Homework Set 10

Math/ECE 430

due Friday, April 12, 2013

The page numbers below refer to the book by Gasquet and Witomski. $\mathcal{F}$ denotes the Fourier transform.

1. p. 274, #29.7. Prove that

$$\sum_{n=-\infty}^{\infty} (-1)^n e^{i\pi n x} = 2 \sum_{n=-\infty}^{\infty} \delta_{2n+1}.$$

   Hint: Use the Fourier series expansion of the function x on (-1,1).

2. p. 295, #31.5 Show that the Fourier transform of the Heaviside function is $\frac{1}{2\pi i}$P.V.$(1/\xi)+\lambda\delta$. Show that $\lambda = 1/2$ by noting that P.V.$(1/x)$ is odd.

3. p. 295, # 31.6. Show that the Fourier transform of P.V. $(1/x)$ is proportional to sgn (ξ), and find the constant of proportionality. Hint: You can compute this directly, without using a test function. Look for the sign function in the change of variables, and use $\int_0^\infty (\sin x)/x dx = \pi/2$.

4. p. 295, #31.8. Show that

$$\mathcal{F}[\cos 2\pi\lambda t] = (\delta_\lambda + \delta_{-\lambda})/2 \qquad \text{and} \qquad \mathcal{F}[\sin 2\pi\lambda t] = (\delta_\lambda - \delta_{-\lambda})/2.$$

5. p. 351, #37.1. a) Use the Fourier transform relation for Dirac's comb to show that

$$\sum_{n=-\infty}^{\infty} \phi\left(\frac{n}{a}\right) = a \sum_{n=-\infty}^{\infty} [\mathcal{F}\phi](na) \qquad (a \neq 0)$$

   b) Deduce from this that

$$\sum_{n=-\infty}^{\infty} e^{-\pi n^2 t} = \frac{1}{\sqrt{t}} \sum_{n=-\infty}^{\infty} e^{-\pi n^2/t}$$