Cubic Surfaces with 27 Lines over GF(8)

Orbiter

April 16, 2019

1 The field of order 8

The field $\mathbb{F}_8$:
polynomial: $X^3 + X^2 + 1 = 13$
$Z_i = \log_\alpha(1 + \alpha^i)$

i	γ_i	$-\gamma_i$	γ_i^{-1}	$\log_\alpha(\gamma_i)$	α^i	Z_i	$\phi(\gamma_i)$	$T(\gamma_i)$	$N(\gamma_i)$
0	$0 = 0$	0	DNE	DNE	1	DNE	0	0	0
1	$1 = 1$	1	1	7	2	5	1	1	1
2	$\alpha = \alpha$	2	6	1	4	3	4	1	1
3	$\alpha + 1 = \alpha^5$	3	4	5	5	2	5	0	1
4	$\alpha^2 = \alpha^2$	4	3	2	7	6	7	1	1
5	$\alpha^2 + 1 = \alpha^3$	5	7	3	3	1	6	0	1
6	$\alpha^2 + \alpha = \alpha^6$	6	2	6	6	4	3	0	1
7	$\alpha^2 + \alpha + 1 = \alpha^4$	7	5	4	1	DNE	2	1	1

2 The groups

2.1 The semilinear group

Group action $\mathrm{P\Gamma L}(4, 8)$ of degree 585
Group order 103675594014720
tl=$(585, 584, 576, 512, 343, 3)$
Base: $(0, 1, 2, 3, 4, 6)$
Strong generators for a group of order 103675594014720:

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_1, \begin{bmatrix}1&0&0&0\\0&\alpha^6&0&0\\0&0&\alpha^6&0\\0&0&0&\alpha^6\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&\alpha&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_0,$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&\alpha&0\\0&0&0&1\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\1&0&0&1\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\\alpha&0&0&1\end{bmatrix}_0,$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\\alpha^2&0&0&1\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&1&0&1\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&\alpha&0&1\end{bmatrix}_0,$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&\alpha^2&0&1\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1&1\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&\alpha&1\end{bmatrix}_0,$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \alpha^2 & 1 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_0,$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_0$$

i	P_i
0	$(1,0,0,0)$
1	$(0,1,0,0)$
2	$(0,0,1,0)$
3	$(0,0,0,1)$
4	$(1,1,1,1)$
5	$(1,1,0,0)$
6	$(2,1,0,0)$
7	$(3,1,0,0)$
8	$(4,1,0,0)$
9	$(5,1,0,0)$

,

i	P_i
10	$(6,1,0,0)$
11	$(7,1,0,0)$
12	$(1,0,1,0)$
13	$(2,0,1,0)$
14	$(3,0,1,0)$
15	$(4,0,1,0)$
16	$(5,0,1,0)$
17	$(6,0,1,0)$
18	$(7,0,1,0)$
19	$(0,1,1,0)$

,

i	P_i
20	$(1,1,1,0)$
21	$(2,1,1,0)$
22	$(3,1,1,0)$
23	$(4,1,1,0)$
24	$(5,1,1,0)$
25	$(6,1,1,0)$
26	$(7,1,1,0)$
27	$(0,2,1,0)$
28	$(1,2,1,0)$
29	$(2,2,1,0)$

,

i	P_i
30	$(3,2,1,0)$
31	$(4,2,1,0)$
32	$(5,2,1,0)$
33	$(6,2,1,0)$
34	$(7,2,1,0)$
35	$(0,3,1,0)$
36	$(1,3,1,0)$
37	$(2,3,1,0)$
38	$(3,3,1,0)$
39	$(4,3,1,0)$

,

i	P_i
40	$(5,3,1,0)$
41	$(6,3,1,0)$
42	$(7,3,1,0)$
43	$(0,4,1,0)$
44	$(1,4,1,0)$
45	$(2,4,1,0)$
46	$(3,4,1,0)$
47	$(4,4,1,0)$
48	$(5,4,1,0)$
49	$(6,4,1,0)$

i	P_i
50	$(7,4,1,0)$
51	$(0,5,1,0)$
52	$(1,5,1,0)$
53	$(2,5,1,0)$
54	$(3,5,1,0)$
55	$(4,5,1,0)$
56	$(5,5,1,0)$
57	$(6,5,1,0)$
58	$(7,5,1,0)$
59	$(0,6,1,0)$

,

i	P_i
60	$(1,6,1,0)$
61	$(2,6,1,0)$
62	$(3,6,1,0)$
63	$(4,6,1,0)$
64	$(5,6,1,0)$
65	$(6,6,1,0)$
66	$(7,6,1,0)$
67	$(0,7,1,0)$
68	$(1,7,1,0)$
69	$(2,7,1,0)$

,

i	P_i
70	$(3,7,1,0)$
71	$(4,7,1,0)$
72	$(5,7,1,0)$
73	$(6,7,1,0)$
74	$(7,7,1,0)$
75	$(1,0,0,1)$
76	$(2,0,0,1)$
77	$(3,0,0,1)$
78	$(4,0,0,1)$
79	$(5,0,0,1)$

,

i	P_i
80	$(6,0,0,1)$
81	$(7,0,0,1)$
82	$(0,1,0,1)$
83	$(1,1,0,1)$
84	$(2,1,0,1)$
85	$(3,1,0,1)$
86	$(4,1,0,1)$
87	$(5,1,0,1)$
88	$(6,1,0,1)$
89	$(7,1,0,1)$

,

i	P_i
90	$(0,2,0,1)$
91	$(1,2,0,1)$
92	$(2,2,0,1)$
93	$(3,2,0,1)$
94	$(4,2,0,1)$
95	$(5,2,0,1)$
96	$(6,2,0,1)$
97	$(7,2,0,1)$
98	$(0,3,0,1)$
99	$(1,3,0,1)$

i	P_i
100	(2, 3, 0, 1)
101	(3, 3, 0, 1)
102	(4, 3, 0, 1)
103	(5, 3, 0, 1)
104	(6, 3, 0, 1)
105	(7, 3, 0, 1)
106	(0, 4, 0, 1)
107	(1, 4, 0, 1)
108	(2, 4, 0, 1)
109	(3, 4, 0, 1)

,

i	P_i
110	(4, 4, 0, 1)
111	(5, 4, 0, 1)
112	(6, 4, 0, 1)
113	(7, 4, 0, 1)
114	(0, 5, 0, 1)
115	(1, 5, 0, 1)
116	(2, 5, 0, 1)
117	(3, 5, 0, 1)
118	(4, 5, 0, 1)
119	(5, 5, 0, 1)

,

i	P_i
120	(6, 5, 0, 1)
121	(7, 5, 0, 1)
122	(0, 6, 0, 1)
123	(1, 6, 0, 1)
124	(2, 6, 0, 1)
125	(3, 6, 0, 1)
126	(4, 6, 0, 1)
127	(5, 6, 0, 1)
128	(6, 6, 0, 1)
129	(7, 6, 0, 1)

,

i	P_i
130	(0, 7, 0, 1)
131	(1, 7, 0, 1)
132	(2, 7, 0, 1)
133	(3, 7, 0, 1)
134	(4, 7, 0, 1)
135	(5, 7, 0, 1)
136	(6, 7, 0, 1)
137	(7, 7, 0, 1)
138	(0, 0, 1, 1)
139	(1, 0, 1, 1)

,

i	P_i
140	(2, 0, 1, 1)
141	(3, 0, 1, 1)
142	(4, 0, 1, 1)
143	(5, 0, 1, 1)
144	(6, 0, 1, 1)
145	(7, 0, 1, 1)
146	(0, 1, 1, 1)
147	(2, 1, 1, 1)
148	(3, 1, 1, 1)
149	(4, 1, 1, 1)

i	P_i
150	(5, 1, 1, 1)
151	(6, 1, 1, 1)
152	(7, 1, 1, 1)
153	(0, 2, 1, 1)
154	(1, 2, 1, 1)
155	(2, 2, 1, 1)
156	(3, 2, 1, 1)
157	(4, 2, 1, 1)
158	(5, 2, 1, 1)
159	(6, 2, 1, 1)

,

i	P_i
160	(7, 2, 1, 1)
161	(0, 3, 1, 1)
162	(1, 3, 1, 1)
163	(2, 3, 1, 1)
164	(3, 3, 1, 1)
165	(4, 3, 1, 1)
166	(5, 3, 1, 1)
167	(6, 3, 1, 1)
168	(7, 3, 1, 1)
169	(0, 4, 1, 1)

,

i	P_i
170	(1, 4, 1, 1)
171	(2, 4, 1, 1)
172	(3, 4, 1, 1)
173	(4, 4, 1, 1)
174	(5, 4, 1, 1)
175	(6, 4, 1, 1)
176	(7, 4, 1, 1)
177	(0, 5, 1, 1)
178	(1, 5, 1, 1)
179	(2, 5, 1, 1)

,

i	P_i
180	(3, 5, 1, 1)
181	(4, 5, 1, 1)
182	(5, 5, 1, 1)
183	(6, 5, 1, 1)
184	(7, 5, 1, 1)
185	(0, 6, 1, 1)
186	(1, 6, 1, 1)
187	(2, 6, 1, 1)
188	(3, 6, 1, 1)
189	(4, 6, 1, 1)

,

i	P_i
190	(5, 6, 1, 1)
191	(6, 6, 1, 1)
192	(7, 6, 1, 1)
193	(0, 7, 1, 1)
194	(1, 7, 1, 1)
195	(2, 7, 1, 1)
196	(3, 7, 1, 1)
197	(4, 7, 1, 1)
198	(5, 7, 1, 1)
199	(6, 7, 1, 1)

i	P_i
200	(7, 7, 1, 1)
201	(0, 0, 2, 1)
202	(1, 0, 2, 1)
203	(2, 0, 2, 1)
204	(3, 0, 2, 1)
205	(4, 0, 2, 1)
206	(5, 0, 2, 1)
207	(6, 0, 2, 1)
208	(7, 0, 2, 1)
209	(0, 1, 2, 1)

,

i	P_i
210	(1, 1, 2, 1)
211	(2, 1, 2, 1)
212	(3, 1, 2, 1)
213	(4, 1, 2, 1)
214	(5, 1, 2, 1)
215	(6, 1, 2, 1)
216	(7, 1, 2, 1)
217	(0, 2, 2, 1)
218	(1, 2, 2, 1)
219	(2, 2, 2, 1)

,

i	P_i
220	(3, 2, 2, 1)
221	(4, 2, 2, 1)
222	(5, 2, 2, 1)
223	(6, 2, 2, 1)
224	(7, 2, 2, 1)
225	(0, 3, 2, 1)
226	(1, 3, 2, 1)
227	(2, 3, 2, 1)
228	(3, 3, 2, 1)
229	(4, 3, 2, 1)

,

i	P_i
230	(5, 3, 2, 1)
231	(6, 3, 2, 1)
232	(7, 3, 2, 1)
233	(0, 4, 2, 1)
234	(1, 4, 2, 1)
235	(2, 4, 2, 1)
236	(3, 4, 2, 1)
237	(4, 4, 2, 1)
238	(5, 4, 2, 1)
239	(6, 4, 2, 1)

,

i	P_i
240	(7, 4, 2, 1)
241	(0, 5, 2, 1)
242	(1, 5, 2, 1)
243	(2, 5, 2, 1)
244	(3, 5, 2, 1)
245	(4, 5, 2, 1)
246	(5, 5, 2, 1)
247	(6, 5, 2, 1)
248	(7, 5, 2, 1)
249	(0, 6, 2, 1)

i	P_i
250	$(1,6,2,1)$
251	$(2,6,2,1)$
252	$(3,6,2,1)$
253	$(4,6,2,1)$
254	$(5,6,2,1)$
255	$(6,6,2,1)$
256	$(7,6,2,1)$
257	$(0,7,2,1)$
258	$(1,7,2,1)$
259	$(2,7,2,1)$

,

i	P_i
260	$(3,7,2,1)$
261	$(4,7,2,1)$
262	$(5,7,2,1)$
263	$(6,7,2,1)$
264	$(7,7,2,1)$
265	$(0,0,3,1)$
266	$(1,0,3,1)$
267	$(2,0,3,1)$
268	$(3,0,3,1)$
269	$(4,0,3,1)$

,

i	P_i
270	$(5,0,3,1)$
271	$(6,0,3,1)$
272	$(7,0,3,1)$
273	$(0,1,3,1)$
274	$(1,1,3,1)$
275	$(2,1,3,1)$
276	$(3,1,3,1)$
277	$(4,1,3,1)$
278	$(5,1,3,1)$
279	$(6,1,3,1)$

,

i	P_i
280	$(7,1,3,1)$
281	$(0,2,3,1)$
282	$(1,2,3,1)$
283	$(2,2,3,1)$
284	$(3,2,3,1)$
285	$(4,2,3,1)$
286	$(5,2,3,1)$
287	$(6,2,3,1)$
288	$(7,2,3,1)$
289	$(0,3,3,1)$

,

i	P_i
290	$(1,3,3,1)$
291	$(2,3,3,1)$
292	$(3,3,3,1)$
293	$(4,3,3,1)$
294	$(5,3,3,1)$
295	$(6,3,3,1)$
296	$(7,3,3,1)$
297	$(0,4,3,1)$
298	$(1,4,3,1)$
299	$(2,4,3,1)$

i	P_i
300	$(3,4,3,1)$
301	$(4,4,3,1)$
302	$(5,4,3,1)$
303	$(6,4,3,1)$
304	$(7,4,3,1)$
305	$(0,5,3,1)$
306	$(1,5,3,1)$
307	$(2,5,3,1)$
308	$(3,5,3,1)$
309	$(4,5,3,1)$

