MODELS FOR AN ANESTHESIA BREATHING CIRCUIT

Paul E. Bigeleisen[†] and Margaret Cheney[††]

[†] Department of Anesthesia, St. Paul Ramsey Medical Center, St. Paul, MN 55101; permanent address: Department of Anesthesiology, University of Rochester, 601 Elmwood Avenue, Box 604, Rochester, NY 14642.
[††] Institute for Mathematics and Its Applications, University of Minnesota, Minneapolis, MN 55455; permanent address: Department of Mathematical Sciences, Rensselaer Polytechnic Institute, Troy, NY 12180; chenem@rpi.edu.

Address for correspondence: Margaret Cheney, Department of Mathematical Sciences, Amos Eaton Hall, Rensselaer Polytechnic Institute, 110 8th Street, Troy, NY 12180; chenem@rpi.edu; fax: (518) 276-4824.

ABSTRACT

This paper presents a mathematical model of a semi-closed anesthesia breathing circuit. The model comprises four perfect-mixing compartments (corresponding to the lung, the reservoir bag, and the two absorber cannisters), together with four tubes through which gas flows without mixing (corresponding to the inspiratory tube, the expiratory tube, the arm leading to the reservoir bag, and the anatomical dead space).

For simplicity, only the case of spontaneous breathing is considered, and only three species of gases are included, namely oxygen, nitrogen, and carbon dioxide.

The model's predictions are compared with experiment, and some of the parameters are adjusted to get a better fit. The resulting model is used to compare the effectiveness of different denitrogenation protocols.

Two appendices contain discussions of technical details and modifications needed to include other gases.

Keywords: denitrogenation, preoxygenation, ventilation, simulation, semi-closed, breathing circuit

1. INTRODUCTION

A critical time in the administration of anesthesia is the period called induction, when the patient has received a drug dose sufficient to paralyze the muscles of respiration but before receiving an endotracheal tube and before mechanical ventilation has commenced. So that the anesthetist will have the most time possible to accomplish intubation before the patient becomes hypoxic, the patient should start with the greatest possible oxygen reservoir. In particular, before induction, the nitrogen normally in the lungs should be replaced with oxygen. This process, called preoxygenation or denitrogenation, is accomplished a few minutes before the anesthetic drugs are given, by having the patient breathe pure oxygen through a mask connected to the breathing circuit. This circuit is also connected to the anesthesia vapor machine and is used to administer anesthetic gases after induction and intubation.

Under standard conditions, once the nitrogen level in the lung is reduced to less than about 15% of the total volume, the effect of nitrogen diffusion into the lungs from body stores becomes significant, and denitrogenation proceeds more slowly. Thus, although reducing the nitrogen concentration below 15% would be desirable, it would take more time. Moreover, reducing the nitrogen to 15% of the total volume, for a patient whose lung volume is 2 liters, gives the patient a lung oxygen volume of 1600ml. If the patient consumes 250ml of oxygen per minute, this 1600ml of oxygen provides the anesthetist an additional 5 to 6 minutes to begin mechanical ventilation. Thus we will refer to 15% as a "safe" or "clinically acceptable" level of lung nitrogen.

There has been considerable debate within the anesthesia community as to the most efficient method to accomplish denitrogenation. In 1981 Gold et al. [GDM], using blood gas samples, showed that four deep breaths produced an oxygen concentration in the blood equivalent to that produced by breathing normally for five minutes from the same circle system. Thereafter, it became standard practice to have the patient take four deep breaths prior to the induction of anesthesia. Further support for this practice was provided by other studies based on blood oxygen concentrations [MR, GNLMS, ND, NKTG]. The study [BGT] used on-line mass spectrometry to reach a similar conclusion. Other recent studies [GHF, CCCP, RSSB], however, which used pulse oximetry and in-line mass spectrometry, have suggested that four deep breaths do not denitrogenate as well as normal tidal breathing for three minutes. These authors suggest that inefficient denitrogenation is due to the fact that large breaths induce greater rebreathing of nitrogen from the circle system. With rebreathing, exhaled gas (which includes nitrogen) travels around the circle system and is re-inhaled. Inhaling more nitrogen slows down the elimination of nitrogen from the lungs.

There are a number of factors that might explain the contradictory nature of various denitrogenation studies. First, the paper [VMM] suggests a reason why studies based on blood oxygen saturation have shown that four deep breaths are as good as several minutes of normal breathing: because of hemoglobin's preferential affinity for oxygen, 100% oxygen saturation might be reached in the blood even though a significant volume of nitrogen remains in the lungs. For safety during induction, it is the lung nitrogen volume that is significant.

This would not explain those studies that compared the effects of different denitrogenation protocols on blood plasma oxygen concentrations. However, blood plasma concentrations may be nonuniform throughout the body until an equilibrium is reached, which could take several minutes. Thus blood plasma concentrations measured after four deep breaths might not reflect nitrogen levels in the lungs.

In order to determine the most efficient method of denitrogenation, we developed a model of the circle system and compared its predictions to experimental data we collected. We adjusted some of the parameters in the model to obtain a good fit. The model is explained in section 2; section 3 contains a summary of the computational methods we used to implement the model as well as a description of our methods for collecting experimental data; section 4 discusses our computational and experimental results; section 5 outlines the conclusions that can be drawn from this study. The appendices contain further technical details.

This model may be useful for other purposes as well. It can be used as well for the cases of assisted or controlled ventilation and could be modified to track other gases (see Appendix 2). A model of this type could perhaps be incorporated into simulators that presently model physiological uptake in the body, such as SIM I [DA], Anesthesia Simulator Consultant (ASC) [SO], CASE [GD] CAE, Gasman, BODY, METI, Loral, and others [GG]. These simulators model in detail the uptake, distribution, metabolism, and exhalation of gases in the body, but do not take into account the detailed flow of these gases through the anesthesia machine.

2. THE MODEL

Assumptions.

Some of our assumptions are based on the observation that our measurements would necessarily be limited in their precision. Consequently, our model ignores the effects of diffusion of nitrogen, carbon dioxide, and water vapor from body stores.

We assume that the pressure and temperature in the system is constant, so that fixed volumes of gas all contain the same number of molecules.

We ignore effects of body temperature. We assume that the patient is using glucose rather than fat for fuel, so that the volume of oxygen consumed is equal to the volume of carbon dioxide produced.

Throughout we assume that the fresh gas flow is composed entirely of oxygen.

We have also made some simplifying assumptions about the gas dynamics. In particular, we assume that gas mixes perfectly in the lung, reservoir bag, and absorber cannisters, and that these compartments are connected by tubes in which the gas flows but does not mix. We assume that carbon dioxide is absorbed immediately upon entering the first absorber cannister. We have ignored the 300 ml/min drawn off by the gas analyzer.

The Breathing Waveform.

Because we are interested in the process of preoxygenation, during which the breathing is under control of the patient, we assume that the lung volume $L(t)$ changes periodically in a known way. In particular, we assume throughout that the lung volume is given by

$$L(t) = FRC + T(1 - \cos(2\pi rt))/2, \tag{2.1}$$

where FRC denotes the functional residual capacity, T the tidal volume, and r the respiration rate. Thus the lung volume oscillates between FRC and $FRC + T$. In this paper, we assume that T and r are constant. In general, they could be functions of time, but this would make the computations of delays in Appendix 1 more complicated.

