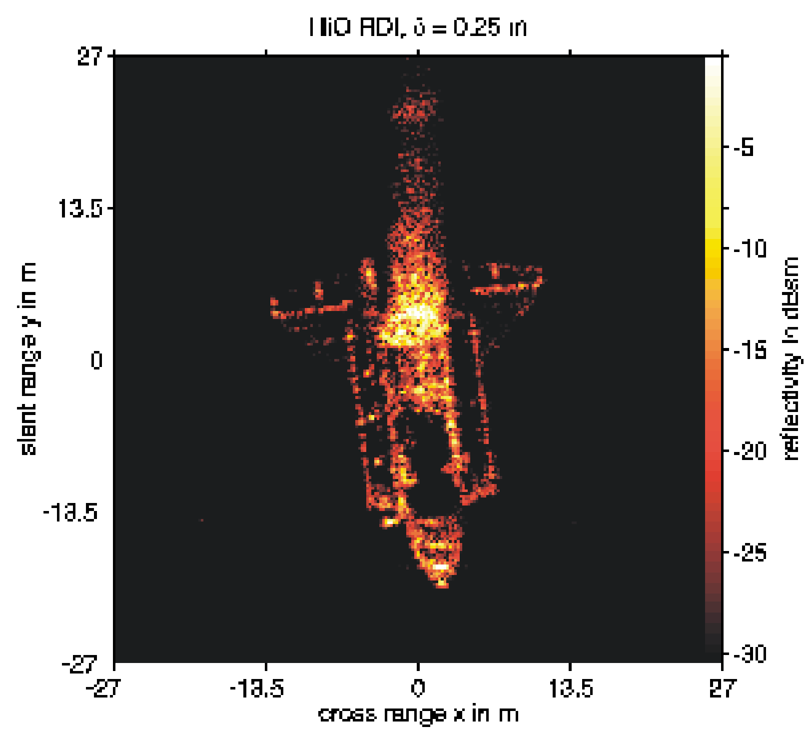
An ISAR image of a Boeing 727 taking off from LAX