,

i	P_i
310	$(5,5,3,1)$
311	$(6,5,3,1)$
312	$(7,5,3,1)$
313	$(0,6,3,1)$
314	$(1,6,3,1)$
315	$(2,6,3,1)$
316	$(3,6,3,1)$
317	$(4,6,3,1)$
318	$(5,6,3,1)$
319	$(6,6,3,1)$

,

i	P_i
320	$(7,6,3,1)$
321	$(0,7,3,1)$
322	$(1,7,3,1)$
323	$(2,7,3,1)$
324	$(3,7,3,1)$
325	$(4,7,3,1)$
326	$(5,7,3,1)$
327	$(6,7,3,1)$
328	$(7,7,3,1)$
329	$(0,0,4,1)$

,

i	P_i
330	$(1,0,4,1)$
331	$(2,0,4,1)$
332	$(3,0,4,1)$
333	$(4,0,4,1)$
334	$(5,0,4,1)$
335	$(6,0,4,1)$
336	$(7,0,4,1)$
337	$(0,1,4,1)$
338	$(1,1,4,1)$
339	$(2,1,4,1)$

,

i	P_i
340	$(3,1,4,1)$
341	$(4,1,4,1)$
342	$(5,1,4,1)$
343	$(6,1,4,1)$
344	$(7,1,4,1)$
345	$(0,2,4,1)$
346	$(1,2,4,1)$
347	$(2,2,4,1)$
348	$(3,2,4,1)$
349	$(4,2,4,1)$

i	P_i
350	$(5,2,4,1)$
351	$(6,2,4,1)$
352	$(7,2,4,1)$
353	$(0,3,4,1)$
354	$(1,3,4,1)$
355	$(2,3,4,1)$
356	$(3,3,4,1)$
357	$(4,3,4,1)$
358	$(5,3,4,1)$
359	$(6,3,4,1)$

,

i	P_i
360	$(7,3,4,1)$
361	$(0,4,4,1)$
362	$(1,4,4,1)$
363	$(2,4,4,1)$
364	$(3,4,4,1)$
365	$(4,4,4,1)$
366	$(5,4,4,1)$
367	$(6,4,4,1)$
368	$(7,4,4,1)$
369	$(0,5,4,1)$

,

i	P_i
370	$(1,5,4,1)$
371	$(2,5,4,1)$
372	$(3,5,4,1)$
373	$(4,5,4,1)$
374	$(5,5,4,1)$
375	$(6,5,4,1)$
376	$(7,5,4,1)$
377	$(0,6,4,1)$
378	$(1,6,4,1)$
379	$(2,6,4,1)$

,

i	P_i
380	$(3,6,4,1)$
381	$(4,6,4,1)$
382	$(5,6,4,1)$
383	$(6,6,4,1)$
384	$(7,6,4,1)$
385	$(0,7,4,1)$
386	$(1,7,4,1)$
387	$(2,7,4,1)$
388	$(3,7,4,1)$
389	$(4,7,4,1)$

,

i	P_i
390	$(5,7,4,1)$
391	$(6,7,4,1)$
392	$(7,7,4,1)$
393	$(0,0,5,1)$
394	$(1,0,5,1)$
395	$(2,0,5,1)$
396	$(3,0,5,1)$
397	$(4,0,5,1)$
398	$(5,0,5,1)$
399	$(6,0,5,1)$

i	P_i
400	$(7,0,5,1)$
401	$(0,1,5,1)$
402	$(1,1,5,1)$
403	$(2,1,5,1)$
404	$(3,1,5,1)$
405	$(4,1,5,1)$
406	$(5,1,5,1)$
407	$(6,1,5,1)$
408	$(7,1,5,1)$
409	$(0,2,5,1)$

i	P_i
410	$(1,2,5,1)$
411	$(2,2,5,1)$
412	$(3,2,5,1)$
413	$(4,2,5,1)$
414	$(5,2,5,1)$
415	$(6,2,5,1)$
416	$(7,2,5,1)$
417	$(0,3,5,1)$
418	$(1,3,5,1)$
419	$(2,3,5,1)$

i	P_i
420	$(3,3,5,1)$
421	$(4,3,5,1)$
422	$(5,3,5,1)$
423	$(6,3,5,1)$
424	$(7,3,5,1)$
425	$(0,4,5,1)$
426	$(1,4,5,1)$
427	$(2,4,5,1)$
428	$(3,4,5,1)$
429	$(4,4,5,1)$

i	P_i
430	$(5,4,5,1)$
431	$(6,4,5,1)$
432	$(7,4,5,1)$
433	$(0,5,5,1)$
434	$(1,5,5,1)$
435	$(2,5,5,1)$
436	$(3,5,5,1)$
437	$(4,5,5,1)$
438	$(5,5,5,1)$
439	$(6,5,5,1)$

i	P_i
440	$(7,5,5,1)$
441	$(0,6,5,1)$
442	$(1,6,5,1)$
443	$(2,6,5,1)$
444	$(3,6,5,1)$
445	$(4,6,5,1)$
446	$(5,6,5,1)$
447	$(6,6,5,1)$
448	$(7,6,5,1)$
449	$(0,7,5,1)$

i	P_i
450	$(1,7,5,1)$
451	$(2,7,5,1)$
452	$(3,7,5,1)$
453	$(4,7,5,1)$
454	$(5,7,5,1)$
455	$(6,7,5,1)$
456	$(7,7,5,1)$
457	$(0,0,6,1)$
458	$(1,0,6,1)$
459	$(2,0,6,1)$

i	P_i
460	$(3,0,6,1)$
461	$(4,0,6,1)$
462	$(5,0,6,1)$
463	$(6,0,6,1)$
464	$(7,0,6,1)$
465	$(0,1,6,1)$
466	$(1,1,6,1)$
467	$(2,1,6,1)$
468	$(3,1,6,1)$
469	$(4,1,6,1)$

i	P_i
470	$(5,1,6,1)$
471	$(6,1,6,1)$
472	$(7,1,6,1)$
473	$(0,2,6,1)$
474	$(1,2,6,1)$
475	$(2,2,6,1)$
476	$(3,2,6,1)$
477	$(4,2,6,1)$
478	$(5,2,6,1)$
479	$(6,2,6,1)$

i	P_i
480	$(7,2,6,1)$
481	$(0,3,6,1)$
482	$(1,3,6,1)$
483	$(2,3,6,1)$
484	$(3,3,6,1)$
485	$(4,3,6,1)$
486	$(5,3,6,1)$
487	$(6,3,6,1)$
488	$(7,3,6,1)$
489	$(0,4,6,1)$

i	P_i
490	$(1,4,6,1)$
491	$(2,4,6,1)$
492	$(3,4,6,1)$
493	$(4,4,6,1)$
494	$(5,4,6,1)$
495	$(6,4,6,1)$
496	$(7,4,6,1)$
497	$(0,5,6,1)$
498	$(1,5,6,1)$
499	$(2,5,6,1)$

i	P_i
500	$(3,5,6,1)$
501	$(4,5,6,1)$
502	$(5,5,6,1)$
503	$(6,5,6,1)$
504	$(7,5,6,1)$
505	$(0,6,6,1)$
506	$(1,6,6,1)$
507	$(2,6,6,1)$
508	$(3,6,6,1)$
509	$(4,6,6,1)$

i	P_i
510	$(5,6,6,1)$
511	$(6,6,6,1)$
512	$(7,6,6,1)$
513	$(0,7,6,1)$
514	$(1,7,6,1)$
515	$(2,7,6,1)$
516	$(3,7,6,1)$
517	$(4,7,6,1)$
518	$(5,7,6,1)$
519	$(6,7,6,1)$

i	P_i
520	$(7,7,6,1)$
521	$(0,0,7,1)$
522	$(1,0,7,1)$
523	$(2,0,7,1)$
524	$(3,0,7,1)$
525	$(4,0,7,1)$
526	$(5,0,7,1)$
527	$(6,0,7,1)$
528	$(7,0,7,1)$
529	$(0,1,7,1)$

i	P_i
530	$(1,1,7,1)$
531	$(2,1,7,1)$
532	$(3,1,7,1)$
533	$(4,1,7,1)$
534	$(5,1,7,1)$
535	$(6,1,7,1)$
536	$(7,1,7,1)$
537	$(0,2,7,1)$
538	$(1,2,7,1)$
539	$(2,2,7,1)$

i	P_i
540	$(3,2,7,1)$
541	$(4,2,7,1)$
542	$(5,2,7,1)$
543	$(6,2,7,1)$
544	$(7,2,7,1)$
545	$(0,3,7,1)$
546	$(1,3,7,1)$
547	$(2,3,7,1)$
548	$(3,3,7,1)$
549	$(4,3,7,1)$

i	P_i
550	(5,3,7,1)
551	(6,3,7,1)
552	(7,3,7,1)
553	(0,4,7,1)
554	(1,4,7,1)
555	(2,4,7,1)
556	(3,4,7,1)
557	(4,4,7,1)
558	(5,4,7,1)
559	(6,4,7,1)

,

i	P_i
560	(7,4,7,1)
561	(0,5,7,1)
562	(1,5,7,1)
563	(2,5,7,1)
564	(3,5,7,1)
565	(4,5,7,1)
566	(5,5,7,1)
567	(6,5,7,1)
568	(7,5,7,1)
569	(0,6,7,1)

,

i	P_i
570	(1,6,7,1)
571	(2,6,7,1)
572	(3,6,7,1)
573	(4,6,7,1)
574	(5,6,7,1)
575	(6,6,7,1)
576	(7,6,7,1)
577	(0,7,7,1)
578	(1,7,7,1)
579	(2,7,7,1)

,

i	P_i
580	(3,7,7,1)
581	(4,7,7,1)
582	(5,7,7,1)
583	(6,7,7,1)
584	(7,7,7,1)

2.2 The orthogonal group

Group action $P\Gamma L(4,8)Wedge$ of degree 37449
Does not have strong generators.

2.3 The group stabilizing the fixed line

Group action $P\Gamma L(4,8)Wedgeres648$ of degree 648
Does not have strong generators.

i	P_i
0	(0,1,0,0,0,0)
1	(0,0,1,0,0,0)
2	(0,0,0,1,0,0)
3	(0,0,0,0,1,0)
4	(1,1,0,0,0,0)
5	(2,1,0,0,0,0)
6	(3,1,0,0,0,0)
7	(4,1,0,0,0,0)
8	(5,1,0,0,0,0)
9	(6,1,0,0,0,0)

,

i	P_i
10	(7,1,0,0,0,0)
11	(1,0,1,0,0,0)
12	(2,0,1,0,0,0)
13	(3,0,1,0,0,0)
14	(4,0,1,0,0,0)
15	(5,0,1,0,0,0)
16	(6,0,1,0,0,0)
17	(7,0,1,0,0,0)
18	(0,1,1,0,0,0)
19	(1,1,1,0,0,0)

,

i	P_i
20	(2,1,1,0,0,0)
21	(3,1,1,0,0,0)
22	(4,1,1,0,0,0)
23	(5,1,1,0,0,0)
24	(6,1,1,0,0,0)
25	(7,1,1,0,0,0)
26	(0,2,1,0,0,0)
27	(1,2,1,0,0,0)
28	(2,2,1,0,0,0)
29	(3,2,1,0,0,0)

,

i	P_i
30	(4,2,1,0,0,0)
31	(5,2,1,0,0,0)
32	(6,2,1,0,0,0)
33	(7,2,1,0,0,0)
34	(0,3,1,0,0,0)
35	(1,3,1,0,0,0)
36	(2,3,1,0,0,0)
37	(3,3,1,0,0,0)
38	(4,3,1,0,0,0)
39	(5,3,1,0,0,0)

,

i	P_i
40	(6,3,1,0,0,0)
41	(7,3,1,0,0,0)
42	(0,4,1,0,0,0)
43	(1,4,1,0,0,0)
44	(2,4,1,0,0,0)
45	(3,4,1,0,0,0)
46	(4,4,1,0,0,0)
47	(5,4,1,0,0,0)
48	(6,4,1,0,0,0)
49	(7,4,1,0,0,0)

i	P_i
50	$(0,5,1,0,0,0)$
51	$(1,5,1,0,0,0)$
52	$(2,5,1,0,0,0)$
53	$(3,5,1,0,0,0)$
54	$(4,5,1,0,0,0)$
55	$(5,5,1,0,0,0)$
56	$(6,5,1,0,0,0)$
57	$(7,5,1,0,0,0)$
58	$(0,6,1,0,0,0)$
59	$(1,6,1,0,0,0)$

,

i	P_i
60	$(2,6,1,0,0,0)$
61	$(3,6,1,0,0,0)$
62	$(4,6,1,0,0,0)$
63	$(5,6,1,0,0,0)$
64	$(6,6,1,0,0,0)$
65	$(7,6,1,0,0,0)$
66	$(0,7,1,0,0,0)$
67	$(1,7,1,0,0,0)$
68	$(2,7,1,0,0,0)$
69	$(3,7,1,0,0,0)$