The rate of volume increase is the time derivative (denoted by dot) of (2.1):

$$\dot{L}(t) = Tr\pi \sin(2\pi rt). \tag{2.2}$$

Except under circumstances explained in Appendix 2, this is the flow through the trachea. Periods when $\dot{L}(t)$ is greater than zero are periods of inhalation; periods when it is less than zero are exhalations.

The Patterns of Flow.

Figures 1, 2, and 3 show the gas flow in the semi-closed circle system. In this system, the pattern of gas flow is different during inhalation and exhalation. During exhalation (Figure 3), exhaled gas from the lungs flows through the anatomical dead space, past the expiratory valve and through the expiratory tube to the "arm junction". This arm junction is where the expiratory tube joins the entrance to the absorber and the arm leading to the reservoir bag. At this junction, the exhaled gas joins the gas which has been pushed through the absorber by the fresh gas flow. The gas at the arm junction passes through the arm and enters the reservoir bag, unless the bag is full, in which case the gas at the arm junction flows out into the scavenging system.

During inhalation, the pattern of gas flow depends on the relative magnitudes of the inhalation flow rate and the fresh gas flow rate. During the short period when the inhalation rate is less than the fresh gas flow (Figure 1), some of the fresh gas flows directly through the inspiratory tube, past the inspiratory valve, through the anatomical dead space, and into the lungs. The rest of the fresh gas flows through the absorber, through the bag arm, and into the reservoir bag, until the bag is full, when gas at the arm junction exits to the scavenging system. The more typical inhalation flow pattern occurs when the inhalation rate is greater than the fresh gas flow (Figure 2). In this case, gas flows out of the reservoir bag, through the arm, and through the absorber. After it passes out of the absorber, it is joined by fresh gas, and the mixture flows through the inspiratory tube, past the inspiratory valve, through the anatomical dead space, and into the lungs.

To model this system, we track the gas concentrations in each of four compartments: the lung, the reservoir bag, and the two absorber cannisters. In addition, we track only three gases: oxygen, nitrogen, and carbon dioxide. Finally, we ignore the effects of nitrogen and carbon dioxide diffusion out of body tissues.

Notation.

We denote the lung volumes of oxygen, nitrogen, and carbon dioxide by L_{O_2}, L_{N_2}, and L_{CO_2}, respectively. We denote the volumes of these three gases in the reservoir bag by B_{O_2}, B_{N_2}, and B_{CO_2}, and the volumes of oxygen and nitrogen in the absorber cannisters by $A^1_{O_2}$, $A^1_{N_2}$, $A^2_{O_2}$, and $A^2_{N_2}$, respectively.

In the lungs, oxygen is consumed and carbon dioxide produced. Because we assume that the patient is using glucose rather than fat for fuel, the volume of carbon dioxide produced is equal to the volume of oxygen consumed. This process occurs at the constant metabolic rate M.

We assume that the carbon dioxide is absorbed immediately as it enters the first absorber cannister, so that $A^1_{O_2} + A^1_{N_2} = A$, where A is the (constant) volume of each absorber cannister. Similarly, $A^2_{O_2} + A^2_{N_2} = A$. In each cannister, we therefore need only track A_{O_2}.

The model is easier to understand if we also introduce notation for the gas concentrations at the three three-way junctions in the system; namely the "fresh gas junction", where the fresh gas inlet connects to the absorber and the inspiratory tube; the mouthpiece, where the inspiratory and expiratory tubes join the anatomical dead space; and the arm junction, where the expiratory tube and entrance to the absorber join the arm leading to the reservoir bag. We denote the gas concentrations at the fresh gas junction by $\psi_{O_2}(t)$, $\psi_{N_2}(t)$, and $\psi_{CO_2}(t)$, the concentrations at the mouthpiece by $\phi_{O_2}(t)$, $\phi_{N_2}(t)$, and $\phi_{CO_2}(t)$, and the concentrations at the arm junction by $\xi_{O_2}(t)$, $\xi_{N_2}(t)$, and $\xi_{CO_2}(t)$.

The circle system contains tubes in which we assume the gas flows without mixing. In such tubes, the gas that exits is the same gas that entered at an earlier time. In other words, these tubes appear in the model as delays. How these delays can be computed is discussed in Appendix 1. Here we simply note that the entering time for gas exiting at time t depends on the tube volume and the flow history for that particular tube. For an exiting time t, we will write $\tau_I(t)$ for the entering time for the inspiratory tube, $\tau_E(t)$ for the expiratory tube, $\tau_D(t)$ for the dead space, and $\tau_A(t)$ for the bag arm. We denote the volumes of these tubes by V_I, V_E, V_D, and V_A, respectively.

We have seen that there are three different flow patterns, one for exhalation when $\dot{L}(t)$ is negative, one when the inhalation rate $\dot{L}(t)$ is greater than the fresh gas flow F, and one when it is less. Consequently many of the equations take different forms during these three types of time periods.

The Lung Equations.

During exhalations (times when $\dot{L}(t) < 0$) we assume that the gas leaving the lung is perfectly mixed, and that oxygen is converted into carbon dioxide at the rate M:

$$\dot{L}_{O_2}(t) = -M + \frac{L_{O_2}}{L}(t)\dot{L}(t) \tag{2.3}$$

$$\dot{L}_{N_2}(t) = \frac{L_{N_2}}{L}(t)\dot{L}(t) \tag{2.4}$$

$$\dot{L}_{CO_2}(t) = M + \frac{L_{CO_2}}{L}(t)\dot{L}(t). \tag{2.5}$$

Inhalations are more complicated, because of the dead space. As an inhalation begins, the first gas breathed into the lung was the last gas exhaled. If we denote the beginning of the inhalation by t^I, then gas inhaled at time t was the gas exhaled at time $t^I - (t - t^I)$. This gas is alveolar gas, which is assumed perfectly mixed. After the dead space gas has all passed into the lungs, the next gas to enter is gas that a few moments before was at the mouthpiece. It has been delayed by the amount of time it took to traverse the dead space;

thus it is gas that was at the mouthpiece at time $\tau_D(t)$. The lung inhalation equations thus take the form

$$\dot{L}_{O_2}(t) = \begin{cases} \frac{L_{O_2}}{L}(t^I - (t - t^I))\dot{L}(t) - M & L(t) - FRC < V_D \\ \phi_{O_2}(\tau_D(t))\dot{L}(t) - M & L(t) - FRC \geq V_D \end{cases} \tag{2.6}$$

$$\dot{L}_{N_2}(t) = \begin{cases} \frac{L_{N_2}}{L}(t^I - (t - t^I))\dot{L}(t) & L(t) - FRC < V_D \\ \phi_{N_2}(\tau_D(t))\dot{L}(t) & L(t) - FRC \geq V_D \end{cases} \tag{2.7}$$

$$\dot{L}_{CO_2}(t) = \begin{cases} \frac{L_{CO_2}}{L}(t^I - (t - t^I))\dot{L}(t) + M & L(t) - FRC < V_D \\ \phi_{CO_2}(\tau_D(t))\dot{L}(t) + M & L(t) - FRC \geq V_D, \end{cases} \tag{2.8}$$

where $L(t) - FRC$ is the volume of gas inhaled since the beginning of the present inhalation.