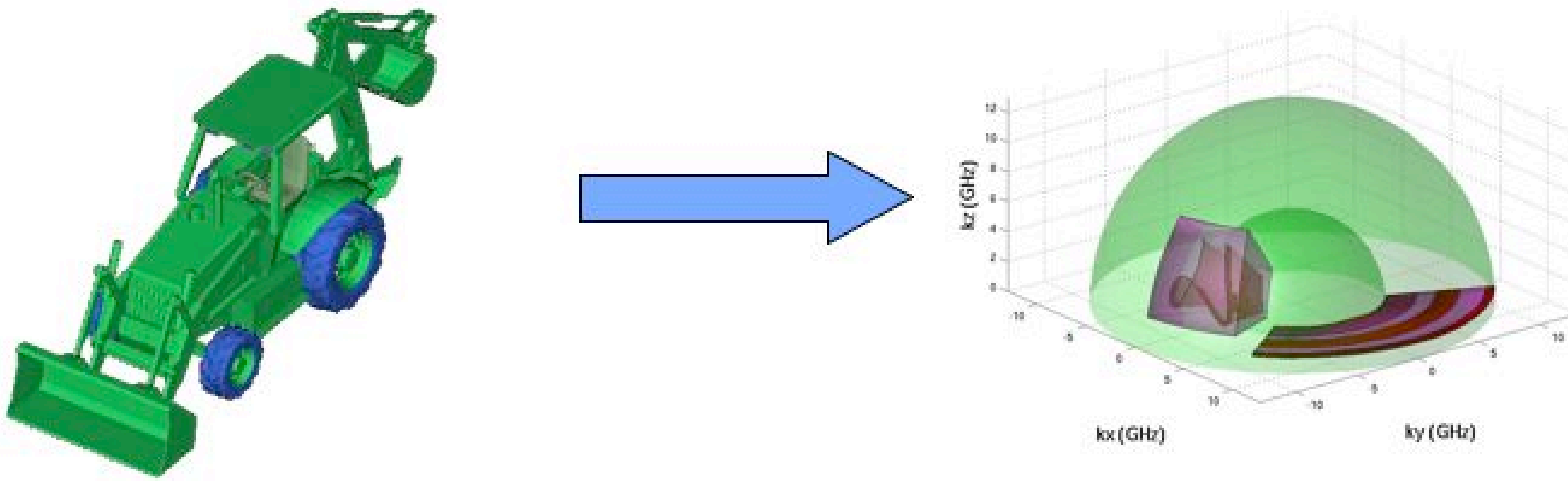


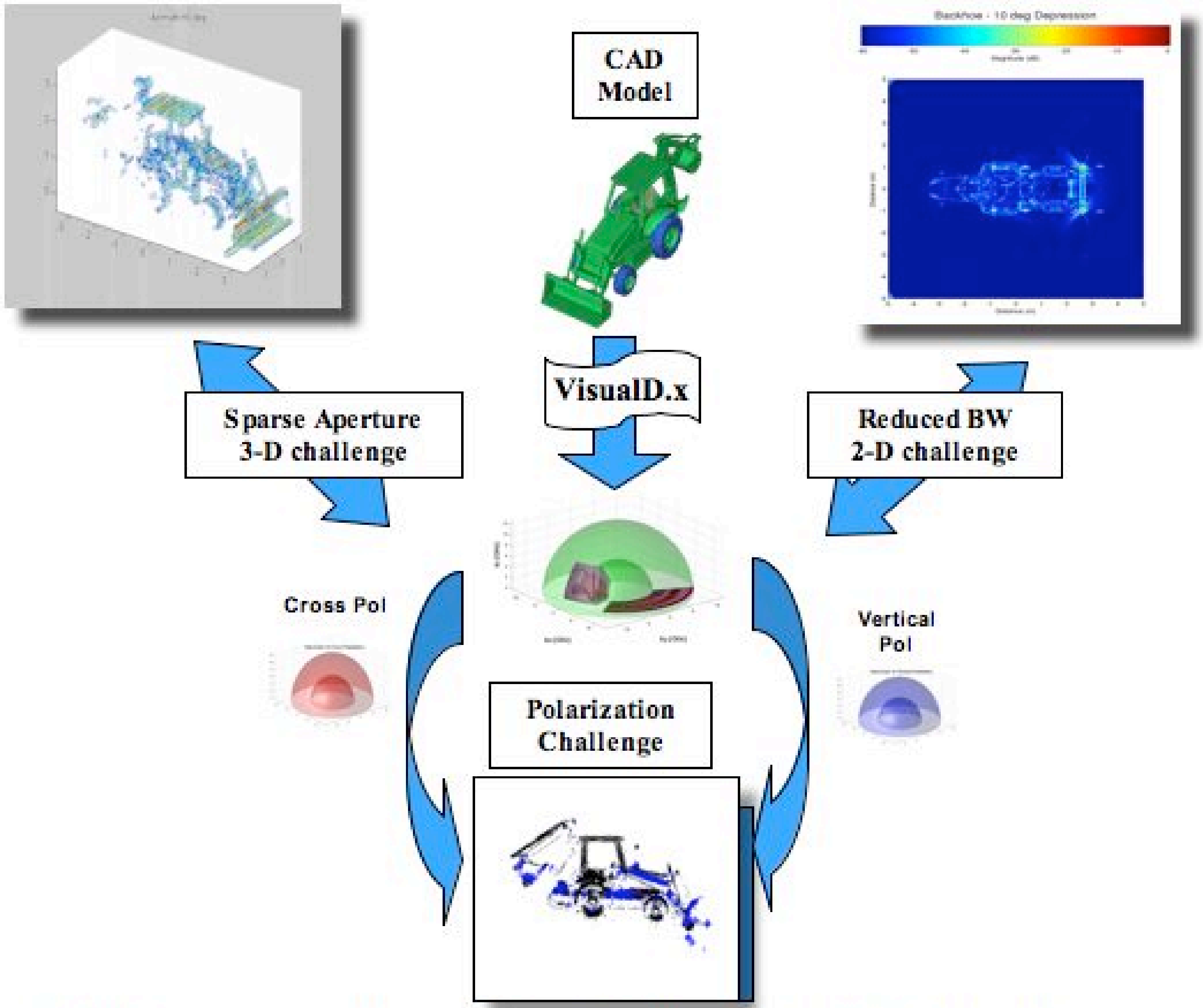


Mir


HiQ RDI, δ = 0.25 m
27
13.5
0
-13.5
-27
slant range y in m
-27
-13.5
0
13.5
27
cross range x in m
-5
-10
-15
-20
-25
-30
reflectivity in dBsm