,

i	P_i
70	$(4,7,1,0,0,0)$
71	$(5,7,1,0,0,0)$
72	$(6,7,1,0,0,0)$
73	$(7,7,1,0,0,0)$
74	$(1,0,0,1,0,0)$
75	$(2,0,0,1,0,0)$
76	$(3,0,0,1,0,0)$
77	$(4,0,0,1,0,0)$
78	$(5,0,0,1,0,0)$
79	$(6,0,0,1,0,0)$

,

i	P_i
80	$(7,0,0,1,0,0)$
81	$(0,1,0,1,0,0)$
82	$(1,1,0,1,0,0)$
83	$(2,1,0,1,0,0)$
84	$(3,1,0,1,0,0)$
85	$(4,1,0,1,0,0)$
86	$(5,1,0,1,0,0)$
87	$(6,1,0,1,0,0)$
88	$(7,1,0,1,0,0)$
89	$(0,2,0,1,0,0)$

,

i	P_i
90	$(1,2,0,1,0,0)$
91	$(2,2,0,1,0,0)$
92	$(3,2,0,1,0,0)$
93	$(4,2,0,1,0,0)$
94	$(5,2,0,1,0,0)$
95	$(6,2,0,1,0,0)$
96	$(7,2,0,1,0,0)$
97	$(0,3,0,1,0,0)$
98	$(1,3,0,1,0,0)$
99	$(2,3,0,1,0,0)$

i	P_i
100	$(3,3,0,1,0,0)$
101	$(4,3,0,1,0,0)$
102	$(5,3,0,1,0,0)$
103	$(6,3,0,1,0,0)$
104	$(7,3,0,1,0,0)$
105	$(0,4,0,1,0,0)$
106	$(1,4,0,1,0,0)$
107	$(2,4,0,1,0,0)$
108	$(3,4,0,1,0,0)$
109	$(4,4,0,1,0,0)$

,

i	P_i
110	$(5,4,0,1,0,0)$
111	$(6,4,0,1,0,0)$
112	$(7,4,0,1,0,0)$
113	$(0,5,0,1,0,0)$
114	$(1,5,0,1,0,0)$
115	$(2,5,0,1,0,0)$
116	$(3,5,0,1,0,0)$
117	$(4,5,0,1,0,0)$
118	$(5,5,0,1,0,0)$
119	$(6,5,0,1,0,0)$

,

i	P_i
120	$(7,5,0,1,0,0)$
121	$(0,6,0,1,0,0)$
122	$(1,6,0,1,0,0)$
123	$(2,6,0,1,0,0)$
124	$(3,6,0,1,0,0)$
125	$(4,6,0,1,0,0)$
126	$(5,6,0,1,0,0)$
127	$(6,6,0,1,0,0)$
128	$(7,6,0,1,0,0)$
129	$(0,7,0,1,0,0)$

,

i	P_i
130	$(1,7,0,1,0,0)$
131	$(2,7,0,1,0,0)$
132	$(3,7,0,1,0,0)$
133	$(4,7,0,1,0,0)$
134	$(5,7,0,1,0,0)$
135	$(6,7,0,1,0,0)$
136	$(7,7,0,1,0,0)$
137	$(1,0,0,0,1,0)$
138	$(2,0,0,0,1,0)$
139	$(3,0,0,0,1,0)$

,

i	P_i
140	$(4,0,0,0,1,0)$
141	$(5,0,0,0,1,0)$
142	$(6,0,0,0,1,0)$
143	$(7,0,0,0,1,0)$
144	$(0,0,1,0,1,0)$
145	$(1,0,1,0,1,0)$
146	$(2,0,1,0,1,0)$
147	$(3,0,1,0,1,0)$
148	$(4,0,1,0,1,0)$
149	$(5,0,1,0,1,0)$

i	P_i
150	$(6,0,1,0,1,0)$
151	$(7,0,1,0,1,0)$
152	$(0,0,2,0,1,0)$
153	$(1,0,2,0,1,0)$
154	$(2,0,2,0,1,0)$
155	$(3,0,2,0,1,0)$
156	$(4,0,2,0,1,0)$
157	$(5,0,2,0,1,0)$
158	$(6,0,2,0,1,0)$
159	$(7,0,2,0,1,0)$

,

i	P_i
160	$(0,0,3,0,1,0)$
161	$(1,0,3,0,1,0)$
162	$(2,0,3,0,1,0)$
163	$(3,0,3,0,1,0)$
164	$(4,0,3,0,1,0)$
165	$(5,0,3,0,1,0)$
166	$(6,0,3,0,1,0)$
167	$(7,0,3,0,1,0)$
168	$(0,0,4,0,1,0)$
169	$(1,0,4,0,1,0)$

,

i	P_i
170	$(2,0,4,0,1,0)$
171	$(3,0,4,0,1,0)$
172	$(4,0,4,0,1,0)$
173	$(5,0,4,0,1,0)$
174	$(6,0,4,0,1,0)$
175	$(7,0,4,0,1,0)$
176	$(0,0,5,0,1,0)$
177	$(1,0,5,0,1,0)$
178	$(2,0,5,0,1,0)$
179	$(3,0,5,0,1,0)$

,

i	P_i
180	$(4,0,5,0,1,0)$
181	$(5,0,5,0,1,0)$
182	$(6,0,5,0,1,0)$
183	$(7,0,5,0,1,0)$
184	$(0,0,6,0,1,0)$
185	$(1,0,6,0,1,0)$
186	$(2,0,6,0,1,0)$
187	$(3,0,6,0,1,0)$
188	$(4,0,6,0,1,0)$
189	$(5,0,6,0,1,0)$

,

i	P_i
190	$(6,0,6,0,1,0)$
191	$(7,0,6,0,1,0)$
192	$(0,0,7,0,1,0)$
193	$(1,0,7,0,1,0)$
194	$(2,0,7,0,1,0)$
195	$(3,0,7,0,1,0)$
196	$(4,0,7,0,1,0)$
197	$(5,0,7,0,1,0)$
198	$(6,0,7,0,1,0)$
199	$(7,0,7,0,1,0)$

i	P_i	i	P_i	i	P_i	i	P_i	i	P_i
200	(0, 0, 0, 1, 1, 0)	210	(2, 1, 1, 1, 1, 0)	220	(4, 2, 2, 1, 1, 0)	230	(6, 3, 3, 1, 1, 0)	240	(0, 5, 5, 1, 1, 0)
201	(1, 0, 0, 1, 1, 0)	211	(3, 1, 1, 1, 1, 0)	221	(5, 2, 2, 1, 1, 0)	231	(7, 3, 3, 1, 1, 0)	241	(1, 5, 5, 1, 1, 0)
202	(2, 0, 0, 1, 1, 0)	212	(4, 1, 1, 1, 1, 0)	222	(6, 2, 2, 1, 1, 0)	232	(0, 4, 4, 1, 1, 0)	242	(2, 5, 5, 1, 1, 0)
203	(3, 0, 0, 1, 1, 0)	213	(5, 1, 1, 1, 1, 0)	223	(7, 2, 2, 1, 1, 0)	233	(1, 4, 4, 1, 1, 0)	243	(3, 5, 5, 1, 1, 0)
204	(4, 0, 0, 1, 1, 0)	214	(6, 1, 1, 1, 1, 0)	224	(0, 3, 3, 1, 1, 0)	234	(2, 4, 4, 1, 1, 0)	244	(4, 5, 5, 1, 1, 0)
205	(5, 0, 0, 1, 1, 0)	215	(7, 1, 1, 1, 1, 0)	225	(1, 3, 3, 1, 1, 0)	235	(3, 4, 4, 1, 1, 0)	245	(5, 5, 5, 1, 1, 0)
206	(6, 0, 0, 1, 1, 0)	216	(0, 2, 2, 1, 1, 0)	226	(2, 3, 3, 1, 1, 0)	236	(4, 4, 4, 1, 1, 0)	246	(6, 5, 5, 1, 1, 0)
207	(7, 0, 0, 1, 1, 0)	217	(1, 2, 2, 1, 1, 0)	227	(3, 3, 3, 1, 1, 0)	237	(5, 4, 4, 1, 1, 0)	247	(7, 5, 5, 1, 1, 0)
208	(0, 1, 1, 1, 1, 0)	218	(2, 2, 2, 1, 1, 0)	228	(4, 3, 3, 1, 1, 0)	238	(6, 4, 4, 1, 1, 0)	248	(0, 6, 6, 1, 1, 0)
209	(1, 1, 1, 1, 1, 0)	219	(3, 2, 2, 1, 1, 0)	229	(5, 3, 3, 1, 1, 0)	239	(7, 4, 4, 1, 1, 0)	249	(1, 6, 6, 1, 1, 0)

i	P_i	i	P_i	i	P_i	i	P_i	i	P_i
250	(2, 6, 6, 1, 1, 0)	260	(4, 7, 7, 1, 1, 0)	270	(6, 0, 0, 2, 1, 0)	280	(0, 4, 2, 2, 1, 0)	290	(2, 6, 3, 2, 1, 0)
251	(3, 6, 6, 1, 1, 0)	261	(5, 7, 7, 1, 1, 0)	271	(7, 0, 0, 2, 1, 0)	281	(1, 4, 2, 2, 1, 0)	291	(3, 6, 3, 2, 1, 0)
252	(4, 6, 6, 1, 1, 0)	262	(6, 7, 7, 1, 1, 0)	272	(0, 2, 1, 2, 1, 0)	282	(2, 4, 2, 2, 1, 0)	292	(4, 6, 3, 2, 1, 0)
253	(5, 6, 6, 1, 1, 0)	263	(7, 7, 7, 1, 1, 0)	273	(1, 2, 1, 2, 1, 0)	283	(3, 4, 2, 2, 1, 0)	293	(5, 6, 3, 2, 1, 0)
254	(6, 6, 6, 1, 1, 0)	264	(0, 0, 0, 2, 1, 0)	274	(2, 2, 1, 2, 1, 0)	284	(4, 4, 2, 2, 1, 0)	294	(6, 6, 3, 2, 1, 0)
255	(7, 6, 6, 1, 1, 0)	265	(1, 0, 0, 2, 1, 0)	275	(3, 2, 1, 2, 1, 0)	285	(5, 4, 2, 2, 1, 0)	295	(7, 6, 3, 2, 1, 0)
256	(0, 7, 7, 1, 1, 0)	266	(2, 0, 0, 2, 1, 0)	276	(4, 2, 1, 2, 1, 0)	286	(6, 4, 2, 2, 1, 0)	296	(0, 5, 4, 2, 1, 0)
257	(1, 7, 7, 1, 1, 0)	267	(3, 0, 0, 2, 1, 0)	277	(5, 2, 1, 2, 1, 0)	287	(7, 4, 2, 2, 1, 0)	297	(1, 5, 4, 2, 1, 0)
258	(2, 7, 7, 1, 1, 0)	268	(4, 0, 0, 2, 1, 0)	278	(6, 2, 1, 2, 1, 0)	288	(0, 6, 3, 2, 1, 0)	298	(2, 5, 4, 2, 1, 0)
259	(3, 7, 7, 1, 1, 0)	269	(5, 0, 0, 2, 1, 0)	279	(7, 2, 1, 2, 1, 0)	289	(1, 6, 3, 2, 1, 0)	299	(3, 5, 4, 2, 1, 0)

i	P_i	i	P_i	i	P_i	i	P_i	i	P_i
300	(4, 5, 4, 2, 1, 0)	310	(6, 7, 5, 2, 1, 0)	320	(0, 3, 7, 2, 1, 0)	330	(2, 0, 0, 3, 1, 0)	340	(4, 3, 1, 3, 1, 0)
301	(5, 5, 4, 2, 1, 0)	311	(7, 7, 5, 2, 1, 0)	321	(1, 3, 7, 2, 1, 0)	331	(3, 0, 0, 3, 1, 0)	341	(5, 3, 1, 3, 1, 0)
302	(6, 5, 4, 2, 1, 0)	312	(0, 1, 6, 2, 1, 0)	322	(2, 3, 7, 2, 1, 0)	332	(4, 0, 0, 3, 1, 0)	342	(6, 3, 1, 3, 1, 0)
303	(7, 5, 4, 2, 1, 0)	313	(1, 1, 6, 2, 1, 0)	323	(3, 3, 7, 2, 1, 0)	333	(5, 0, 0, 3, 1, 0)	343	(7, 3, 1, 3, 1, 0)
304	(0, 7, 5, 2, 1, 0)	314	(2, 1, 6, 2, 1, 0)	324	(4, 3, 7, 2, 1, 0)	334	(6, 0, 0, 3, 1, 0)	344	(0, 6, 2, 3, 1, 0)
305	(1, 7, 5, 2, 1, 0)	315	(3, 1, 6, 2, 1, 0)	325	(5, 3, 7, 2, 1, 0)	335	(7, 0, 0, 3, 1, 0)	345	(1, 6, 2, 3, 1, 0)
306	(2, 7, 5, 2, 1, 0)	316	(4, 1, 6, 2, 1, 0)	326	(6, 3, 7, 2, 1, 0)	336	(0, 3, 1, 3, 1, 0)	346	(2, 6, 2, 3, 1, 0)
307	(3, 7, 5, 2, 1, 0)	317	(5, 1, 6, 2, 1, 0)	327	(7, 3, 7, 2, 1, 0)	337	(1, 3, 1, 3, 1, 0)	347	(3, 6, 2, 3, 1, 0)
308	(4, 7, 5, 2, 1, 0)	318	(6, 1, 6, 2, 1, 0)	328	(0, 0, 0, 3, 1, 0)	338	(2, 3, 1, 3, 1, 0)	348	(4, 6, 2, 3, 1, 0)
309	(5, 7, 5, 2, 1, 0)	319	(7, 1, 6, 2, 1, 0)	329	(1, 0, 0, 3, 1, 0)	339	(3, 3, 1, 3, 1, 0)	349	(5, 6, 2, 3, 1, 0)

i	P_i
350	$(6,6,2,3,1,0)$
351	$(7,6,2,3,1,0)$
352	$(0,5,3,3,1,0)$
353	$(1,5,3,3,1,0)$
354	$(2,5,3,3,1,0)$
355	$(3,5,3,3,1,0)$
356	$(4,5,3,3,1,0)$
357	$(5,5,3,3,1,0)$
358	$(6,5,3,3,1,0)$
359	$(7,5,3,3,1,0)$