Equations for the Reservoir Bag.

The flow $f(t)$ in the arm leading to the reservoir bag is determined by the difference between the fresh gas flow F and the flow $\dot{L}(t)$ induced by breathing. When $F - \dot{L}(t) > 0$, gas flows into the bag at the rate $f(t) = F - \dot{L}(t)$ until the bag is full, at which point the gas exits through the scavenging system; when $F - \dot{L}(t) < 0$, gas flows out of the bag, until the bag is empty. We use t_s to denote the time at which the flow out of the bag begins and t_q for the time at which it ends.

We consider first the high-flow phase of inhalation, when gas is sucked out of the reservoir bag. The gas flowing out of the bag is assumed to be perfectly mixed. However, the rate at which gas flows out of the bag is not obvious, because part of the flow, namely the carbon dioxide, simply disappears in the absorber on its way to the lung. Since the positive quantity $-f(t)$ denotes the gas flow out of the bag, we have for the bag equations

$$\dot{B}_{O_2}(t) = \begin{cases} \frac{B_{O_2}}{B}(t)f(t) & \text{if } B(t) > 0 \\ 0 & \text{if } B(t) \leq 0 \end{cases} \tag{2.9}$$

$$\dot{B}_{N_2}(t) = \begin{cases} \frac{B_{N_2}}{B}(t)f(t) & \text{if } B(t) > 0 \\ 0 & \text{if } B(t) \leq 0 \end{cases} \tag{2.10}$$

$$\dot{B}_{CO_2}(t) = \begin{cases} \frac{B_{CO_2}}{B}(t)f(t) & \text{if } B(t) > 0 \\ 0 & \text{if } B(t) \leq 0. \end{cases} \tag{2.11}$$

Here $B(t)$ is the total volume of the reservoir bag:

$$B(t) = B_{O_2}(t) + B_{N_2}(t) + B_{CO_2}(t). \tag{2.12}$$

To determine the flow $f(t)$ when $F - \dot{L}(t) < 0$, we note that the total flow out of the absorber must be enough to make up the difference between the inhalation rate and the fresh gas flow rate. In other words, the flow out of the absorber is $\dot{L} - F$, and since the absorber has constant volume, the flow into the absorber from the reservoir bag must be the same. The flow into the absorber is gas from the arm junction, with concentrations denoted by ξ. Because we assume for simplicity that the carbon dioxide disappears as it enters the absorber, this entering flow is

$$\xi_{O_2}(-f) + \xi_{N_2}(-f) = \dot{L} - F. \tag{2.13}$$

Solving this equation for the bag outflow $(-f)$, using the fact that the concentrations ξ sum to one, and combining the result with the known flows when $F - \dot{L} > 0$, we have

$$f(t) = \begin{cases} F - \dot{L}(t) & \text{when } F - \dot{L}(t) > 0 \text{ and } B(t) < C \\ 0 & \text{when } F - \dot{L}(t) > 0 \text{ and } B(t) \geq C \\ \frac{F-\dot{L}(t)}{1-\xi_{CO_2}(t)} & \text{when } F - \dot{L}(t) < 0 \\ 0 & \text{when } F - \dot{L}(t) < 0 \text{ and } B(t) \leq 0. \end{cases} \tag{2.14}$$

Here, as before, B is the volume of the reservoir bag; C is its capacity.

When gas is flowing into the bag, the amount that has entered during the present cycle is $V(t) = \int_{t_q}^{t} (F - \dot{L}(t'))dt'$. If this volume is less than the volume of the arm, then the gas entering the bag is the same gas that recently exited the bag. The time at which it exited was $\tau_B(t)$, where $\int_{\tau_B(t)}^{t_q} f(t')dt' = V(t)$. Thus when $F - \dot{L} > 0$ and $V(t) < V_A$, the bag equations are

$$\dot{B}_{O_2}(t) = \frac{B_{O_2}}{B}(\tau_B(t)) \tag{2.15}$$

$$\dot{B}_{N_2}(t) = \frac{B_{N_2}}{B}(\tau_B(t)) \tag{2.16}$$

$$\dot{B}_{CO_2}(t) = \frac{B_{CO_2}}{B}(\tau_B(t)) \tag{2.17}$$

After the gas present in the arm is exhausted, the gas entering the bag is gas that was at the arm junction at the delayed time $\tau_A(t)$. The bag equations during time intervals for which $F - \dot{L} > 0$ and $V(t) \geq V_A$ are thus

$$\dot{B}_{O_2}(t) = \begin{cases} \xi_{O_2}(\tau_A(t))(F - \dot{L}(t)) & \text{if } B(t) < C \\ 0 & \text{if } B(t) \geq C \end{cases} \tag{2.18}$$

$$\dot{B}_{N_2}(t) = \begin{cases} \xi_{N_2}(\tau_A(t))(F - \dot{L}(t)) & \text{if } B(t) < C \\ 0 & \text{if } B(t) \geq C \end{cases} \tag{2.19}$$

$$\dot{B}_{CO_2}(t) = \begin{cases} \xi_{CO_2}(\tau_A)(F - \dot{L}(t)) & \text{if } B(t) < C \\ 0 & \text{if } B(t) \geq C. \end{cases} \tag{2.20}$$

Equations for the Absorber.

When $F - \dot{L} < 0$, gas flows out of the reservoir bag at the rate $-f$ of (2.14) and flows into the first absorber cannister. Perfectly mixed gas flows out of the first cannister and into the second; perfectly mixed gas flows out of the second cannister. The equations for the oxygen in the absorber cannisters are then

$$\dot{A}^1_{O_2} = \Big(- \frac{A^1_{O_2}}{A} + \frac{\xi_{O_2}}{1 - \xi_{CO_2}} \Big)(\dot{L} - F) \tag{2.21}$$

$$\dot{A}^2_{O_2} = \Big(-\frac{A^2_{O_2}}{A} + \frac{A^1_{O_2}}{A} \Big)(\dot{L} - F). \tag{2.22}$$

When $F - \dot{L} > 0$, fresh gas flows into the first absorber cannister, mixed gas flows out and into the second cannister, from which mixed gas flows out. The equations are thus

$$\dot{A}^1_{O_2} = \Big(\frac{A^2_{O_2}}{A} - \frac{A^1_{O_2}}{A} \Big)(\dot{L} - F). \tag{2.22}$$

$$\dot{A}^2_{O_2} = \Big(1 - \frac{A^2_{O_2}}{A} \Big)(\dot{L} - F). \tag{2.22}$$

The reason we included both absorber cannisters was to slow down the rate at which exhaled gases traverse the model circuit and reappear at the mouthpiece. This is not a big effect, but the inclusion of both separate absorber cannisters does not significantly complicate the model.

This completes the set of equations for the compartments, except for the determination of the concentrations at the junctions.

Concentrations at the Fresh Gas Junction.