Backhoe Sample Public Release VISUAL-D Challenge Problem





The material on this DVD was made possible with help from the following VISUAL-D DARPA seedling team members and their affiliations: Gerard Titi of DARPA; Edmund Zelnio, Ronald Dilsavor, Michael Minardi, Kiranmai Naidu, Tom Majumder, Steve Sawtelle and 2nd Lt. Bill Keichel of AFRL/SNAS; Rhonda Vickery and John Nehrbass of ASC/HPTI; Wayne Williams, Eric Keydel, Luke Lin, and Rajan Bhalla, of SAIC-Champaign and SAIC-Ann Arbor; Joe Burns of Altarum; Lee Potter, Randy Moses, and Emre Ertin of the Ohio State University; Donna Fitzgerald and Dave Doty of General Dynamics; and Leroy Gorham of MRC.

ISAR Resolution – Notation

Assume turntable geometry

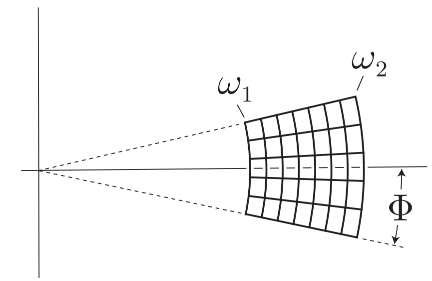
Take $\boldsymbol{x}^0 = (1,0,0)$, write $\hat{\boldsymbol{e}}_\theta = \mathcal{O}(\theta)\boldsymbol{x}^0$.

$$\tilde{D}(k,\theta) = \int e^{-2ik(y_1\cos\theta + y_2\sin\theta)} q(y_1,y_2,y_3)\,dy_1\,dy_2\,dy_3$$

$$\int e^{-2ik(y_1\cos\theta + y_2\sin\theta)} \underbrace{\int q(y_1,y_2,y_3)\,dy_3}_{\tilde{q}(y_1,y_2)}\,dy_1\,dy_2$$

get 2D image of $\tilde{q}$ = target projected onto plane containing radar

ISAR Resolution, continued



$$\tilde{D}(k,\theta) = \chi_\Omega(k\hat{\boldsymbol{e}}_\theta)\mathcal{F}[q](k\hat{\boldsymbol{e}}_\theta)$$

to form image, take 2D inverse Fourier transform

$$I(\mathbf{x}) = \int e^{i\mathbf{x}\cdot k\hat{\boldsymbol{e}}_\theta}\tilde{D}(k,\theta)kdkd\theta = \int e^{i\mathbf{x}\cdot k\hat{\boldsymbol{e}}_\theta}\int_\Omega e^{-i\mathbf{y}\cdot k\hat{\boldsymbol{e}}_\theta}\tilde{q}(\mathbf{y})d\mathbf{y}kdkd\theta$$

$$= \int \underbrace{\int_\Omega e^{i(\mathbf{x}-\mathbf{y})\cdot k\hat{\boldsymbol{e}}_\theta}kdkd\theta}_{K(\mathbf{x}-\mathbf{y})}\tilde{q}(\mathbf{y})d^2\mathbf{y}$$

$\uparrow$

point spread function (PSF), imaging kernel, ambiguity function

ISAR Resolution: analysis of PSF

$$K(\mathbf{z}) = \int_\Omega e^{i\mathbf{z}\cdot k\hat{\boldsymbol{e}}_\theta} k dk d\theta$$

Take $\hat{\boldsymbol{e}}_\theta = (\cos\theta, \sin\theta) \approx (1, \theta)$ (small-angle approx.)