,

i	P_i
360	$(0,1,4,3,1,0)$
361	$(1,1,4,3,1,0)$
362	$(2,1,4,3,1,0)$
363	$(3,1,4,3,1,0)$
364	$(4,1,4,3,1,0)$
365	$(5,1,4,3,1,0)$
366	$(6,1,4,3,1,0)$
367	$(7,1,4,3,1,0)$
368	$(0,2,5,3,1,0)$
369	$(1,2,5,3,1,0)$

,

i	P_i
370	$(2,2,5,3,1,0)$
371	$(3,2,5,3,1,0)$
372	$(4,2,5,3,1,0)$
373	$(5,2,5,3,1,0)$
374	$(6,2,5,3,1,0)$
375	$(7,2,5,3,1,0)$
376	$(0,7,6,3,1,0)$
377	$(1,7,6,3,1,0)$
378	$(2,7,6,3,1,0)$
379	$(3,7,6,3,1,0)$

,

i	P_i
380	$(4,7,6,3,1,0)$
381	$(5,7,6,3,1,0)$
382	$(6,7,6,3,1,0)$
383	$(7,7,6,3,1,0)$
384	$(0,4,7,3,1,0)$
385	$(1,4,7,3,1,0)$
386	$(2,4,7,3,1,0)$
387	$(3,4,7,3,1,0)$
388	$(4,4,7,3,1,0)$
389	$(5,4,7,3,1,0)$

,

i	P_i
390	$(6,4,7,3,1,0)$
391	$(7,4,7,3,1,0)$
392	$(0,0,0,4,1,0)$
393	$(1,0,0,4,1,0)$
394	$(2,0,0,4,1,0)$
395	$(3,0,0,4,1,0)$
396	$(4,0,0,4,1,0)$
397	$(5,0,0,4,1,0)$
398	$(6,0,0,4,1,0)$
399	$(7,0,0,4,1,0)$

i	P_i
400	$(0,4,1,4,1,0)$
401	$(1,4,1,4,1,0)$
402	$(2,4,1,4,1,0)$
403	$(3,4,1,4,1,0)$
404	$(4,4,1,4,1,0)$
405	$(5,4,1,4,1,0)$
406	$(6,4,1,4,1,0)$
407	$(7,4,1,4,1,0)$
408	$(0,5,2,4,1,0)$
409	$(1,5,2,4,1,0)$

,

i	P_i
410	$(2,5,2,4,1,0)$
411	$(3,5,2,4,1,0)$
412	$(4,5,2,4,1,0)$
413	$(5,5,2,4,1,0)$
414	$(6,5,2,4,1,0)$
415	$(7,5,2,4,1,0)$
416	$(0,1,3,4,1,0)$
417	$(1,1,3,4,1,0)$
418	$(2,1,3,4,1,0)$
419	$(3,1,3,4,1,0)$

,

i	P_i
420	$(4,1,3,4,1,0)$
421	$(5,1,3,4,1,0)$
422	$(6,1,3,4,1,0)$
423	$(7,1,3,4,1,0)$
424	$(0,7,4,4,1,0)$
425	$(1,7,4,4,1,0)$
426	$(2,7,4,4,1,0)$
427	$(3,7,4,4,1,0)$
428	$(4,7,4,4,1,0)$
429	$(5,7,4,4,1,0)$

,

i	P_i
430	$(6,7,4,4,1,0)$
431	$(7,7,4,4,1,0)$
432	$(0,3,5,4,1,0)$
433	$(1,3,5,4,1,0)$
434	$(2,3,5,4,1,0)$
435	$(3,3,5,4,1,0)$
436	$(4,3,5,4,1,0)$
437	$(5,3,5,4,1,0)$
438	$(6,3,5,4,1,0)$
439	$(7,3,5,4,1,0)$

,

i	P_i
440	$(0,2,6,4,1,0)$
441	$(1,2,6,4,1,0)$
442	$(2,2,6,4,1,0)$
443	$(3,2,6,4,1,0)$
444	$(4,2,6,4,1,0)$
445	$(5,2,6,4,1,0)$
446	$(6,2,6,4,1,0)$
447	$(7,2,6,4,1,0)$
448	$(0,6,7,4,1,0)$
449	$(1,6,7,4,1,0)$

i	P_i
450	$(2,6,7,4,1,0)$
451	$(3,6,7,4,1,0)$
452	$(4,6,7,4,1,0)$
453	$(5,6,7,4,1,0)$
454	$(6,6,7,4,1,0)$
455	$(7,6,7,4,1,0)$
456	$(0,0,0,5,1,0)$
457	$(1,0,0,5,1,0)$
458	$(2,0,0,5,1,0)$
459	$(3,0,0,5,1,0)$

,

i	P_i
460	$(4,0,0,5,1,0)$
461	$(5,0,0,5,1,0)$
462	$(6,0,0,5,1,0)$
463	$(7,0,0,5,1,0)$
464	$(0,5,1,5,1,0)$
465	$(1,5,1,5,1,0)$
466	$(2,5,1,5,1,0)$
467	$(3,5,1,5,1,0)$
468	$(4,5,1,5,1,0)$
469	$(5,5,1,5,1,0)$

,

i	P_i
470	$(6,5,1,5,1,0)$
471	$(7,5,1,5,1,0)$
472	$(0,7,2,5,1,0)$
473	$(1,7,2,5,1,0)$
474	$(2,7,2,5,1,0)$
475	$(3,7,2,5,1,0)$
476	$(4,7,2,5,1,0)$
477	$(5,7,2,5,1,0)$
478	$(6,7,2,5,1,0)$
479	$(7,7,2,5,1,0)$

,

i	P_i
480	$(0,2,3,5,1,0)$
481	$(1,2,3,5,1,0)$
482	$(2,2,3,5,1,0)$
483	$(3,2,3,5,1,0)$
484	$(4,2,3,5,1,0)$
485	$(5,2,3,5,1,0)$
486	$(6,2,3,5,1,0)$
487	$(7,2,3,5,1,0)$
488	$(0,3,4,5,1,0)$
489	$(1,3,4,5,1,0)$

,

i	P_i
490	$(2,3,4,5,1,0)$
491	$(3,3,4,5,1,0)$
492	$(4,3,4,5,1,0)$
493	$(5,3,4,5,1,0)$
494	$(6,3,4,5,1,0)$
495	$(7,3,4,5,1,0)$
496	$(0,6,5,5,1,0)$
497	$(1,6,5,5,1,0)$
498	$(2,6,5,5,1,0)$
499	$(3,6,5,5,1,0)$

i	P_i	i	P_i	i	P_i	i	P_i	i	P_i
500	$(4,6,5,5,1,0)$	510	$(6,4,6,5,1,0)$	520	$(0,0,0,6,1,0)$	530	$(2,6,1,6,1,0)$	540	$(4,1,2,6,1,0)$
501	$(5,6,5,5,1,0)$	511	$(7,4,6,5,1,0)$	521	$(1,0,0,6,1,0)$	531	$(3,6,1,6,1,0)$	541	$(5,1,2,6,1,0)$
502	$(6,6,5,5,1,0)$	512	$(0,1,7,5,1,0)$	522	$(2,0,0,6,1,0)$	532	$(4,6,1,6,1,0)$	542	$(6,1,2,6,1,0)$
503	$(7,6,5,5,1,0)$	513	$(1,1,7,5,1,0)$	523	$(3,0,0,6,1,0)$	533	$(5,6,1,6,1,0)$	543	$(7,1,2,6,1,0)$
504	$(0,4,6,5,1,0)$	514	$(2,1,7,5,1,0)$	524	$(4,0,0,6,1,0)$	534	$(6,6,1,6,1,0)$	544	$(0,7,3,6,1,0)$
505	$(1,4,6,5,1,0)$	515	$(3,1,7,5,1,0)$	525	$(5,0,0,6,1,0)$	535	$(7,6,1,6,1,0)$	545	$(1,7,3,6,1,0)$
506	$(2,4,6,5,1,0)$	516	$(4,1,7,5,1,0)$	526	$(6,0,0,6,1,0)$	536	$(0,1,2,6,1,0)$	546	$(2,7,3,6,1,0)$
507	$(3,4,6,5,1,0)$	517	$(5,1,7,5,1,0)$	527	$(7,0,0,6,1,0)$	537	$(1,1,2,6,1,0)$	547	$(3,7,3,6,1,0)$
508	$(4,4,6,5,1,0)$	518	$(6,1,7,5,1,0)$	528	$(0,6,1,6,1,0)$	538	$(2,1,2,6,1,0)$	548	$(4,7,3,6,1,0)$
509	$(5,4,6,5,1,0)$	519	$(7,1,7,5,1,0)$	529	$(1,6,1,6,1,0)$	539	$(3,1,2,6,1,0)$	549	$(5,7,3,6,1,0)$

i	P_i	i	P_i	i	P_i	i	P_i	i	P_i
550	$(6,7,3,6,1,0)$	560	$(0,4,5,6,1,0)$	570	$(2,3,6,6,1,0)$	580	$(4,5,7,6,1,0)$	590	$(6,0,0,7,1,0)$
551	$(7,7,3,6,1,0)$	561	$(1,4,5,6,1,0)$	571	$(3,3,6,6,1,0)$	581	$(5,5,7,6,1,0)$	591	$(7,0,0,7,1,0)$
552	$(0,2,4,6,1,0)$	562	$(2,4,5,6,1,0)$	572	$(4,3,6,6,1,0)$	582	$(6,5,7,6,1,0)$	592	$(0,7,1,7,1,0)$
553	$(1,2,4,6,1,0)$	563	$(3,4,5,6,1,0)$	573	$(5,3,6,6,1,0)$	583	$(7,5,7,6,1,0)$	593	$(1,7,1,7,1,0)$
554	$(2,2,4,6,1,0)$	564	$(4,4,5,6,1,0)$	574	$(6,3,6,6,1,0)$	584	$(0,0,0,7,1,0)$	594	$(2,7,1,7,1,0)$
555	$(3,2,4,6,1,0)$	565	$(5,4,5,6,1,0)$	575	$(7,3,6,6,1,0)$	585	$(1,0,0,7,1,0)$	595	$(3,7,1,7,1,0)$
556	$(4,2,4,6,1,0)$	566	$(6,4,5,6,1,0)$	576	$(0,5,7,6,1,0)$	586	$(2,0,0,7,1,0)$	596	$(4,7,1,7,1,0)$
557	$(5,2,4,6,1,0)$	567	$(7,4,5,6,1,0)$	577	$(1,5,7,6,1,0)$	587	$(3,0,0,7,1,0)$	597	$(5,7,1,7,1,0)$
558	$(6,2,4,6,1,0)$	568	$(0,3,6,6,1,0)$	578	$(2,5,7,6,1,0)$	588	$(4,0,0,7,1,0)$	598	$(6,7,1,7,1,0)$
559	$(7,2,4,6,1,0)$	569	$(1,3,6,6,1,0)$	579	$(3,5,7,6,1,0)$	589	$(5,0,0,7,1,0)$	599	$(7,7,1,7,1,0)$

i	P_i	i	P_i	i	P_i	i	P_i	i	P_i
600	$(0,3,2,7,1,0)$	610	$(2,4,3,7,1,0)$	620	$(4,6,4,7,1,0)$	630	$(6,1,5,7,1,0)$	640	$(0,2,7,7,1,0)$
601	$(1,3,2,7,1,0)$	611	$(3,4,3,7,1,0)$	621	$(5,6,4,7,1,0)$	631	$(7,1,5,7,1,0)$	641	$(1,2,7,7,1,0)$
602	$(2,3,2,7,1,0)$	612	$(4,4,3,7,1,0)$	622	$(6,6,4,7,1,0)$	632	$(0,5,6,7,1,0)$	642	$(2,2,7,7,1,0)$
603	$(3,3,2,7,1,0)$	613	$(5,4,3,7,1,0)$	623	$(7,6,4,7,1,0)$	633	$(1,5,6,7,1,0)$	643	$(3,2,7,7,1,0)$
604	$(4,3,2,7,1,0)$	614	$(6,4,3,7,1,0)$	624	$(0,1,5,7,1,0)$	634	$(2,5,6,7,1,0)$	644	$(4,2,7,7,1,0)$
605	$(5,3,2,7,1,0)$	615	$(7,4,3,7,1,0)$	625	$(1,1,5,7,1,0)$	635	$(3,5,6,7,1,0)$	645	$(5,2,7,7,1,0)$
606	$(6,3,2,7,1,0)$	616	$(0,6,4,7,1,0)$	626	$(2,1,5,7,1,0)$	636	$(4,5,6,7,1,0)$	646	$(6,2,7,7,1,0)$
607	$(7,3,2,7,1,0)$	617	$(1,6,4,7,1,0)$	627	$(3,1,5,7,1,0)$	637	$(5,5,6,7,1,0)$	647	$(7,2,7,7,1,0)$
608	$(0,4,3,7,1,0)$	618	$(2,6,4,7,1,0)$	628	$(4,1,5,7,1,0)$	638	$(6,5,6,7,1,0)$		
609	$(1,4,3,7,1,0)$	619	$(3,6,4,7,1,0)$	629	$(5,1,5,7,1,0)$	639	$(7,5,6,7,1,0)$		

Strong generators for a group of order 21849440256:

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_1, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&\alpha&0\\0&0&0&1\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&\alpha&0\\0&0&0&\alpha^6\end{bmatrix}_1,$$

$$\begin{bmatrix}1&0&0&0\\0&\alpha&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\1&0&0&1\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1&1\end{bmatrix}_0,$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&\alpha^2&1\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&\alpha&0&1\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\\alpha&0&0&1\end{bmatrix}_0,$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&\alpha&1&0\\0&0&0&1\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\\alpha&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_0,$$

$$\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_0$$

Poset classification up to depth 5

3 The orbits

3.1 Number of orbits at depth

Depth	Nb of orbits
0	1
1	1
2	1
3	1
4	5
5	18

3.2 Orbit representatives: overview

N = node
D = depth or level
O = orbit with a level
Rep = orbit representative
SO = (order of stabilizer, orbit length)
L = number of live points
F = number of flags
Gen = number of generators for the stabilizer of the orbit rep.