The concentrations at the fresh gas inlet are not too difficult to determine. During exhalations, there is no flow through the inspiratory tube, and the fresh gas junction is flooded with fresh gas, which flows into the absorber. During the low-flow phase of inhalation, some of the fresh gas goes into the inspiratory tube and the rest goes into the absorber. It is only during the high-flow phase of inhalation that the flow through the absorber reverses, so that gas from the absorber appears at the fresh gas junction. The junction concentrations are thus

$$\psi_{O_2}(t) = \begin{cases} 1 & \dot{L}(t) < F \\ \frac{F}{\dot{L}(t)} + \frac{A^2_{O_2}(t)}{A}\big(1 - \frac{F}{\dot{L}(t)}\big) & F < \dot{L}(t) \end{cases} \tag{2.23}$$

$$\psi_{N_2}(t) = \begin{cases} 0 & \dot{L}(t) < F \\ \frac{F}{\dot{L}(t)} + \big(1 - \frac{A^2_{O_2}(t)}{A}\big)\big(1 - \frac{F}{\dot{L}(t)}\big) & F < \dot{L}(t) \end{cases} \tag{2.24}$$

$$\psi_{CO_2}(t) = 0. \tag{2.25}$$

The Mouthpiece Concentrations.

The gas at the mouthpiece originates in different parts of the system, depending on the present flow pattern. During inhalation, gas at the mouthpiece at time t comes from the fresh gas junction at the delayed time $\tau_I(t)$. During exhalation, gas can originate in two different places. In particular, suppose an exhalation begins at time t^E. During the first part of the exhalation, the gas appearing at the mouthpiece is gas that entered the dead space most recently. If a time $t - t^E$ has elapsed since the beginning of the exhalation, then the gas appearing at the mouthpiece was gas that passed the mouthpiece before, at time $t^E-(t-t^E)$. This gas entered the dead space after traveling through the inspiratory tube, where it was delayed by the amount of time it took to traverse the tube. The time at which it entered the inspiratory tube was $\tau_I(t^E-(t-t^E))$. When this gas entered the inspiratory tube, it was at the junction between the fresh gas inlet and the absorber outlet. In other words the composition of this gas is described by the functions ψ at time $\tau_I(t^E-(t-t^E))$. After this initial part of the exhalation, the dead space gas is exhausted, and gas from the lung appears. This gas has simply traveled through a tube whose volume is the dead space volume V_D. The mouthpiece concentrations are thus

$$\phi_{O_2}(t)=\begin{cases}\psi_{O_2}(\tau_I(t)) & \dot{L}(t)>0\\ \psi_{O_2}\big(\tau_I(t^E-(t-t^E))\big) & \dot{L}(t)<0 \text{ and } L(t^E)-L(t)<V_D\\ \frac{L_{O_2}}{L}(\tau_D(t)) & \dot{L}(t)<0 \text{ and } L(t^E)-L(t)\geq V_D\end{cases} \tag{2.26}$$

$$\phi_{N_2}(t)=\begin{cases}\psi_{N_2}(\tau_I(t)) & \dot{L}(t)>0\\ \psi_{N_2}\big(\tau_I(t^E-(t-t^E))\big) & \dot{L}(t)<0 \text{ and } L(t^E)-L(t)<V_D\\ \frac{L_{N_2}}{L}(\tau_D(t)) & \dot{L}(t)<0 \text{ and } L(t^E)-L(t)\geq V_D\end{cases} \tag{2.27}$$

$$\phi_{CO_2}(t)=\begin{cases}0 & \dot{L}(t)>0\\ 0 & \dot{L}(t)<0 \text{ and } L(t^E)-L(t)<V_D\\ \frac{L_{CO_2}}{L}(\tau_D(t)) & \dot{L}(t)<0 \text{ and } L(t^E)-L(t)\geq V_D\end{cases} \tag{2.28}$$

Concentrations at the Arm Junction.

The pattern of flow determines which gas appears at the arm junction. During exhalations, the gas is a combination of absorber gas and gas that is exiting the expiratory tube. This exiting gas was at the mouthpiece at a delayed time. During inhalations, on the other hand, gas at the arm junction can come from either of two sources. During the low-flow part of an inhalation, gas at the arm junction is gas from the absorber. During the high-flow part of an inhalation, gas at the arm junction comes from the arm. Suppose the gas starts to flow out of the arm at time t_s. The first gas to appear is gas that previously appeared at the arm junction as it flowed into the arm. We denote the time of its previous appearance by $\tau_a(t)$. After all the gas stored in the arm has been exhausted, gas from the reservoir bag appears. The arm junction concentrations are thus

$$\xi_{O_2}(t)=\begin{cases}A^1_{O_2}(t)/A & \text{for } F>\dot{L}(t)>0\\ \xi_{O_2}(\tau_a(t)) & \text{for } \dot{L}(t)>F \text{ and } \int_{t_s}^t f(t')dt'<V_A\\ \frac{B_{O_2}}{B}(\tau_A(t)) & \text{for } \dot{L}(t)>F \text{ and } \int_{t_s}^t f(t')dt'\geq V_A\\ \phi_{O_2}(\tau_E(t))\frac{|\dot{L}(t)|}{F+|\dot{L}(t)|}+\frac{A^1_{O_2}(t)}{A}\frac{F}{F+|\dot{L}(t)|} & \text{for } \dot{L}(t)<0\end{cases} \tag{2.29}$$

$$\xi_{N_2}(t) = \begin{cases} 1 - A^1_{O_2}(t)/A & \text{for } F > \dot{L}(t) > 0 \\ \xi_{N_2}(\tau_a(t)) & \text{for } \dot{L}(t) > F \text{ and } \int_{t_s}^{t} f(t')dt' < V_A \\ \frac{B_{N_2}}{B}(\tau_A(t)) & \text{for } \dot{L}(t) > F \text{ and } \int_{t_s}^{t} f(t')dt' \geq V_A \\ \phi_{N_2}(\tau_E(t))\frac{|\dot{L}(t)|}{F+|\dot{L}(t)|} + \frac{A-A^1_{O_2}(t)}{A}\frac{F}{F+|\dot{L}(t)|} & \text{for } \dot{L}(t) < 0 \end{cases} \tag{2.30}$$

$$\xi_{CO_2}(t) = \begin{cases} 0 & \text{for } F > \dot{L}(t) > 0 \\ \xi_{CO_2}(\tau_a(t)) & \text{for } \dot{L}(t) > F \text{ and } \int_{t_s}^{t} f(t')dt' < V_A \\ \frac{B_{CO_2}}{B}(\tau_A(t)) & \text{for } \dot{L}(t) > F \text{ and } \int_{t_s}^{t} f(t')dt' \geq V_A \\ \phi_{CO_2}(\tau_E(t))\frac{|\dot{L}(t)|}{F+|\dot{L}(t)|} & \text{for } \dot{L}(t) < 0 \end{cases} \tag{2.31}$$

Here $\tau_a(t)$ is defined by

$$\int_{\tau_a}^{t_s} f(t')dt' = \int_{t_s}^{t} f(t')dt'; \tag{2.32}$$

this time occurs either during exhalation or during the low-flow phase of inhalation.