$\mathbf{z} = r(\cos\psi, \sin\psi)$ $\qquad b = k_2 - k_1$ $\qquad k_c = \omega_c / c$

Down-range resolution

$$K(r, 0) \approx \frac{2\Phi}{i}\frac{d}{dr}\left[e^{ik_c r}\frac{b}{2}\mathrm{sinc}(br/2)\right]$$

$$K(r, 0) \approx b\omega_c\Phi\, e^{ik_c r}\mathrm{sinc}\frac{br}{2}$$

down-range resolution is $4\pi/b$

note oscillatory factor

Cross-range Resolution

$$K(0,r) \approx 2bk_c\Phi \,\mathrm{sinc}\left(\frac{br\Phi}{2}\right)\mathrm{sinc}(k_c r\Phi)$$

$k_c \gg b \;\Rightarrow$

$$K(r,0) \approx 2bk_c\Phi \,\mathrm{sinc}(k_c r\Phi)$$

cross-range resolution is $2\pi/(k_c\Phi)$

ISAR in the time domain

$$D_B(\omega,\theta_n) \approx \frac{e^{2ik|\boldsymbol{x}^0|}}{32\pi^2|\boldsymbol{x}^0|^2} k^2 |P(\omega)|^2 \underbrace{\int e^{2ik\ \mathcal{O}(\theta_n)\widehat{\boldsymbol{x}^0}\cdot\boldsymbol{y}} q(\ \boldsymbol{y}) d\boldsymbol{y}}_{\mathcal{F}[q](2k\mathcal{O}(\theta_n)\widehat{\boldsymbol{x}^0})}$$

$\downarrow$ Fourier transform

$$\eta_B(t,\theta_n) \propto \int\int e^{-i\omega(t-2|\boldsymbol{x}^0|/c+2\mathcal{O}(\theta_n)\widehat{\boldsymbol{x}^0}\cdot\boldsymbol{y}/c)} \omega^2 |P(\omega)|^2 d\omega\ q(\boldsymbol{y}) d\boldsymbol{y}$$

$$\eta_B\left(t+\frac{2|\boldsymbol{x}^0|}{c},\theta_n\right) = \int \underbrace{\int e^{-i\omega(t+\overbrace{2\mathcal{O}(\theta_n)\boldsymbol{x}^0\cdot\boldsymbol{y}/c}^{-\tau})} \omega^2 |P(\omega)|^2 d\omega}_{\int \delta(s-\tau) \underbrace{\int e^{-i\omega(t-s)}\omega^2|P(\omega)|^2 d\omega}_{\chi(t-s)}\, ds}\ q(\boldsymbol{y}) d\boldsymbol{y}$$

$$\eta_B\left(t + \frac{2|\boldsymbol{x}^0|}{c}, \theta_n\right) = \int \chi(t-s) \underbrace{\int \delta\left(s - \frac{2\mathcal{O}(\theta_n)\widehat{\boldsymbol{x}^0}}{c} \cdot \boldsymbol{y}\right) q(\boldsymbol{y}) d\boldsymbol{y}}_{\mathcal{R}[q]\left(s, \frac{2\mathcal{O}(\theta_n)\widehat{\boldsymbol{x}^0}}{c}\right)} \, ds$$

$$= \chi * \mathcal{R}[q]\left(\frac{2\mathcal{O}(\theta_n)\widehat{\boldsymbol{x}^0}}{c}\right)$$

where $\mathcal{R}[q](s, \widehat{\boldsymbol{\mu}}) = \int \delta(s - \widehat{\boldsymbol{\mu}} \cdot \boldsymbol{x}) q(\boldsymbol{x}) d\boldsymbol{x}$ is the *Radon transform*

For ISAR, use waveform so that $\chi \approx \delta$ (high range-resolution waveform)

