Problem 1

Suppose your best friend won $1000 in the lottery. Knowing that you are taking calculus, your friend comes to you for advice. The lottery folks are giving your friend a choice:

(a) $700 upfront, deposited into an account for 10 years before it can be touched

(b) $200 every two years on January 1, deposited into an account for 10 years

(c) $100 per year on January 1, deposited into an account for 10 years

or (d) $100 "trickled" into an account per year for 10 years, totaling $1000.

The account the lottery folks will use has an interest rate of 7%, compounded continuously. Which option results in the most money for your friend after the 10 years have passed?

Option (b)

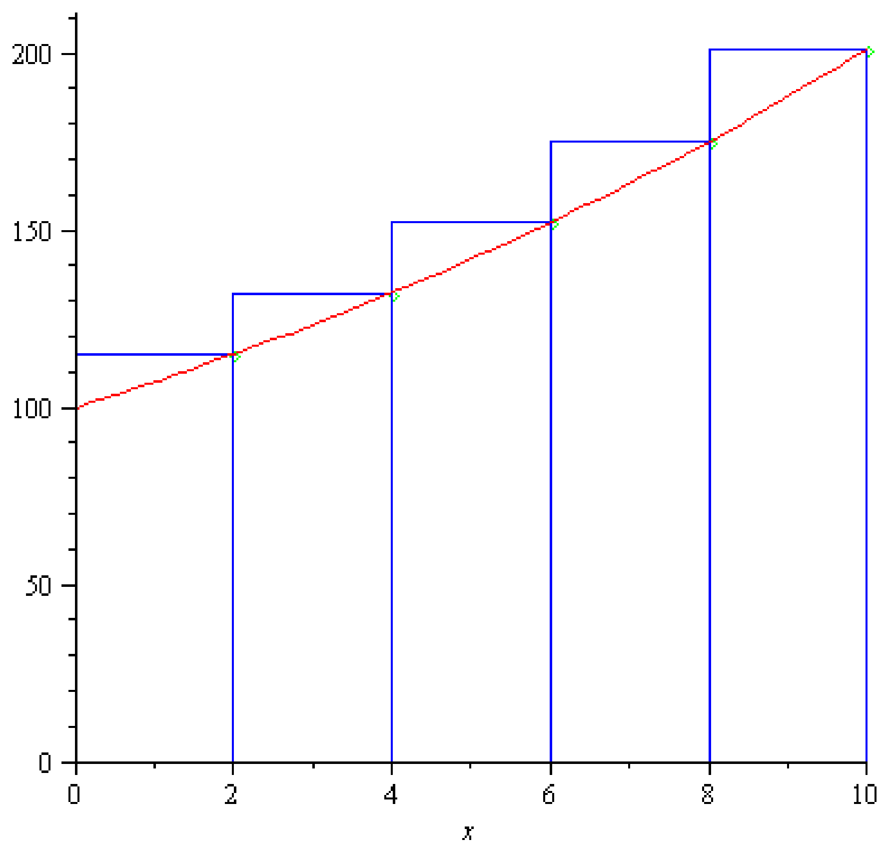
Date of Deposit	Time invested	Total Value
Jan. 1, 2007	10 years	$A = 200e^{(.07)(10)} = 402.75$
Jan. 1, 2009	8 years	$A = 200e^{(.07)(8)} = 350.13$
Jan. 1, 2011	6 years	$A = 200e^{(.07)(6)} = 304.39$
Jan. 1, 2013	4 years	$A = 200e^{(.07)(4)} = 264.63$
Jan. 1, 2015	2 years	$A = 200e^{(.07)(2)} = 230.05$

Total Value After 10 Years = Sum of third column

$$= 200e^{(.07)(2)} + 200e^{(.07)(4)} + 200e^{(.07)(6)} + 200e^{(.07)(8)} + 200e^{(.07)(10)}$$

$$= 230.05 + 264.63 + 304.39 + 350.13 + 402.75$$

$$= \$1551.95$$



Let $\Delta t = 2$ (the time between deposits).