3. EXPERIMENTAL AND COMPUTATIONAL METHODS

All experiments were done with the standard semi-closed circle system (model: Ohmeda Ohio Modulus II) and on-line Raman spectroscopy gas analyzer (model: Ohmeda Rascal II) that are used in the operating rooms at Ramsey Medical Center. The gas analyzer continuously removes 300 ml/min of gas from the mouthpiece, an effect that is ignored in the model. This instrument uses the carbon dioxide concentration to discriminate between inhalations and exhalations. The expiratory concentrations displayed are those occurring at the time of the highest carbon dioxide level. The inspiratory levels displayed are those occurring at the time of the lowest carbon dioxide level. Both inspiratory and expiratory displays are updated shortly after the end of an exhalation.

Respiratory volumes were measured with an electronic spirometer incorporated into the expiratory limb of the circle system. This instrument measures the exhalation flow and, after each breath, displays the tidal volume and respiration rate.

Choice of Model Parameters.

The mathematical model requires the volumes of the various components of the circle system. We determined these volumes as follows.

We measured the volumes of some components by filling them with water, which we then poured into a graduated beaker. We used this method to find the volumes of the corrugated tubes connecting the circle system to the patient (both 400 ml). These tubes and the other components of the system are joined via a system of connecting compartments in the absorber head. The volumes of these compartments we also determined by the method of volumetric displacement.

We determined the volumes of some components by measuring the external dimensions. We used this system for estimating the volume of the arm leading to the reservoir bag (585 ml), for the tube leading from the bottom of the absorber to the fresh gas compartment (175 ml), and for the total volume contained within each absorber cannister (1400 ml).

The absorber, however, is full of sodalyme crystals, so the volume of gas in each cannister is considerably less than 1400 ml. We estimated the gas volume within each absorber cannister by measuring the amount of water displaced by the crystals and subtracting this volume from the total volume of the cannister. By this method we found that the volume permeable to water in each cannister was 700 ml, half of the total volume.

The total volume we used for the inspiratory tube was the sum of the volume of the corrugated inspiratory tube, the connecting compartment in the absorber head, and the tube connecting this compartment to the the bottom of the absorber; this sum we found to be 880 ml. Similarly, the total volume for the expiratory tube was 435 ml. The volume of the reservoir bag was 3000 ml.

We found that the model agreed better with the experimental data when we modified some of the estimated volumes. In particular, we used 900 ml for the volume of each absorber cannister. It is possible that the volume permeable to gas is significantly larger than the volume permeable to water. Similarly, we used half the original estimate of 585 ml for the volume of the bag arm. We believe that measuring the external dimensions led to a significant overestimate of the actual volume.

The functional residual capacity of the subject (who was one of the authors) was measured in the pulmonary laboratory at Ramsey Hospital by the methods of body plethysmography and nitrogen wash-out. The values obtained ranged from 2850 ml to 3370 ml; in the model we used 2800 ml because smaller values gave results that agreed more closely with the data. For the dead space volume we used 150 ml; for the metabolic rate we used 300 ml/min.

TABLE OF MODEL PARAMETERS

Functional Residual Capacity	FRC = 2800 ml
maximum capacity of reservoir bag	$C = 3000$ ml
gas volume in each absorber cannister	$A = 900$ ml
dead space volume	$V_D = 150$ ml
volume of inspiratory tube	$V_I = 880$ ml
volume of expiratory tube	$V_E = 435$ ml
volume of bag arm	$V_A = 292.5$ ml
metabolic rate	$M = 300$ ml/min

Numerical Implementation.

The model was implemented in Matlab, with some minor modifications of the Runge-Kutta routine ODE23 to allow for use of previously computed values of the solution. Running the model out to two minutes took several minutes of computation on a Sun workstation.

Experimental Method.

We began each experiment by setting the fresh gas flow to 5 l/min and flushing the system with oxygen. Then the subject (the second author), in a sitting position, pinched her nose shut, and began breathing through the mouthpiece to the circle system.

We did two types of experiments. In the first, the subject breathed "normally"; we recorded by hand the inspired and expired nitrogen concentrations every ten seconds (as timed by a wristwatch), and recorded by hand the reported tidal volumes and respiration rate every time these quantities were updated. In the second type of experiment, the subject purposely breathed deeply, inhaling for 5 seconds, then exhaling for 5 seconds, according to a wristwatch. After every complete breath, we recorded by hand the inspired and expired nitrogen concentrations and the tidal volume. Each experiment ran for about 2 minutes. Between experiments, we waited for at least 20 minutes to allow the lung gas concentrations to return to those of room air.

We found apparent uncertainties in both the timing information and in the readings from the Raman spectrometer. The nitrogen concentrations generally seemed to be reported about 5 seconds after the end of an exhalation, but in some cases the readings changed at other times.

Controlling tidal volumes proved difficult in all the experiments; in Figures 5, 6, 8, and 9, we have shown results for several experiments with roughly similar minute ventilations. The subject had particular difficulty maintaining constant tidal volumes during the "normal breathing" experiments: in each case the initial tidal volumes were around 1.5 l and later decreased to well under 1 l.

4. RESULTS

We used the output from our model to simulate the gas analyzer readings as follows. We computed the simulated inspiratory readouts to be the average nitrogen concentration during each inspiration. In Figures 4–9, each simulated inspiratory readout is plotted as a circle at the end of the following expiration (which is when the gas analyzer reports the value). The inspired nitrogen corresponds clinically to rebreathed nitrogen.

The simulated expiratory readout is computed as the final nitrogen concentration at end expiration (end tidal nitrogen). Because the expired nitrogen concentration is very nearly constant, end-expiratory values are essentially identical to average values. In Figures 4–9 each simulated expiratory readout is shown as a circled star at the end of the expiration.

Figures 4 and 7 show the relation between the simulated concentration at the mouthpiece and the corresponding simulated gas analyzer readouts. Figure 4 does this for a hypothetical patient with an average tidal volume of 2.66 l and a respiratory rate of 6 breaths per minute; Figure 7 shows the same thing for a hypothetical patient with an average tidal volume of 1.03 l and a respiratory rate of about 9 breaths per minute.

Figures 4 and 7 can be understood as follows. During the initial inhalation, only pure oxygen is inhaled from the system, so the nitrogen concentration begins at zero. The first exhalation, on the other hand, would be about 80% nitrogen if not for the fact that pure oxygen had just been inhaled. The mixture of 80% nitrogen and a deep breath of pure oxygen brings the initial exhaled concentration down to 42%. On the second and subsequent inhalations, some exhaled gas has traveled around the circle system and is rebreathed. The downward spikes during inhalations are due to the fact that during the initial and final phases of inhalation, the flow into the lungs is very small, and during these short times, pure oxygen flows into the inspiratory tube from the fresh gas inlet. This causes the nitrogen concentration of the inspired gas to be zero during short intervals. These intervals are so short that sometimes the numerical sampling misses them, which is why some of the spikes do not go to zero. When the inspiratory flow is larger, this inspiratory gas contains gas from the absorber and is no longer pure oxygen.

We found that the model predictions agreed quite well with our experimental data in the deep-breathing case. This is illustrated in Figures 5 and 6, which show the same simulated inspiratory and expiratory readouts as in Figure 4, plotted together with the experimental data.