Table 1: Orbit Representatives

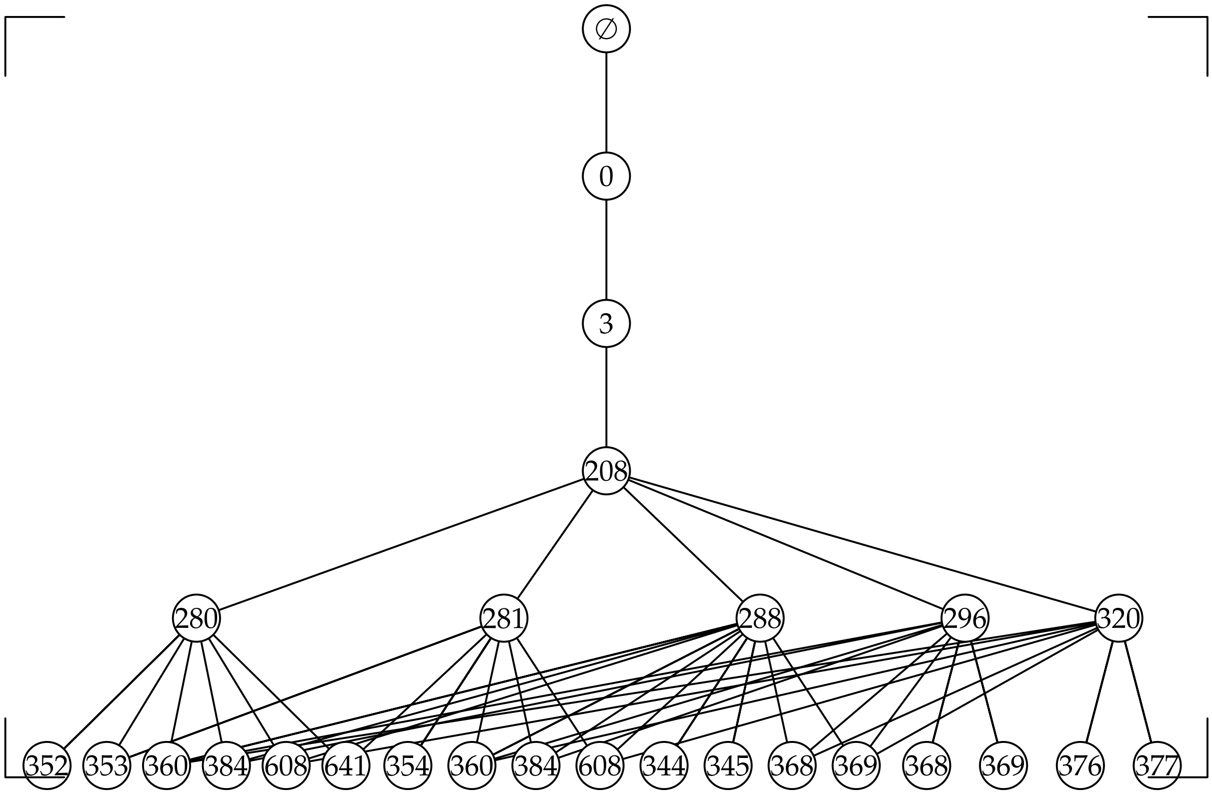
N	D	O	Rep	SO	L	F	Gen
0	0	0	{ }	(21849440256, 1)	648	1	13
1	1	0	{ 0 }	(33718272, 648)	512	1	10
2	2	0	{ 0, 3 }	(131712, 165888)	392	1	8
3	3	0	{ 0, 3, 208 }	(1008, 21676032)	288	5	8
4	4	0	{ 0, 3, 208, 280 }	(672, 32514048)	200	7	5
5	4	1	{ 0, 3, 208, 281 }	(96, 227598336)	200	9	6
6	4	2	{ 0, 3, 208, 288 }	(28, 780337152)	200	14	3
7	4	3	{ 0, 3, 208, 296 }	(84, 260112384)	200	10	3
8	4	4	{ 0, 3, 208, 320 }	(84, 260112384)	200	10	4

Continued on next page

Table 1 – continued from previous page

N	D	O	Rep	SO	L	F	Gen
9	5	0	{ 0, 3, 208, 280, 352 }	(672, 32514048)			5
10	5	1	{ 0, 3, 208, 280, 353 }	(24, 910393344)			4
11	5	2	{ 0, 3, 208, 280, 360 }	(7, 3121348608)			1
12	5	3	{ 0, 3, 208, 280, 384 }	(21, 1040449536)			2
13	5	4	{ 0, 3, 208, 280, 608 }	(21, 1040449536)			2
14	5	5	{ 0, 3, 208, 280, 641 }	(96, 227598336)			5
15	5	6	{ 0, 3, 208, 281, 354 }	(24, 910393344)			4
16	5	7	{ 0, 3, 208, 281, 360 }	(1, 21849440256)			0
17	5	8	{ 0, 3, 208, 281, 384 }	(3, 7283146752)			1
18	5	9	{ 0, 3, 208, 281, 608 }	(3, 7283146752)			1
19	5	10	{ 0, 3, 208, 288, 344 }	(28, 780337152)			3
20	5	11	{ 0, 3, 208, 288, 345 }	(4, 5462360064)			2
21	5	12	{ 0, 3, 208, 288, 368 }	(21, 1040449536)			2
22	5	13	{ 0, 3, 208, 288, 369 }	(3, 7283146752)			1
23	5	14	{ 0, 3, 208, 296, 368 }	(84, 260112384)			3
24	5	15	{ 0, 3, 208, 296, 369 }	(12, 1820786688)			3
25	5	16	{ 0, 3, 208, 320, 376 }	(84, 260112384)			4
26	5	17	{ 0, 3, 208, 320, 377 }	(12, 1820786688)			3

4 The poset of orbits



5 Stabilizers and Schreier trees

5.1 Stabilizers and Schreier trees at level 0

Node 0 at Level 0 Orbit 0 / 1

$$\{\}_{21849440256}$$

Strong generators for a group of order 21849440256:

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_1, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&\alpha&0\\0&0&0&1\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&\alpha&0\\0&0&0&\alpha^6\end{bmatrix}_1,$$

$$\begin{bmatrix}1&0&0&0\\0&\alpha&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\1&0&0&1\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1&1\end{bmatrix}_0,$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&\alpha^2&1\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&\alpha&0&1\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\\alpha&0&0&1\end{bmatrix}_0,$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&\alpha&1&0\\0&0&0&1\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\\alpha&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_0,$$

$$\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_0$$

There are 1 extensions
Number of generators 13
Generators for the Schreier trees:
Generators for a group of order 21849440256:

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_1 \begin{bmatrix}0&1&0&0\\1&0&0&0\\\alpha&0&\alpha&1\\0&0&\alpha&0\end{bmatrix}_0 \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&\alpha&0\\0&0&0&\alpha^6\end{bmatrix}_1 \begin{bmatrix}1&0&0&0\\0&\alpha&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_0$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\1&0&0&1\end{bmatrix}_0 \begin{bmatrix}1&1&0&0\\\alpha^3&0&0&0\\\alpha^4&0&\alpha^6&\alpha^2\\0&0&\alpha^4&0\end{bmatrix}_1 \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&\alpha^2&1\end{bmatrix}_0 \begin{bmatrix}1&\alpha&0&0\\0&1&0&0\\0&\alpha&\alpha^6&1\\0&0&\alpha&0\end{bmatrix}_0$$

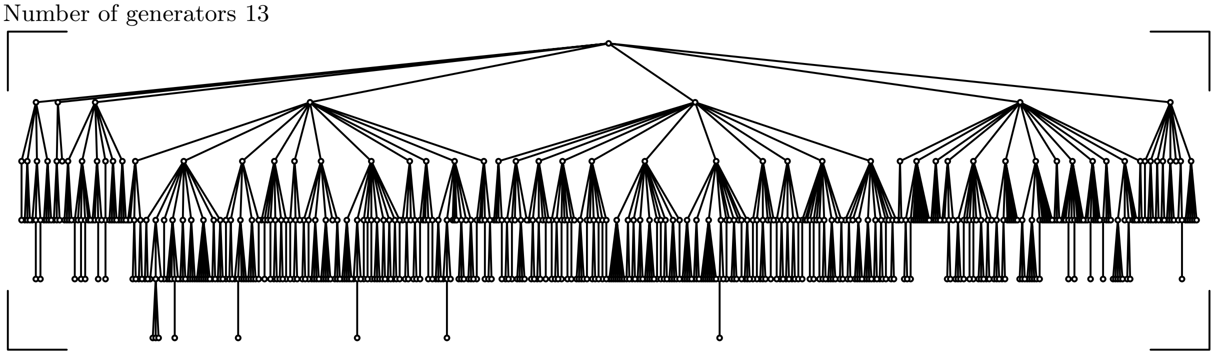
$$\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&\alpha&\alpha^6\\0&0&\alpha&0\end{bmatrix}_1 \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&\alpha&1&0\\0&0&0&1\end{bmatrix}_0 \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}_0 \begin{bmatrix}1&0&0&0\\\alpha&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_0$$

$$\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_0$$

Orbit 0 / 1: Point 0 lies in an orbit of length 641 with average word length 4.39002 $H_{13} = 3.0965$

Node 0 at Level 0 Orbit 0 / 1 Tree 0 / 1

Number of generators 13



Extension number 0
Orbit representative 0
Flag orbit length 648
Flag orbit is defining new orbit 1 at level 1

5.2 Stabilizers and Schreier trees at level 1

Node 1 at Level 1 Orbit 0 / 1

$$\{0\}_{33718272}$$

Strong generators for a group of order 33718272:

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_1, \begin{bmatrix}1&0&0&0\\0&\alpha^3&0&0\\0&0&\alpha^6&0\\0&0&0&\alpha^5\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&\alpha^3&0&0\\0&0&\alpha^2&0\\0&0&0&\alpha^5\end{bmatrix}_1,$$

$$\begin{bmatrix}1&0&0&0\\0&\alpha^5&0&0\\0&0&\alpha^2&0\\0&0&0&1\end{bmatrix}_0,\begin{bmatrix}1&0&0&0\\0&\alpha^3&0&0\\0&0&\alpha^6&0\\0&0&0&\alpha^4\end{bmatrix}_1,\begin{bmatrix}1&0&0&0\\0&\alpha^6&0&0\\0&0&\alpha^3&0\\0&0&\alpha^5&\alpha^5\end{bmatrix}_0,$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&\alpha^5&0\\\alpha^5&\alpha^3&\alpha^4&\alpha^2\end{bmatrix}_0,\begin{bmatrix}1&0&0&0\\0&1&0&0\\\alpha^2&0&\alpha^3&0\\1&\alpha^6&\alpha&\alpha^4\end{bmatrix}_0,\begin{bmatrix}1&0&0&0\\\alpha^6&1&0&0\\0&0&\alpha^4&0\\\alpha&\alpha^2&0&\alpha^5\end{bmatrix}_2,$$

$$\begin{bmatrix}1&0&0&0\\\alpha^6&\alpha^5&0&0\\1&0&\alpha^2&0\\0&\alpha^6&\alpha^2&\alpha^2\end{bmatrix}_2$$

There are 1 extensions
Number of generators 10
Generators for the Schreier trees:
Generators for a group of order 33718272:

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_1\begin{bmatrix}1&0&0&0\\0&\alpha^3&0&0\\0&0&\alpha^6&0\\0&0&0&\alpha^5\end{bmatrix}_0\begin{bmatrix}1&0&0&0\\\alpha^6&\alpha^3&0&0\\\alpha^5&0&\alpha^2&0\\0&\alpha^2&\alpha^5&\alpha^4\end{bmatrix}_2\begin{bmatrix}1&0&0&0\\0&\alpha^5&0&0\\0&0&\alpha^2&0\\0&0&0&1\end{bmatrix}_0$$

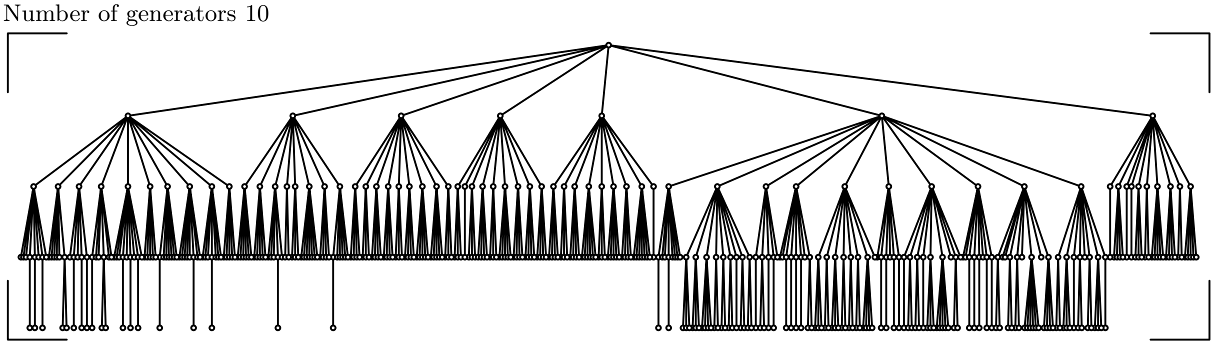
$$\begin{bmatrix}1&0&0&0\\\alpha^3&\alpha^5&0&0\\0&0&\alpha&0\\\alpha&0&0&\alpha^3\end{bmatrix}_2\begin{bmatrix}1&0&0&0\\0&\alpha^6&0&0\\0&0&\alpha^3&0\\0&0&\alpha^5&\alpha^5\end{bmatrix}_0\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&\alpha^5&0\\\alpha^5&\alpha^3&\alpha^4&\alpha^2\end{bmatrix}_0\begin{bmatrix}1&0&0&0\\0&1&0&0\\\alpha^2&0&\alpha^3&0\\1&\alpha^6&\alpha&\alpha^4\end{bmatrix}_0$$

$$\begin{bmatrix}1&0&0&0\\\alpha^3&\alpha^3&0&0\\0&0&\alpha^6&0\\\alpha^6&\alpha^6&\alpha&\alpha^6\end{bmatrix}_0\begin{bmatrix}1&0&0&0\\\alpha^5&\alpha^3&0&0\\\alpha&0&\alpha^5&0\\\alpha^6&1&\alpha^4&1\end{bmatrix}_0$$

Orbit 0 / 1: Point 3 lies in an orbit of length 512 with average word length 4.08594 $H_{10} = 3.32056$

Node 1 at Level 1 Orbit 0 / 1 Tree 0 / 1

Number of generators 10



Extension number 0
Orbit representative 3
Flag orbit length 512
Flag orbit is defining new orbit 2 at level 2

5.3 Stabilizers and Schreier trees at level 2

Node 2 at Level 2 Orbit 0 / 1

$$\{0,3\}_{131712}$$

Strong generators for a group of order 131712:

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_1,\begin{bmatrix}1&0&0&0\\0&\alpha&0&0\\0&0&\alpha^4&0\\0&0&0&\alpha^3\end{bmatrix}_1,\begin{bmatrix}1&0&0&0\\0&\alpha^2&0&0\\0&0&\alpha&0\\0&0&0&\alpha\end{bmatrix}_1,$$

$$\begin{bmatrix}1&0&0&0\\0&\alpha^4&0&0\\0&0&\alpha^4&0\\0&0&0&\alpha^2\end{bmatrix}_1,\begin{bmatrix}1&0&0&0\\0&\alpha^5&0&0\\0&0&\alpha^4&0\\0&0&0&\alpha\end{bmatrix}_1,\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&\alpha^3&0&\alpha\end{bmatrix}_1,$$

$$\begin{bmatrix}1&0&0&0\\0&\alpha&0&0\\\alpha^2&0&\alpha^4&0\\0&\alpha^3&0&1\end{bmatrix}_2,\begin{bmatrix}0&1&0&0\\\alpha^5&0&0&0\\0&\alpha^2&0&1\\\alpha^5&0&1&0\end{bmatrix}_2$$

There are 1 extensions
Number of generators 8
Generators for the Schreier trees:
Generators for a group of order 131712:

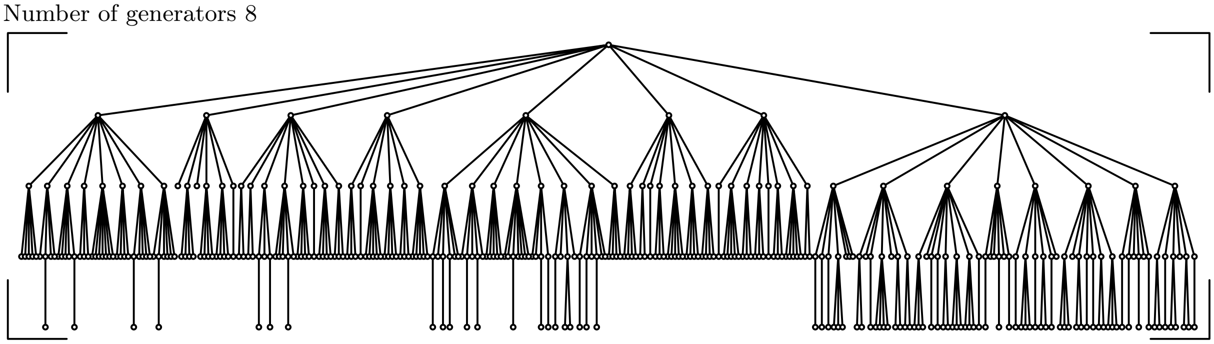
$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\1&0&\alpha^4&0\\0&\alpha^2&0&1\end{bmatrix}_0\begin{bmatrix}1&0&0&0\\0&\alpha&0&0\\0&0&\alpha^4&0\\0&0&0&\alpha^3\end{bmatrix}_1\begin{bmatrix}1&0&0&0\\0&\alpha^2&0&0\\0&0&\alpha&0\\0&0&0&\alpha\end{bmatrix}_1\begin{bmatrix}1&0&0&0\\0&\alpha^5&0&0\\0&0&\alpha^2&0\\0&\alpha^5&0&\alpha^5\end{bmatrix}_1$$

$$\begin{bmatrix}0&1&0&0\\\alpha^4&0&0&0\\0&0&0&\alpha^6\\\alpha^3&0&\alpha^3&0\end{bmatrix}_2\begin{bmatrix}0&1&0&0\\\alpha^4&0&0&0\\0&\alpha&0&\alpha^4\\\alpha^6&0&\alpha^3&0\end{bmatrix}_1\begin{bmatrix}1&0&0&0\\0&\alpha&0&0\\\alpha^2&0&\alpha^4&0\\0&\alpha^3&0&1\end{bmatrix}_2\begin{bmatrix}0&1&0&0\\\alpha^5&0&0&0\\0&\alpha^2&0&1\\\alpha^5&0&1&0\end{bmatrix}_2$$

Orbit 0 / 1: Point 208 lies in an orbit of length 392 with average word length 4.02296 $H_8 = 3.54099$

Node 2 at Level 2 Orbit 0 / 1 Tree 0 / 1

Number of generators 8



Extension number 0
Orbit representative 208
Flag orbit length 392
Flag orbit is defining new orbit 3 at level 3

5.4 Stabilizers and Schreier trees at level 3

Node 3 at Level 3 Orbit 0 / 1

$$\{0, 3, 208\}_{1008}$$

Strong generators for a group of order 1008:

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_1, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&\alpha^2&0\\0&0&0&\alpha^2\end{bmatrix}_1, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&\alpha^6&0\\0&0&0&\alpha^6\end{bmatrix}_0,$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\\alpha&0&\alpha&0\\0&\alpha&0&\alpha\end{bmatrix}_2, \begin{bmatrix}1&0&0&0\\0&1&0&0\\\alpha^2&0&\alpha^6&0\\0&\alpha^2&0&\alpha^6\end{bmatrix}_2, \begin{bmatrix}1&0&0&0\\1&1&0&0\\0&0&1&0\\0&0&1&1\end{bmatrix}_0,$$

$$\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&\alpha^3&0&\alpha^5\\\alpha^3&0&\alpha^5&0\end{bmatrix}_2, \begin{bmatrix}1&1&0&0\\0&1&0&0\\\alpha^2&\alpha^2&\alpha^6&\alpha^6\\0&\alpha^2&0&\alpha^6\end{bmatrix}_0$$

There are 5 extensions
Number of generators 8
Generators for the Schreier trees:
Generators for a group of order 1008:

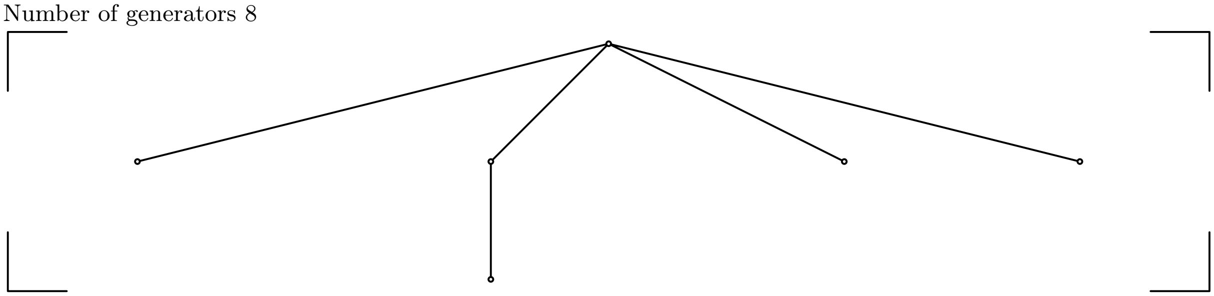
$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_1 \begin{bmatrix}0&1&0&0\\1&0&0&0\\0&\alpha^3&0&\alpha^3\\\alpha^3&0&\alpha^3&0\end{bmatrix}_2 \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&\alpha^6&0\\0&0&0&\alpha^6\end{bmatrix}_0 \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&\alpha^2&0\\0&0&0&\alpha^2\end{bmatrix}_1$$

$$\begin{bmatrix}1&0&0&0\\1&1&0&0\\\alpha^2&0&\alpha^6&0\\\alpha^2&\alpha^2&\alpha^6&\alpha^6\end{bmatrix}_2 \begin{bmatrix}1&0&0&0\\1&1&0&0\\0&0&1&0\\0&0&1&1\end{bmatrix}_0 \begin{bmatrix}0&1&0&0\\1&0&0&0\\0&\alpha^3&0&\alpha^5\\\alpha^3&0&\alpha^5&0\end{bmatrix}_2 \begin{bmatrix}1&1&0&0\\0&1&0&0\\\alpha^2&\alpha^2&\alpha^6&\alpha^6\\0&\alpha^2&0&\alpha^6\end{bmatrix}_0$$

Orbit 0 / 5: Point 280 lies in an orbit of length 6 with average word length 2 $H_8 = 1.19499$
Orbit 1 / 5: Point 281 lies in an orbit of length 42 with average word length 2.88095 $H_8 = 2.30629$
Orbit 2 / 5: Point 288 lies in an orbit of length 144 with average word length 3.71528 $H_8 = 3.02113$
Orbit 3 / 5: Point 296 lies in an orbit of length 48 with average word length 3.14583 $H_8 = 2.4128$
Orbit 4 / 5: Point 320 lies in an orbit of length 48 with average word length 3.14583 $H_8 = 2.4128$

Node 3 at Level 3 Orbit 0 / 1 Tree 0 / 5

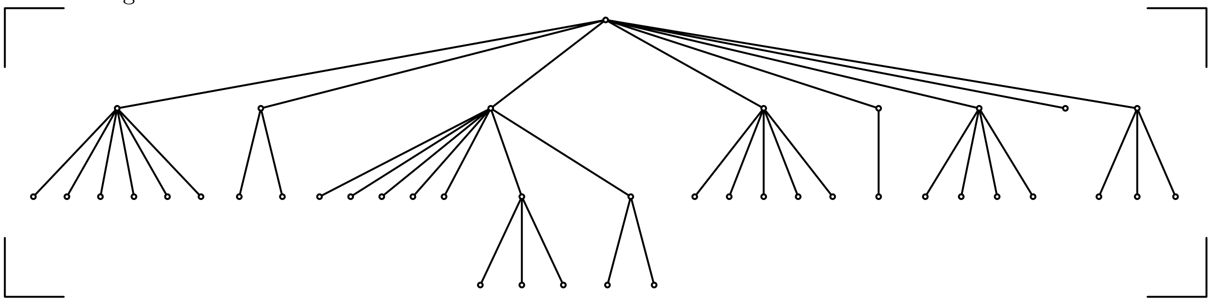
Number of generators 8



Extension number 0
Orbit representative 280
Flag orbit length 6
Flag orbit is defining new orbit 4 at level 4