Total Value After 10 Years

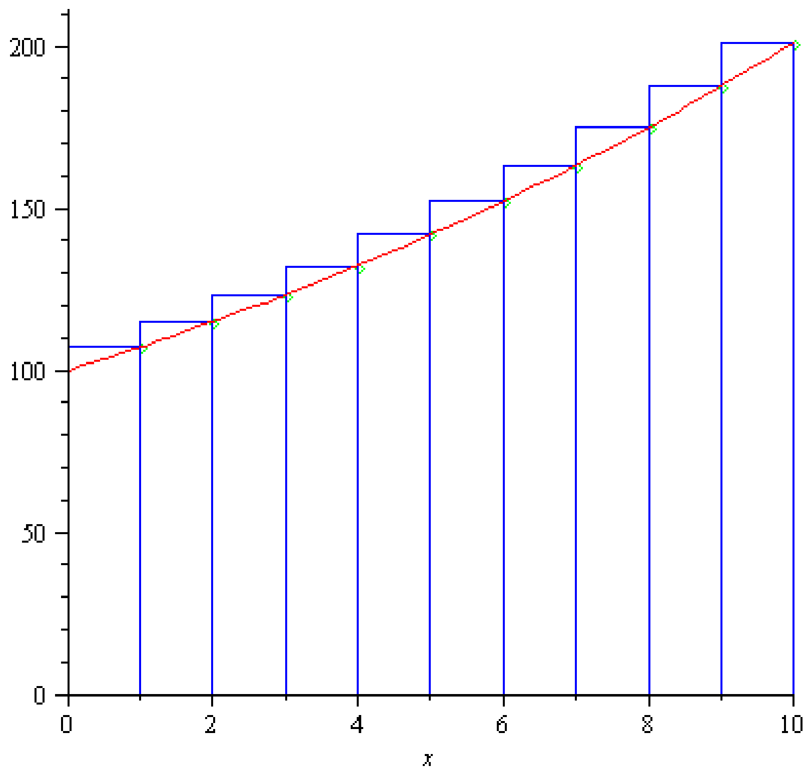
$$= 2 \cdot 100e^{(.07)(2)} + 2 \cdot 100e^{(.07)(4)} + 2 \cdot 100e^{(.07)(6)} + 2 \cdot 100e^{(.07)(8)} + 2 \cdot 100e^{(.07)(10)}$$

$$= \Delta t \cdot 100e^{(.07)(2)} + \Delta t \cdot 100e^{(.07)(4)} + \Delta t \cdot 100e^{(.07)(6)} + \Delta t \cdot 100e^{(.07)(8)} + \Delta t \cdot 100e^{(.07)(10)}$$

= sum of areas of 5 rectangles.

Option (c)

Date of Deposit	Time invested	Total Value
Jan. 1, 2007	10 years	$A = 100e^{(.07)(10)} = 201.38$
Jan. 1, 2008	9 years	$A = 100e^{(.07)(9)} = 187.76$
Jan. 1, 2009	8 years	$A = 100e^{(.07)(8)} = 175.07$
Jan. 1, 2010	7 years	$A = 100e^{(.07)(7)} = 163.23$
Jan. 1, 2011	6 years	$A = 100e^{(.07)(6)} = 152.20$
Jan. 1, 2012	5 years	$A = 100e^{(.07)(5)} = 141.91$
Jan. 1, 2013	4 years	$A = 100e^{(.07)(4)} = 132.31$
Jan. 1, 2014	3 years	$A = 100e^{(.07)(3)} = 123.37$
Jan. 1, 2015	2 years	$A = 100e^{(.07)(2)} = 115.03$
Jan. 1, 2016	1 year	$A = 100e^{(.07)(1)} = 107.25$



Total Value After 10 Years = Sum of third column

$$= 100e^{(.07)(1)} + 100e^{(.07)(2)} + 100e^{(.07)(3)} + 100e^{(.07)(4)} + 100e^{(.07)(5)}$$

$$+ 100e^{(.07)(6)} + 100e^{(.07)(7)} + 100e^{(.07)(8)} + 100e^{(.07)(9)} + 100e^{(.07)(10)}$$

$$= \Delta t \cdot 100e^{(.07)(1)} + \Delta t \cdot 100e^{(.07)(2)} + \Delta t \cdot 100e^{(.07)(3)} + \Delta t \cdot 100e^{(.07)(4)} + \Delta t \cdot 100e^{(.07)(5)}$$

$$+ \Delta t \cdot 100e^{(.07)(6)} + \Delta t \cdot 100e^{(.07)(7)} + \Delta t \cdot 100e^{(.07)(8)} + \Delta t \cdot 100e^{(.07)(9)} + \Delta t \cdot 100e^{(.07)(10)}$$

$$= 107.25 + 115.03 + 123.37 + 132.31 + 141.91 + 152.20 + 163.23 + 175.07 + 187.76 + 201.38$$

$$= \$1499.51$$

=sum of 10 rectangles

Problem 2

Suppose we know that we want the Total Value (amount of continuous money flow) to be $500,000 after 10 years at an interest rate of 7%, compounded continuously.

What should P, the constant rate of investment per year, be? That is, how much money needs to trickle in each year so that we walk away with $500,000 at the end of 10 years?