Radon transform

$$\begin{aligned}\mathcal{R}_{\widehat{\boldsymbol{\mu}}}[q](s) = \mathcal{R}[q](s, \widehat{\boldsymbol{\mu}}) &= \int \delta(s - \widehat{\boldsymbol{\mu}} \cdot \boldsymbol{x}) q(\boldsymbol{x}) d\boldsymbol{x} = \int_{s=\widehat{\boldsymbol{\mu}}\cdot\boldsymbol{x}} q(\boldsymbol{x}) d^n \boldsymbol{x} \\ &= \int_{\boldsymbol{x}' \perp \widehat{\boldsymbol{\mu}}} q(s\widehat{\boldsymbol{\mu}} + \boldsymbol{x}') d^{n-1} \boldsymbol{x}'\end{aligned}$$

Properties

- Projection-slice theorem:

$$\mathcal{F}_{s\to\sigma}(\mathcal{R}_{\widehat{\boldsymbol{\mu}}}[f])(\sigma) = (2\pi)^{(n-1)/2} \mathcal{F}_n[f](\sigma \widehat{\boldsymbol{\mu}})$$

$\uparrow$ 1D Fourier transform $\qquad$ $\uparrow$ nD Fourier transform

here Fourier transforms have symmetric $1/\sqrt{2\pi}$ convention

- Filtered backprojection (FBP):

$$(\mathcal{R}^{\#}h) * f = \mathcal{R}^{\#}(h * \mathcal{R}[f])$$

$$\nearrow \qquad\qquad \uparrow$$

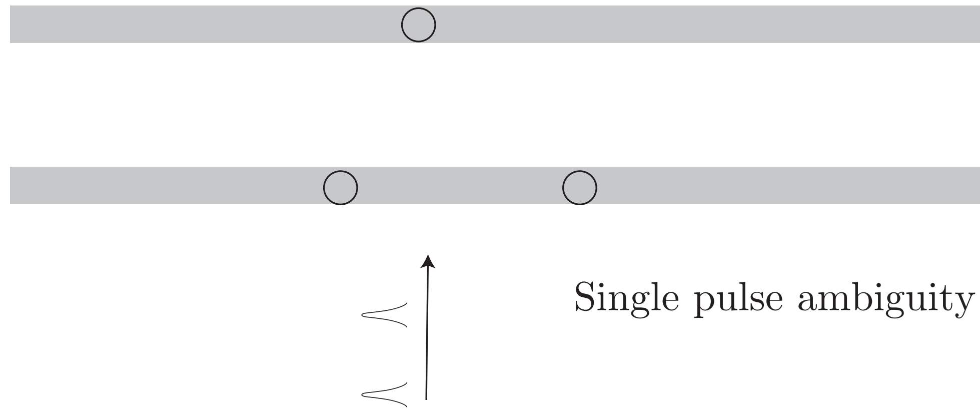
backprojection operator filter

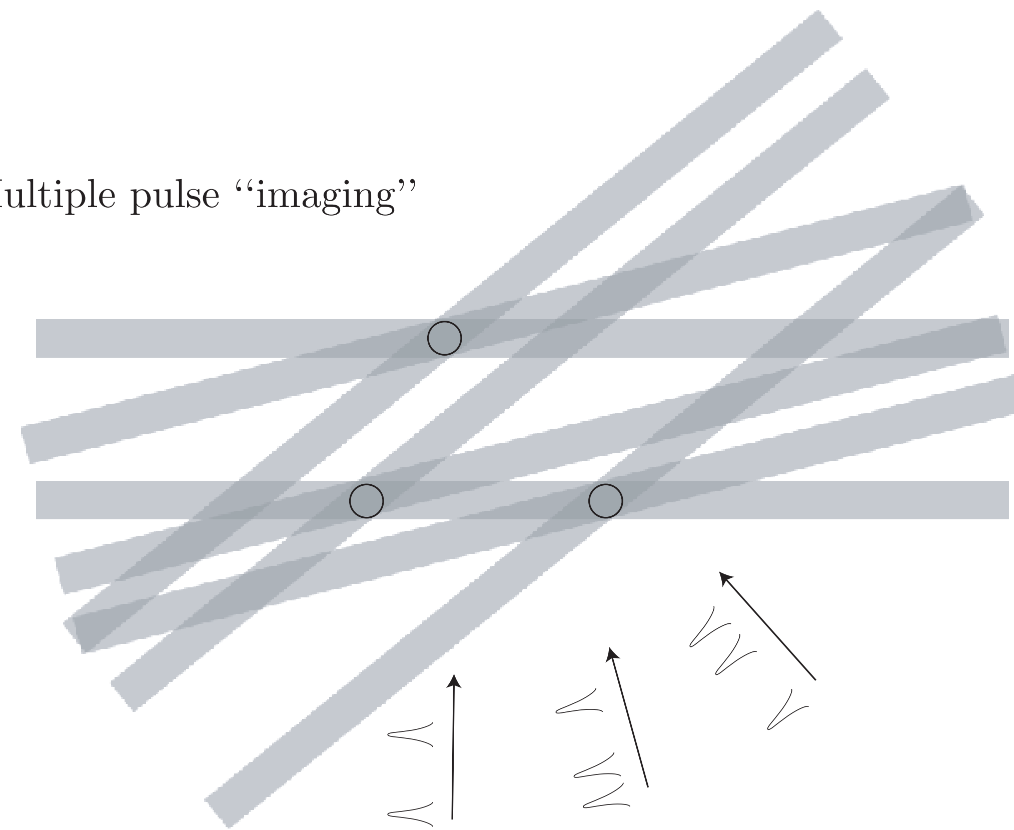
where $\mathcal{R}^{\#}$ operates on $h(s, \boldsymbol{\mu})$ via