Node 3 at Level 3 Orbit 0 / 1 Tree 1 / 5

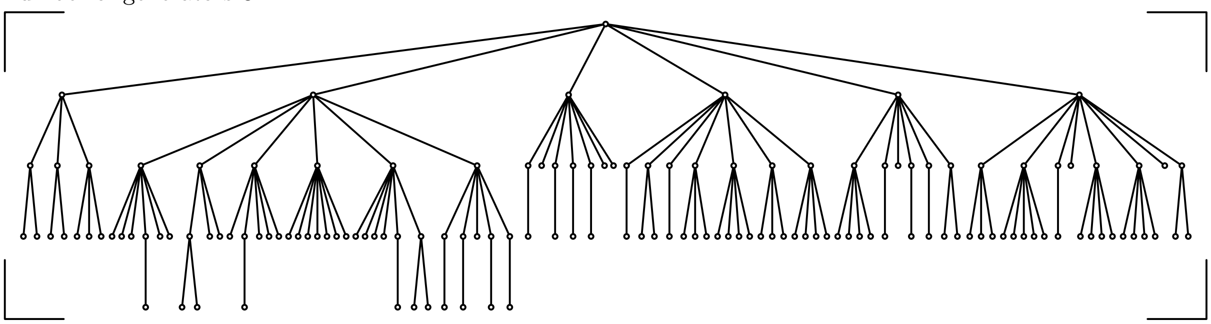
Number of generators 8



Extension number 1
Orbit representative 281
Flag orbit length 42
Flag orbit is defining new orbit 5 at level 4

Node 3 at Level 3 Orbit 0 / 1 Tree 2 / 5

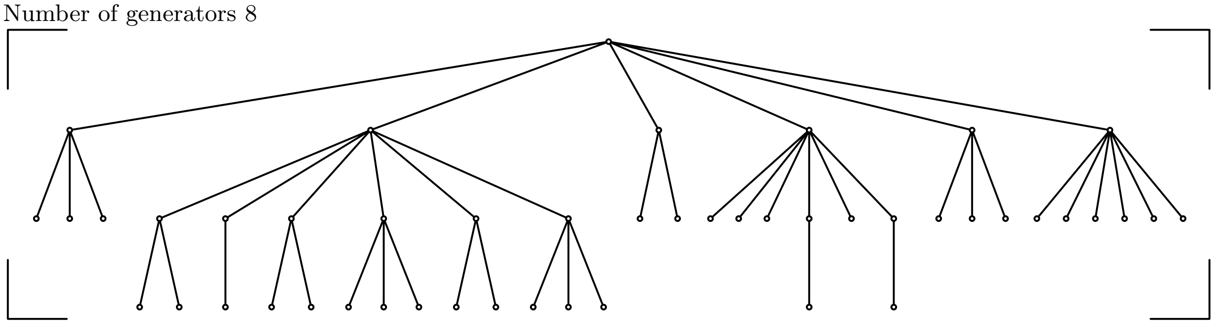
Number of generators 8



Extension number 2
Orbit representative 288
Flag orbit length 144
Flag orbit is defining new orbit 6 at level 4

Node 3 at Level 3 Orbit 0 / 1 Tree 3 / 5

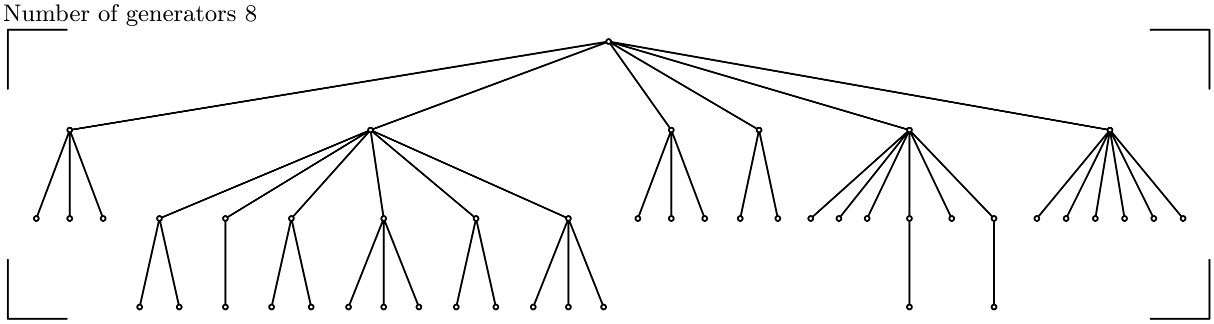
Number of generators 8



Extension number 3
Orbit representative 296
Flag orbit length 48
Flag orbit is defining new orbit 7 at level 4

Node 3 at Level 3 Orbit 0 / 1 Tree 4 / 5

Number of generators 8



Extension number 4
Orbit representative 320
Flag orbit length 48
Flag orbit is defining new orbit 8 at level 4

5.5 Stabilizers and Schreier trees at level 4

Node 4 at Level 4 Orbit 0 / 5

$$\{0, 3, 208, 280\}_{672}$$

Strong generators for a group of order 672:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha^3 & 0 \\ 0 & 0 & 0 & \alpha^3 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \alpha^4 & 0 & \alpha^3 & 0 \\ 0 & \alpha^4 & 0 & \alpha^3 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & \alpha^5 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & \alpha^5 & 1 & \alpha^5 \end{bmatrix}_2,$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & \alpha^3 & 0 & \alpha^4 \\ \alpha^3 & \alpha^3 & \alpha^4 & \alpha^4 \end{bmatrix}_2, \begin{bmatrix} 1 & \alpha^6 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & \alpha^6 & 1 & \alpha^6 \\ 1 & 1 & 1 & 1 \end{bmatrix}_0$$

There are 7 extensions
Number of generators 5
Generators for the Schreier trees:
Generators for a group of order 672:

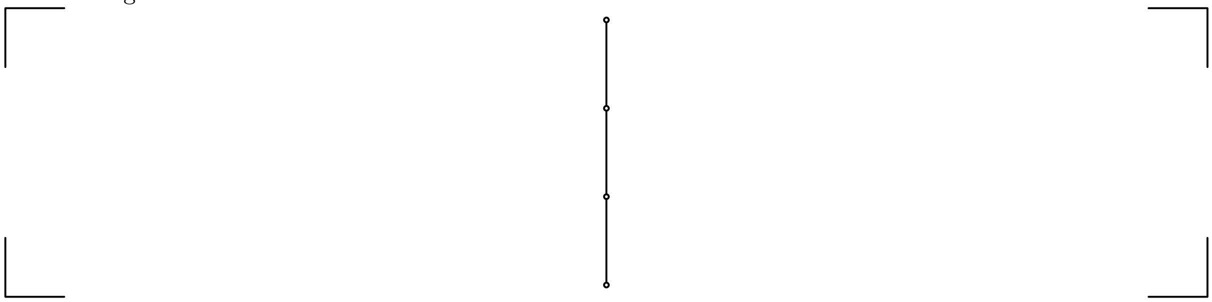
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha^3 & 0 \\ 0 & 0 & 0 & \alpha^3 \end{bmatrix}_0 \begin{bmatrix} 1 & \alpha^6 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & \alpha^6 & 1 & \alpha^6 \\ 1 & 1 & 1 & 1 \end{bmatrix}_0 \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha & \alpha & 0 & 0 \\ \alpha^4 & 0 & \alpha & 0 \\ \alpha^5 & \alpha^5 & \alpha^2 & \alpha^2 \end{bmatrix}_1 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_0$$

$$\begin{bmatrix} 1 & \alpha^6 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ \alpha^6 & \alpha^5 & \alpha^3 & \alpha^2 \\ \alpha^6 & \alpha^6 & \alpha^3 & \alpha^3 \end{bmatrix}_0$$

Orbit 0 / 7: Point 352 lies in an orbit of length 4 with average word length 2.5 $H_5 = 1.43068$
Orbit 1 / 7: Point 353 lies in an orbit of length 28 with average word length 3.5 $H_5 = 2.8488$
Orbit 2 / 7: Point 360 lies in an orbit of length 96 with average word length 4.63542 $H_5 = 3.78895$
Orbit 3 / 7: Point 384 lies in an orbit of length 32 with average word length 3.65625 $H_5 = 2.95891$
Orbit 4 / 7: Point 608 lies in an orbit of length 32 with average word length 3.65625 $H_5 = 2.95891$
Orbit 5 / 7: Point 640 lies in an orbit of length 1 with average word length 1 $H_5 = 0$
Orbit 6 / 7: Point 641 lies in an orbit of length 7 with average word length 2.71429 $H_5 = 1.82948$

Node 4 at Level 4 Orbit 0 / 5 Tree 0 / 7

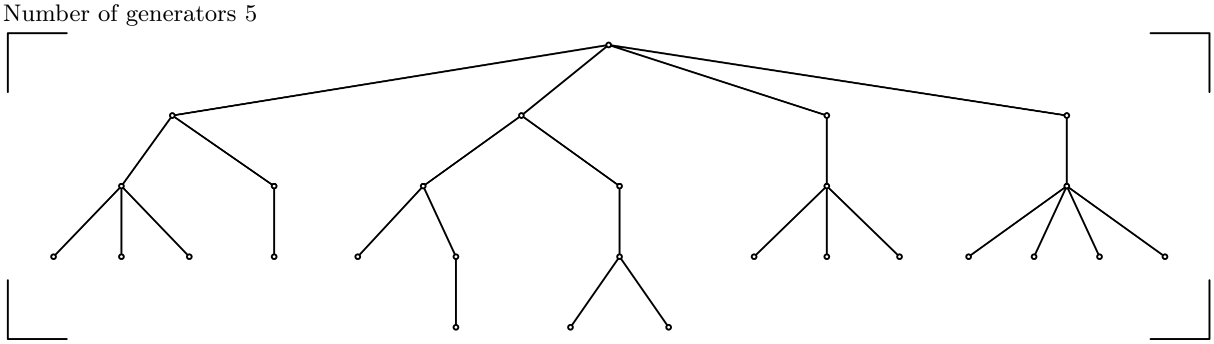
Number of generators 5



Extension number 0
Orbit representative 352
Flag orbit length 4
Flag orbit is defining new orbit 9 at level 5

Node 4 at Level 4 Orbit 0 / 5 Tree 1 / 7

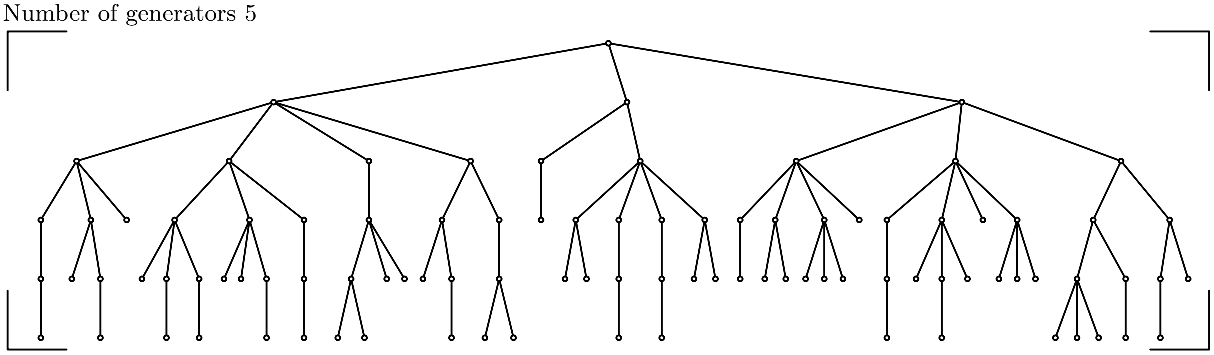
Number of generators 5



Extension number 1
Orbit representative 353
Flag orbit length 28
Flag orbit is defining new orbit 10 at level 5

Node 4 at Level 4 Orbit 0 / 5 Tree 2 / 7

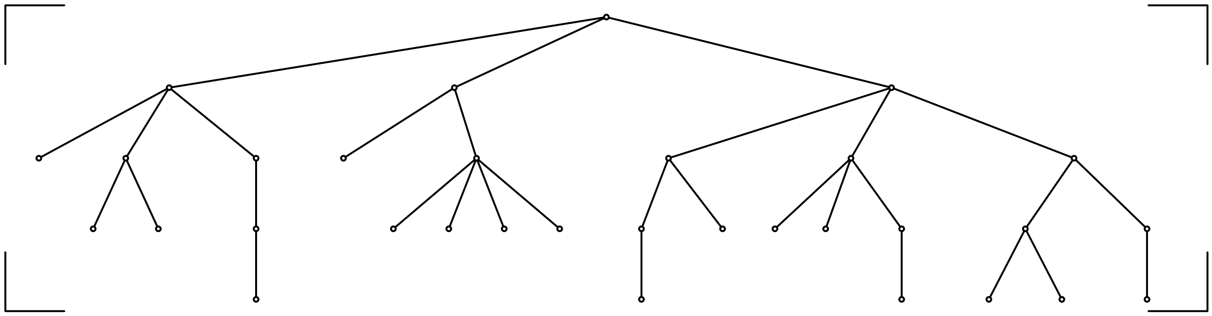
Number of generators 5



Extension number 2
Orbit representative 360
Flag orbit length 96
Flag orbit is defining new orbit 11 at level 5