Figures 8 and 9 show the relation between model predictions and data for "normal breathing". Although the agreement is quite good at 120 seconds, the model predicted a nitrogen washout rate that is slower in the first 60 seconds than that shown by the data. We believe that rather than reflecting a shortcoming of the model, this is probably due to the subject's difficulty in maintaining constant tidal volumes during the experiments. The large initial tidal volumes in the experimental runs would cause a faster nitrogen washout.

Figure 8 shows that in some tests, there was still some nitrogen present in the inspiratory limb of the system, despite our having begun the experiment with an oxygen flush. Figure 8 might seem much noisier than Figure 9, but this is only because of the difference in vertical scales.

Figure 10 compares the simulated denitrogenation data for different minute ventilations (6 l, 12 l, and 16 l, respectively) and different fresh gas flows. When deep breaths and a large minute ventilation are used, denitrogenation to clinically acceptable levels (<15%) are achieved in 60 seconds. When normal tidal breathing is used, this occurs at 120 seconds. Comparison with Figures 4 (tidal volume 2600 ml, minute ventilation about 16 l) and 7 (tidal volume 1030 ml and minute ventilation about 9.5 l) shows that increasing the tidal volume beyond 1000 ml does not result in significantly faster denitrogenation.

Figure 10 also shows that increasing the fresh gas flow from 5l/min to 10 l/min has little effect on denitrogenation when normal tidal breathing is employed. When deep breaths are employed, increasing the fresh gas flow to 10l/min results in clinically acceptable denitrogenation about 20 seconds faster.

We can also use Figure 10 to compare the effectiveness of four deep breaths, if we note that four breaths have been taken in 30 seconds. We see that four deep breaths remove about as much nitrogen as a minute and a half of normal breathing; this reduces the nitrogen in the lungs to about 15%.

The last column of Figure 10 shows the total (integrated) nitrogen inhaled as a percentage of the total ventilation. When normal tidal breathing is employed, only 3.8 % of the exhaled nitrogen is rebreathed at a fresh gas flow of 5l/min. When fresh gas flow is increased to 10l/min, virtually no nitrogen is rebreathed. In the case of a large minute ventilation with deep breaths (2000ml), a significant amount of rebreathing occurs (9%) at a fresh gas flow of 5 l/min. Rebreathing becomes clinically insignificant (3.8 %) when the fresh gas flow is increased to 10l/min.

The top part of Figure 11 shows the simulated denitrogenation data for constant minute ventilation of 6l/min but variable tidal volume and frequency. Increasing the tidal volume and decreasing the frequency hastens the nitrogen washout. At normal tidal volumes (500ml) or greater, denitrogenation is accomplished by 100 seconds. Rebreathed nitrogen peaks at 3.6% when a tidal volume of 500ml and a frequency of 12 are used.

This represents a competition between the efficiency of the circuit in removing exhaled gas and the relatively slower nitrogen removal from the lungs when small tidal volumes are used. In other words, for small tidal volumes, denitrogenation of the lungs is slow because of the dead space in the trachea, but once this nitrogen is exhaled, it is efficiently removed from the circuit because the fresh gas flow washes it out quickly. To test this hypothesis, we did some test runs in which the dead space volume was set to zero. These results are summarized in the bottom part of Figure 11. We see that in this situation where the minute ventilation is held constant, the nitrogen washout is essentially independent of tidal volume.

Both the upper and lower parts of Figure 11 show that there is less rebreathing with large tidal volumes. This is due to the composition of the gas removed by the scavenging system. When the minute ventilation and fresh gas flow are held constant, the total gas volume leaving through the scavenging system is also constant from breath to breath, and is equal to the volume of fresh gas entering the system minus the volume metabolized. When tidal volumes are small, a larger fraction of the exiting gas comes directly from the fresh gas inlet. When the tidal volumes are large, a larger fraction of the scavenged gas is exhaled gas. The nitrogen in this exhaled gas leaves the system and is not available for rebreathing.

Regardless of tidal volume, rebreathing of nitrogen is clinically unimportant when the minute ventilation is 6l/min and the fresh gas flow is 5l/min.

5. DISCUSSION AND CONCLUSIONS

We have constructed a mathematical model which predicts the concentration of nitrogen, oxygen and carbon dioxide in the lung when the patient breathes into a semiclosed anesthetic circle system. We chose the parameters of the model so that its predictions agree well with our experimental data.

The model predicts that four deep breaths are as effective in removing nitrogen from the lungs as is about a minute and a half of normal breathing. Both these protocols reduce the lung nitrogen to about 15%.

Rebreathing of exhaled gases has been given as a reason for not using large breaths during denitrogenation. The simulation data in figure 10, however, show clearly that denitrogenation is faster with a large minute ventilation accomplished by deep breaths. Moreover, even with large minute ventilation, rebreathing of nitrogen is clinically insignificant when a fresh gas flow of 10l/min is used.

In summary, our results predict that denitrogenation is more efficient the larger the minute ventilation and the larger the tidal volume, with a fresh gas flow of at least 10l/min. Tidal volumes above 1000ml do not significantly decrease the time to safe denitrogenation.

ACKNOWLEDGEMENTS

The work of the second author was partially supported by the Institute for Mathematics and Its Applications at the University of Minnesota, by Rensselaer Polytechnic

Institute, by the National Science Foundation, and by the Office of Naval Research. Both authors are grateful to Oberlin College for bringing them together as physics majors in 1974 and 1975.

REFERENCES

[BGT] P.E. Bigeleisen, J.I. Gerson, and G. Threatte, "A comparison of preoxygenation methods in ill surgical patients", Anesthesiology Review 15 (1988) 21–25.

[BH] E. Bowie and L.M. Huffman, The Anesthesia Machine: Essentials for Understanding (Ohmeda, Madison, Wisconsin, 1985).

[BM] C.B. Berry and P.S. Myles, "Preoxygenation in healthy volunteers: a graph of oxygen 'washin' using end-tidal oxygraphy", British J. Anaesthesia 72 (1994) 116–118.

[CCCP] F.J. Carmichael, C.J.E. Cruise, R.R. Crago, and S. Paluck, "Preoxygenation: a study of denitrogenation", Anesth. Analg. 68 (1989) 406–409.

[D] R.D. Driver, Ordinary and Delay Differential Equations, Applied Mathematical Sciences Vol. 20, Springer, New York, 1977.

[DA] J.S. Denson and S. Abrahamson, "A computer-controlled patient simulator", J. Amer. Med. Assoc. 208 (1969) 504–508.

[GD] D.M. Gaba and A. DeAnda, "A comprehensive anesthesia simulation environment: re-creating the operating room for research and training", Anesthesiology 69 (1988) 387–394.

[GDM] M.I. Gold, I. Duarte, and S. Muravchick, "Arterial oxygenation in conscious patients after 5 minutes and after 30 seconds of oxygen breathing", Anesth. Analg. 60 (1981) 313–315.

[GG] M.L. Good and J.S. Gravenstein, "Training for safety on an anesthesia simulator", Seminars in Anesthesia 12 (1993) 235–250.

[GHF] A.M. Gambee, R.E. Hertzka, and D.M. Fisher, "Preoxygenation techniques: comparison of three minutes and four breaths", Anesth. Analg. 66 (1987) 468–470.