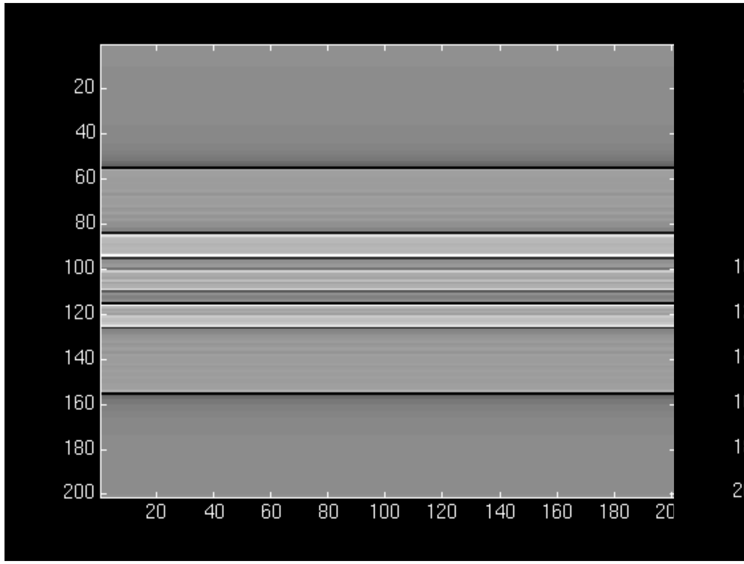
$$(R^{\#}h)(\boldsymbol{x}) = \int_{S^{n-1}} h(\boldsymbol{x} \cdot \widehat{\boldsymbol{\mu}}, \widehat{\boldsymbol{\mu}}) d\widehat{\boldsymbol{\mu}}$$

  - $\mathcal{R}^{\#}$ integrates over part of h corresponding to all lines through $\boldsymbol{x}$
  - $\mathcal{R}^{\#}$ is the formal adjoint of $\mathcal{R}$, i.e.,
    $\langle \mathcal{R}f, h \rangle_{L^2(\mathbb{R} \times S^{n-1})} = \langle f, \mathcal{R}^{\#}h \rangle_{L^2(\mathbb{R}^n)}$

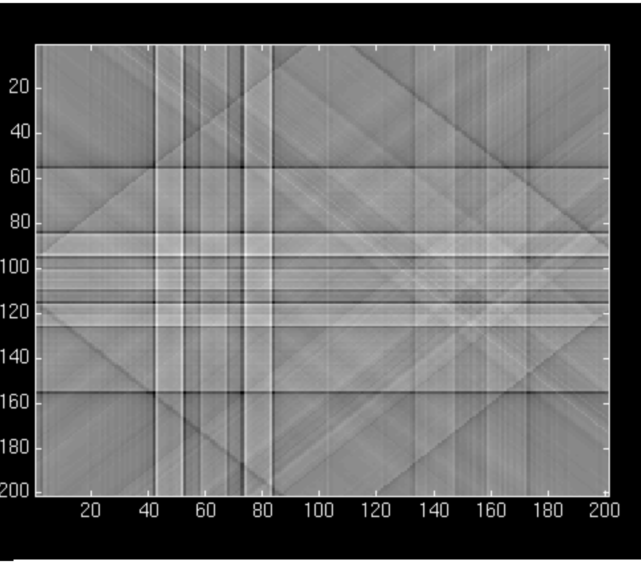
Radon transform



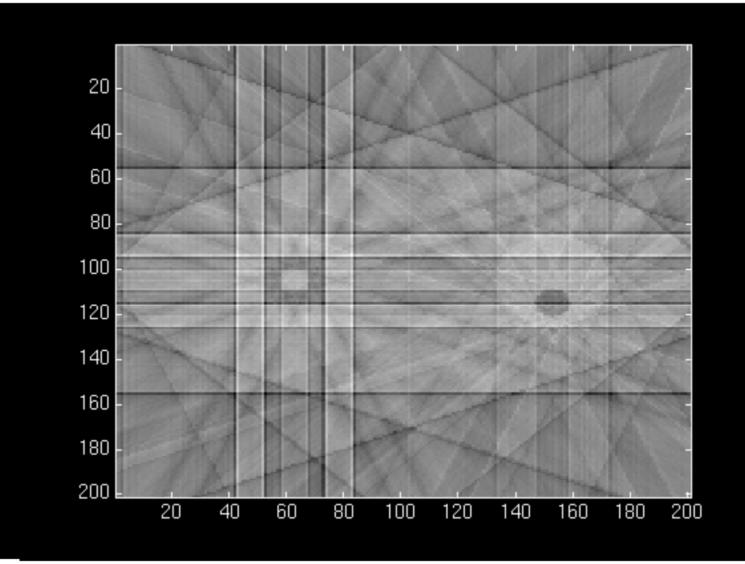




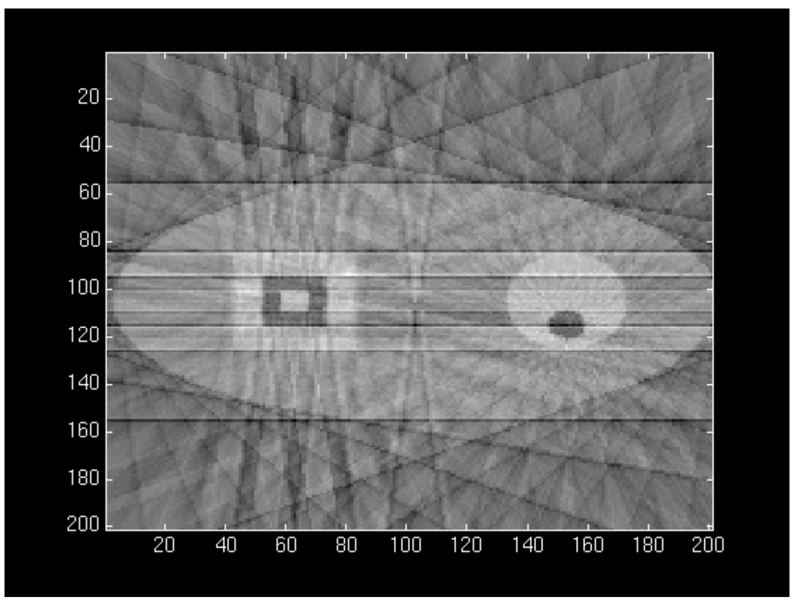
1 view



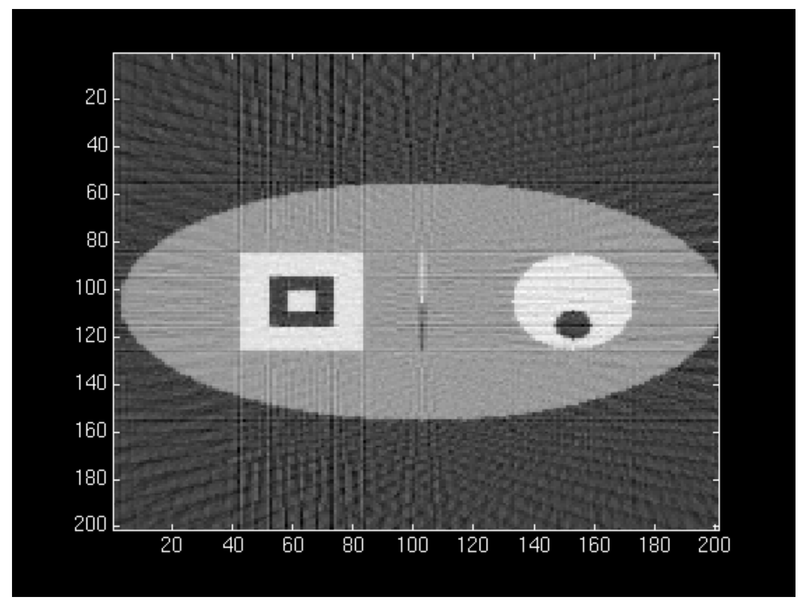
4 views



8 views



16 views



60 views

ISAR summary

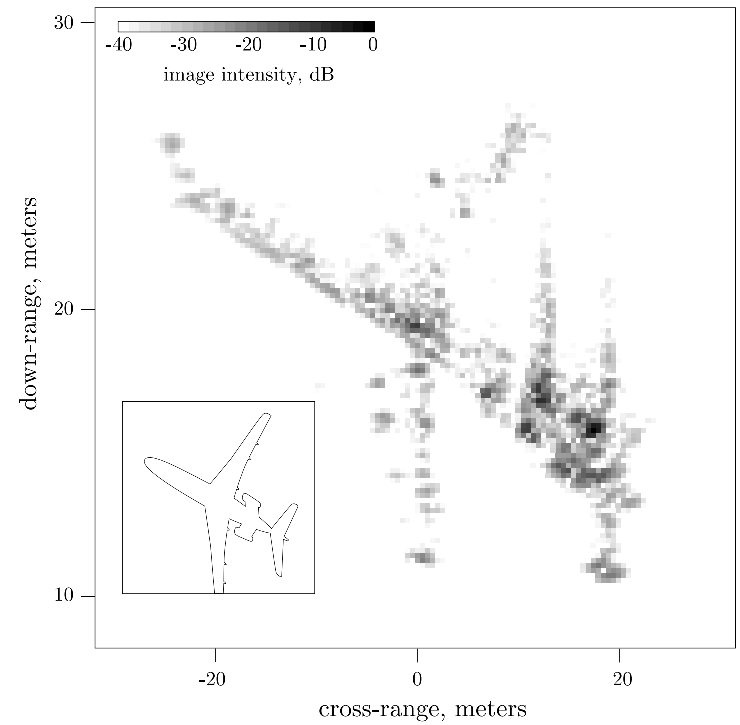
- use high range-resolution waveform (broad frequency band)
- time-domain formulation gives rise to Radon transform
  – invert via FBP or ...
- frequency-domain formulation gives Fourier transform of target on polar grid
  – interpolate to cartesian grid, use FFT
- can make images from an aperture of a few degrees because of high carrier frequency

ISAR is sometimes called range-doppler imaging

Doppler shift can be *inferred* from phase of successive measurements

Where more work is needed on ISAR

- model; Born approximation leaves out:
  - multiple scattering
  - shadowing
  - creeping waves
- find target/sensor motion
- 3D imaging
- template library look-up
- fast algorithms



Outline

1. introduction, history, frequency bands, dB, real-aperture imaging
2. radar systems: stepped-frequency systems, I/Q demodulation
3. 1D scattering by perfect conductor
4. receiver design, matched filtering
5. ambiguity function & its properties
6. range-doppler (unfocused) imaging
7. introduction to 3D scattering
8. ISAR
9. antenna theory
10. spotlight SAR
11. stripmap SAR
12. time permitting: deramp processing, Doppler from successive pulses