[GNLMS] M.E. Goldberg, M.C. Norris, G.E. Larijani, A.T. Marr, and J.L. Seltzer, "Preoxygenation in the morbidly obese: a comparison of two techniques", Anesth. Analg. 68 (1989) 520-522.

[HE] W.K. Hamilton and D.W. Eastwood, "A study of denitrogenation with some inhalation anesthetic systems", Anesthesiology 16 (1955) 861–867.

[MR] G.A. Moyer and P. Rein, "Preoxygenation in the morbidly obese patient", Anesth Analg. 65 (1086) S106.

[ND] M.C. Norris and D.M. Dewan, "Preoxygenation for Cesarean section: a comparison of two techniques", Anesthesiology 62 (1984) 827–829.

[NKTG] M.C. Norris, M.R. Kirkland, M.C. Torjman, and M.E. Goldberg, "A study of denitrogenation during pregnancy", Anesth. Analg. 68 (1989) S213.

[RSSB] G.N. Russel, C.L. Smith, S.L. Snowdon, and T.H.L Bryson, "Preoxygenation and the parturient patient", Anaesthesia 42 (1987) 346–351.

[SM] R.K. Stoelting and R.D. Miller, Basics of Anesthesia, Churchill Livingstone, New York, 1984.

[SO] H.A. Schwid and D. O'Donnell, "Anesthesiologists' management of cimulated critical incidents", Anesthesiology 76 (1992) 495–501; also "Education computer simulation of malignant hyperthermia", J. Clinical Monitoring 8 (1992) 201–208.

[VMM] S.J. Valentine, R. Marjot, and C.R. Monk, “Preoxygenation in the elderly: a comparison of the four-maximal-breath and three-minute techniques”, Anesth. Analg. 71 (1990) 516–519.

APPENDEX 1. COMPUTATION OF DELAYS

The model involves computation of delays for gas flow through various tubes. To compute the entrance time t_{in} for gas that exits at time t_{out} after traveling though a tube, we need to know the volume of the tube, V, and the rate of gas flow $\dot{v}(t)$. The volume filled by a gas flowing into the tube at rate $\dot{v}(t)$ between times t_{in} and t_{out} is the integral of $\dot{v}(t)$ over this time interval, or simply $v(t_{out}) - v(t_{in})$. Therefore, to find the time delay, we must solve the equation

$$v(t_{out}) - v(t_{in}) = V \tag{A1.1}$$

for t_{in}.

Below we find $t_{in} = \tau(t_{out})$ for each of the tubes in the system.

The Inspiratory Tube.

For the case of the tube in the inspiratory limb of the system, the rate of gas flow into the tube is $\dot{v}(t) = \dot{L}(t)$ during inhalations, but zero during exhalations. Thus, in computing the volume of flow through an inspiratory tube, we should integrate $\dot{L}$ only over inspiratory time intervals $(t_{in}, t_1^E), (t_2^I, t_2^E), \ldots, (t_n^I, t_{out})$, where the times at which inhalations begin are denoted with a superscript I and times at which they end (and exhalations begin) are denoted with a superscript E. Equation (A1.1) should therefore be replaced by

$$[L(t_{out}) - L(t_n^I)] + \ldots + [L(t_2^E) - L(t_2^I)] + [L(t_1^E) - L(t_{in})] = V. \tag{A1.2}$$

This expression can be simplified by noting that the difference in lung inflation between the beginning and end of an inspiration is precisely the tidal volume T. Equation (A1.2) therefore reduces to

$$nT + L(t_{out}) - L(t_{in}) = V. \tag{A1.3}$$

In order to solve this equation for $t_{in} = \tau_I(t_{out})$, we must first find n, the number of full breaths during the delay period.

To find n, we solve (A1.3) for $L(t_{in})$ and use the fact that the lung volume $L(t_{in})$ must be greater than the volume at the beginning of an inspiration (i.e., the functional residual capacity FRC), and less than the volume at the end of an inspiration (i.e., $FRC + T$), thus obtaining

$$FRC \leq L(t_{out}) + nT - V \leq FRC + T. \tag{A1.4}$$

Subtracting FRC and nT from each term, and finally dividing by $-T$ gives

$$n - 1 \leq \frac{FRC + V - L(t_{out})}{T} \leq n. \tag{A1.5}$$

This expression shows that we can compute the number of full inhalations during the delay by finding the smallest integer greater than the middle quantity in (A1.5).

Now that we know the number of full breathing cycles for the delay, we can find the precise cycle containing t_{in}. To do this, first we find the number of full breaths N that have elapsed, since time zero, for the time t_{out}. This number N can be computed as the greatest integer less than rt_{out}. Then t_{in} must occur during the cycle number $N - n$.

Once we know which cycle contains t_{in}, we find t_{in} itself, assuming that the breathing waveform is given by (2.1). To do this, we need the fraction Δt_{in} of a breathing cycle that corresponds to t_{in}:

$$\Delta t_{in} = \frac{1}{2\pi r}\mathrm{Arccos}\big(1 - \frac{2}{T}[L(t_{out}) - FRC - V + nT]\big). \tag{A1.6}$$

Finally, we obtain t_{in} as

$$t_{in} = (N - n)/r + \Delta t_{in}. \tag{A1.7}$$

The Expiratory Tube.

The case of the tube in the expiratory limb is similar to that of the inspiratory tube. The flow into the expiratory tube is $\dot{v}(t) = -\dot{L}(t)$ during exhalations and zero during inhalations. The equation to be solved is

$$L(t_{in}) - L(t_{out}) + nT = V; \tag{A1.8}$$

the number of exhalations of delay can be found from

$$n \le \frac{L(t_{out}) - FRC + V}{T} \le n + 1, \tag{A1.9}$$

by computing the greatest integer less than the middle quantity in (A1.9). We find Δt_{in} as

$$\Delta t_{in} = \frac{1}{2\pi r}\bigg(2\pi - \mathrm{Arccos}\big(1 - \frac{2}{T}[L(t_{out}) - FRC + V - nT]\big)\bigg). \tag{A1.10}$$

Here we have subtracted the Arccosine from 2π because exhalation occurs for phases between π and 2π, whereas the Arccosine returns a phase between 0 and π. Finally, $t_{in} = \tau(t_{out})$ is computed as before from (A1.7).

The Anatomical Dead Space.

We can also use the above computations for the dead space delay. This is because, first, the flow in the dead space is precisely $\dot{L}(t)$. Second, the dead space volume cannot be so great relative to the tidal volume that the delay is more than a full breath: this situation could not sustain life. Thus the entrance time is always during the same inspiration or expiration as the exit time. Therefore delays for travel through the dead space can be computed in the same way as for the inspiratory and expiratory tubes.

The Arm Leading to the Reservoir Bag.

The flow in the arm leading to the reservoir bag is given by f of (2.14). This flow, however, depends on the concentration of carbon dioxide at the arm junction, and cannot be predicted precisely in advance. For gas exiting the arm at time t_{out}, the entrance time $t_{in} = \tau_A(t_{out})$ is defined by

$$\int_{t_{in}}^{t_{out}} f(t')dt' = V_A \tag{A1.11}$$

The time t_{in} can be computed numerically by using a trapezoidal rule approximation to the integral.

APPENDEX 2. INCLUDING OTHER EFFECTS AND OTHER GASES

The model of section 2 can be improved in many ways. Other physiological effects can be included, and other gases can be tracked. Adding anesthetic gases, for example, merely requires some additional equations that have a form similar to the equations for oxygen. Including some of the physiological effects, however, affect the total flow of gas in some of the tubes, and this requires that the computation of delays be modified.

Including Water Vapor.

Accounting for the presence of water vapor is a bit complicated. To a good approximation, the concentration of water vapor in the lungs is kept constant by the process of evaporation from the body into the lungs. Because gas is now appearing in the lungs without having passed through the trachea, the inflow of fresh gas through the trachea is no longer precisely the same as the rate of increase in lung volume. This means that $\dot{L}$ in (2.6 – 8) should be replaced by $(1-w)\dot{L}$, where $w = 47/760$ is the water vapor concentration. We can then augment the model with extra equations for the water vapor volume. For the lung, this is simply

$$\dot{L}_{H_2O}(t) = w\dot{L}(t), \tag{A2.1}$$

which is the same for both inhalation and for exhalation. For other compartments, the equations for water vapor follow a pattern similar to the ones for other gas species.

Including Nitrogen Diffusion.

In a similar manner, we can include the effects of nitrogen diffusion from the body into the lungs. If we assume that this diffusion occurs at a rate N, then the inflow of fresh gas through the trachea must be less than $(1-w)\dot{L}$ by an amount equal to the rate of appearance of nitrogen. Thus, among the lung inhalation equations, equations (2.6 – 8), $\dot{L}$ should be replaced by $(1-w)\dot{L} - N$, and an extra term $+N$ should appear in each case on the right side of (2.7). The appended equation for water vapor, (A2.1), remains the same.

For the exhalation equations, we must keep in mind that the change in lung volume is no longer the same as the exhaled flow. Denoting the exhaled flow temporarily by X, we have (A3.2) together with

$$\dot{L}_{O_2} = -M - \frac{L_{O_2}}{L}X \tag{A2.2}$$

$$\dot{L}_{N_2} = N - \frac{L_{N_2}}{L}X \tag{A2.3}$$

$$\dot{L}_{CO_2} = M - \frac{L_{CO_2}}{L}X. \tag{A2.3}$$

Adding equations (A2.1 – 3), we find that

$$\dot{L} = \frac{L_{O_2} + L_{N_2} + L_{CO_2}}{L}X + N + w\dot{L}. \tag{A2.4}$$

The coefficient of X in (A2.4), however, can be replaced by $1 - w$, and the resulting equation solved for X, resulting in

$$X = \dot{L} - \frac{N}{1 - w}. \tag{$A2.5$}$$

This expression for X can be used in (A2.2 – 3) to obtain the equations for exhalation in the presence of nitrogen diffusion.

Other Effects.

A similar modification can be made to model the diffusion of carbon dioxide out of body tissues in situations such as deep breathing when the carbon dioxide concentration in the lungs falls significantly below the concentration in body tissue.

FIGURE CAPTIONS

Figure 1. This is a diagram of the semi-closed breathing circuit, showing the low-flow inspiratory pattern. The fresh gas inlet is labeled "F", the inspiratory tube "I", the lungs "L", the anatomical dead space "DS", the expiratory tube "E", the reservoir bag "B", the arm leading to the bag "A", the exit to the scavenging system "S", and the two absorber cannisters "C_1" and "C_2". The inspiratory and expiratory tubes contain one-way valves that allow gas to circulate only in the counter-clockwise direction in the figure.

Figure 2. This shows the flow pattern during the high-flow inspiratory phase.

Figure 3. This shows the flow during exhalation.

Figure 4. This shows the model predictions for the nitrogen concentration at the mouthpiece in the deep breathing case (respiratory rate 6, tidal volumes 2.66l). Also shown are simulated gas analyzer readouts (average concentrations) for inspirations (circles) and expirations (circled stars), both plotted at the end of each expiration. The inspiratory readouts are shown at the delayed time to facilitate comparison with the experimental data in Figure 5. The horizontal axis is time in seconds; the vertical axis is nitrogen concentration.

Figure 5. This compares the predicted inhaled nitrogen concentrations to the experimental data, for a subject with respiratory rate of 6 breaths per minute and tidal volumes of 2.66l. The horizontal axis is time in seconds and the vertical axis is nitrogen concentration. The circles are the same predicted values shown in Figure 4. The minute ventilations for each curve are as follows: −+− 17.3 (experimental); -·+-· 15.5 (experimental); –x– 15.2 (experimental); solid line with circles 16.0 (calculated).

Figure 6. This compares the predicted exhaled nitrogen concentrations to the experimental data, for the same experiment as shown in Figure 5. The horizontal axis is time in seconds and the vertical axis is nitrogen concentration. The circled stars are the same predicted values, shown at the same times, as in Figure 4. The minute ventilations for each curve are as follows: −+− 17.3 (experimental); -·+-· 15.5 (experimental); –x– 15.2 (experimental); solid line with circled stars 16.0 (calculated).

Figure 7. This shows the model predictions for the nitrogen concentration at the mouthpiece in the "normal" breathing case (respiratory rate 9.15, tidal volumes 1.04l). Also shown are simulated gas analyzer readouts (average concentrations) for inspirations (circles) and expirations (circled stars), both plotted at the end of each expiration. The inspiratory readouts are shown at the delayed time in order to facilitate comparison with the experimental data in Figure 8. The horizontal axis is time in seconds; the vertical axis is nitrogen concentration.

Figure 8. This compares the predicted inhaled nitrogen concentrations to the experimental data, for a subject with respiratory rate of 9.15 breaths per minute and tidal volumes of 1.04l. The horizontal axis is time in seconds and the vertical axis is nitrogen concentration. The circles are the same predicted values, shown at the same times, as in Figure 7. The minute ventilations for each curve are as follows: −+− 9.4 (experimental); –x– 9.7

(experimental); --+-- 9.2 (experimental); --x-- 8.9 (experimental); solid line with circles 9.5 (calculated).

Figure 9. This compares the predicted exhaled nitrogen concentrations to the experimental data, for the same experiment as shown in figure 8. The horizontal axis is time in seconds and the vertical axis is nitrogen concentration. The circled stars are the same predicted values shown in Figure 7. The minute ventilations for each curve are as follows: –+– 9.4 (experimental); –x– 9.7 (experimental); --+-- 9.2 (experimental); --x-- 8.9 (experimental); solid line with circled stars 9.5 (calculated).

Figure 10. This compares simulated denitrogenation data for a patient with normal minute ventilation and a normal tidal volume to that for the same patient with a large minute ventilation and deep breaths. The "Nitrogen washout" numbers are the expiratory readings at the mouthpiece; the "Percent of rebreathed nitrogen" numbers are obtained by integrating the inhaled nitrogen to 2 minutes and dividing by the total ventilation during that period.

Figure 11. This table compares denitrogenation data for the fixed minute ventilation of 6 l/min but different tidal volumes and respiratory rates. The "Nitrogen washout" numbers are the expiratory readings at the mouthpiece; the "Percent of rebreathed nitrogen" numbers are obtained by integrating the inhaled nitrogen to 2 minutes and dividing by the total ventilation, 12 l.