Node 4 at Level 4 Orbit 0 / 5 Tree 3 / 7

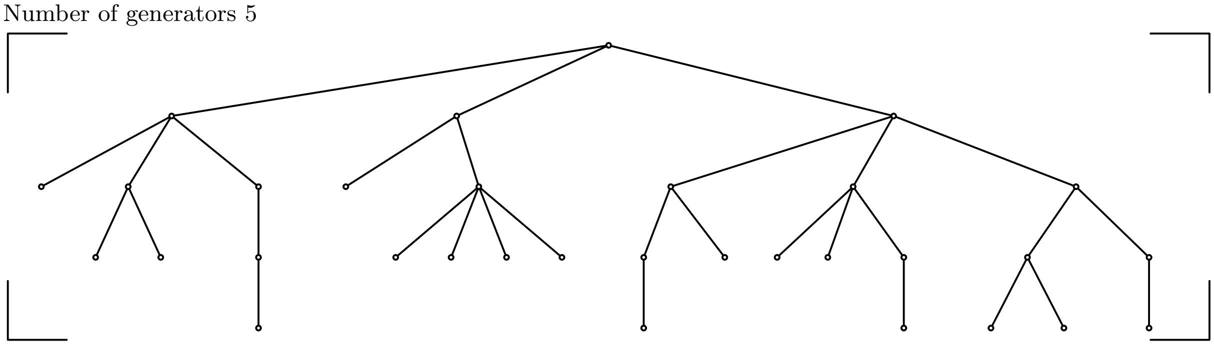
Number of generators 5



Extension number 3
Orbit representative 384
Flag orbit length 32
Flag orbit is defining new orbit 12 at level 5

Node 4 at Level 4 Orbit 0 / 5 Tree 4 / 7

Number of generators 5



Extension number 4
Orbit representative 608
Flag orbit length 32
Flag orbit is defining new orbit 13 at level 5

Node 4 at Level 4 Orbit 0 / 5 Tree 5 / 7

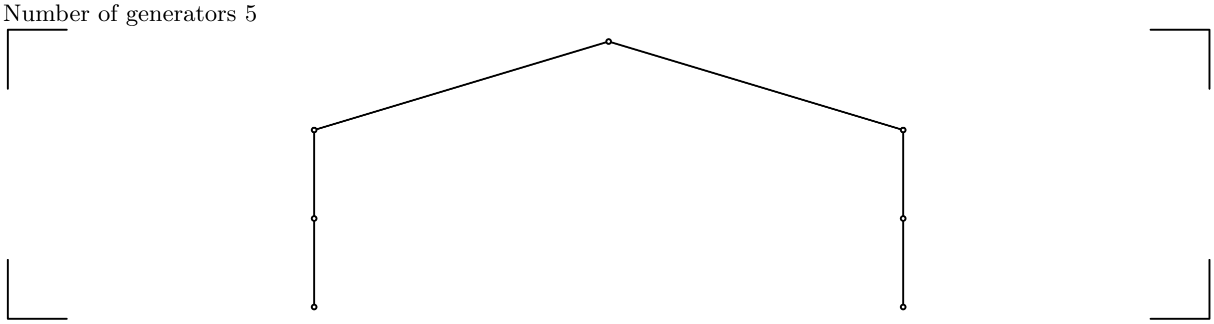
Number of generators 5

Extension number 5
Orbit representative 640
Flag orbit length 1
Flag orbit is fused to node 4 extension 0
Fusion element:

$$\begin{bmatrix} 1 & \alpha & 0 & 0 \\ 0 & \alpha^5 & 0 & 0 \\ \alpha^5 & \alpha^6 & 1 & \alpha \\ 0 & \alpha^3 & 0 & \alpha^5 \end{bmatrix}_1$$

Node 4 at Level 4 Orbit 0 / 5 Tree 6 / 7

Number of generators 5



Extension number 6
Orbit representative 641
Flag orbit length 7
Flag orbit is defining new orbit 14 at level 5

Node 5 at Level 4 Orbit 1 / 5

$$\{0, 3, 208, 281\}_{96}$$

Strong generators for a group of order 96:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \alpha^2 & 0 & 1 & 0 \\ 0 & \alpha^2 & 0 & 1 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \alpha^3 & 0 & 1 & 0 \\ 0 & \alpha^3 & 0 & 1 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \alpha^6 & 0 & 1 & 0 \\ 0 & \alpha^6 & 0 & 1 \end{bmatrix}_0,$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & \alpha^5 & 0 & 0 \\ \alpha^5 & 0 & \alpha^4 & 0 \\ \alpha^5 & \alpha^2 & \alpha^4 & \alpha^2 \end{bmatrix}_2, \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & \alpha^4 & 0 & \alpha^4 \\ \alpha^4 & \alpha^4 & \alpha^4 & \alpha^4 \end{bmatrix}_2, \begin{bmatrix} 1 & \alpha^6 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ \alpha^5 & \alpha^2 & 1 & \alpha^6 \\ \alpha^5 & \alpha^5 & 1 & 1 \end{bmatrix}_0$$

There are 9 extensions
Number of generators 6
Generators for the Schreier trees:
Generators for a group of order 96:

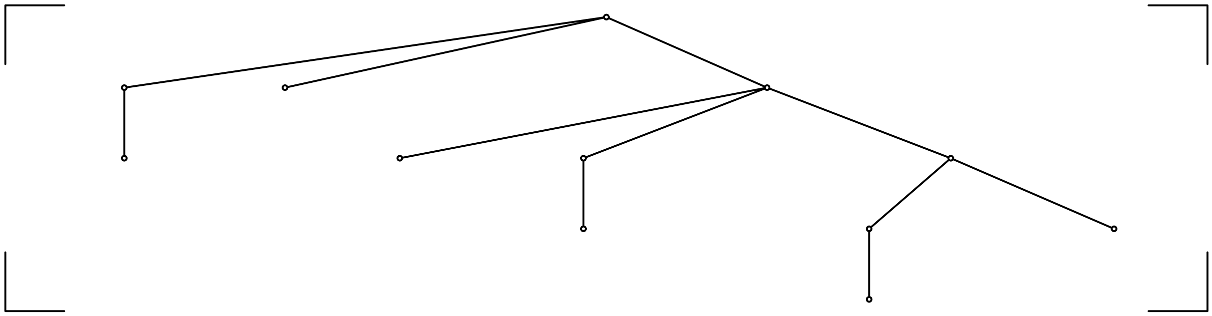
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \alpha^2 & 0 & 1 & 0 \\ 0 & \alpha^2 & 0 & 1 \end{bmatrix}_0 \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & \alpha^4 \\ 0 & 0 & \alpha^4 & \alpha^4 \end{bmatrix}_2 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_0 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & \alpha^5 & 0 & 0 \\ \alpha^5 & 0 & \alpha^4 & 0 \\ \alpha^5 & \alpha^2 & \alpha^4 & \alpha^2 \end{bmatrix}_2$$

$$\begin{bmatrix} 1 & \alpha^5 & 0 & 0 \\ \alpha & 0 & 0 & 0 \\ \alpha & \alpha^4 & \alpha^4 & \alpha^2 \\ \alpha^3 & 0 & \alpha^5 & 0 \end{bmatrix}_2 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \alpha^6 & 0 & 1 & 0 \\ 0 & \alpha^6 & 0 & 1 \end{bmatrix}_0$$

Orbit 0 / 9: Point 352 lies in an orbit of length 12 with average word length 3 $H_6 = 2$
Orbit 1 / 9: Point 353 lies in an orbit of length 4 with average word length 2 $H_6 = 1.16056$
Orbit 2 / 9: Point 354 lies in an orbit of length 4 with average word length 2 $H_6 = 1.16056$
Orbit 3 / 9: Point 355 lies in an orbit of length 12 with average word length 3 $H_6 = 2$
Orbit 4 / 9: Point 360 lies in an orbit of length 96 with average word length 4.125 $H_6 = 3.33829$
Orbit 5 / 9: Point 384 lies in an orbit of length 32 with average word length 3.125 $H_6 = 2.57019$
Orbit 6 / 9: Point 608 lies in an orbit of length 32 with average word length 3.125 $H_6 = 2.57019$
Orbit 7 / 9: Point 640 lies in an orbit of length 4 with average word length 2 $H_6 = 1.16056$
Orbit 8 / 9: Point 641 lies in an orbit of length 4 with average word length 1.75 $H_6 = 1.08603$

Node 5 at Level 4 Orbit 1 / 5 Tree 0 / 9

Number of generators 6

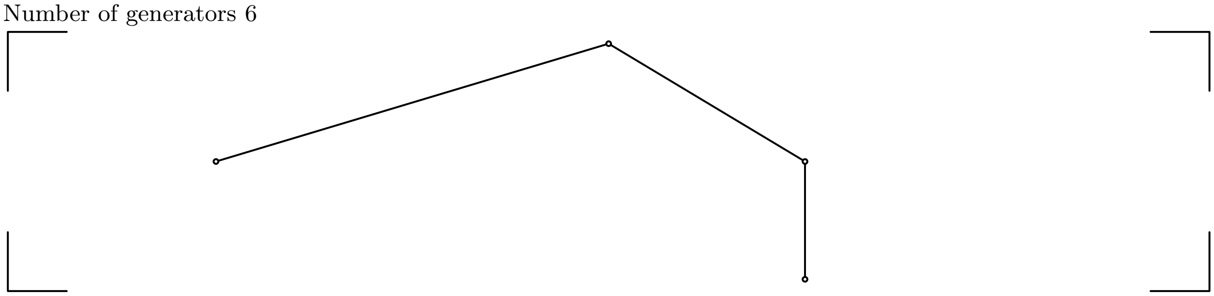


Extension number 0
Orbit representative 352
Flag orbit length 12
Flag orbit is fused to node 4 extension 1
Fusion element:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^5 & \alpha & 0 & 0 \\ \alpha^2 & 0 & \alpha^6 & 0 \\ 1 & \alpha^3 & \alpha^4 & 1 \end{bmatrix}_1$$

Node 5 at Level 4 Orbit 1 / 5 Tree 1 / 9

Number of generators 6

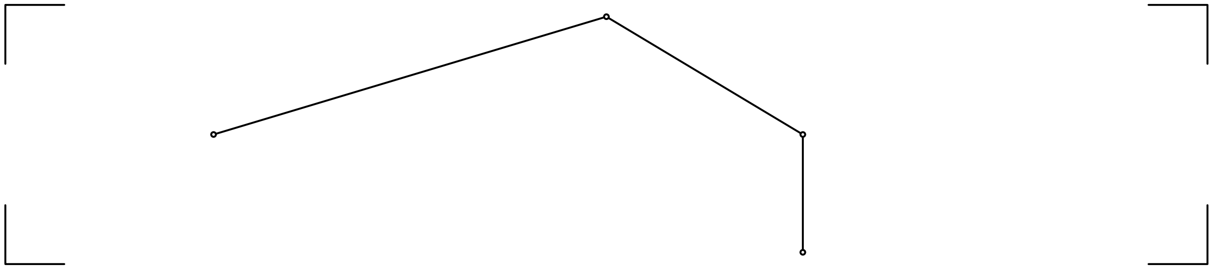


Extension number 1
Orbit representative 353
Flag orbit length 4
Flag orbit is fused to node 4 extension 6
Fusion element:

$$\begin{bmatrix} 1 & \alpha^6 & 0 & 0 \\ 0 & \alpha^4 & 0 & 0 \\ \alpha^5 & \alpha^2 & 1 & \alpha^6 \\ 0 & \alpha^4 & 0 & \alpha^4 \end{bmatrix}_2$$

Node 5 at Level 4 Orbit 1 / 5 Tree 2 / 9

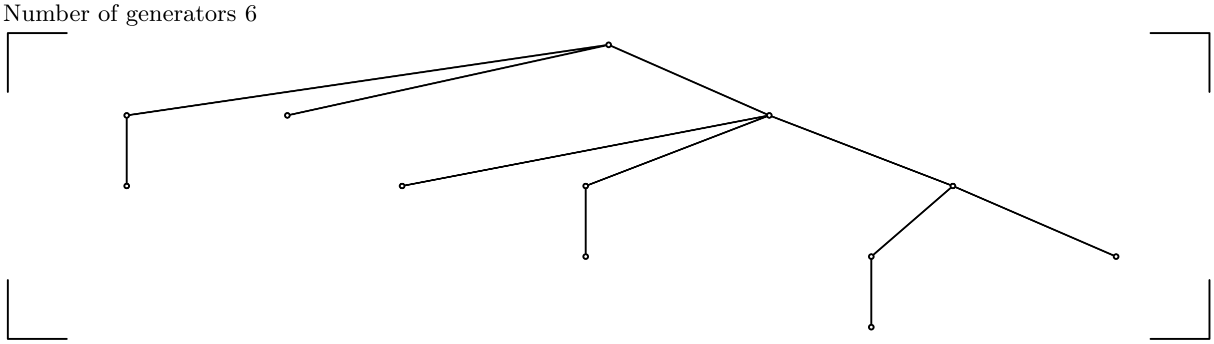
Number of generators 6



Extension number 2
Orbit representative 354
Flag orbit length 4
Flag orbit is defining new orbit 15 at level 5

Node 5 at Level 4 Orbit 1 / 5 Tree 3 / 9

Number of generators 6

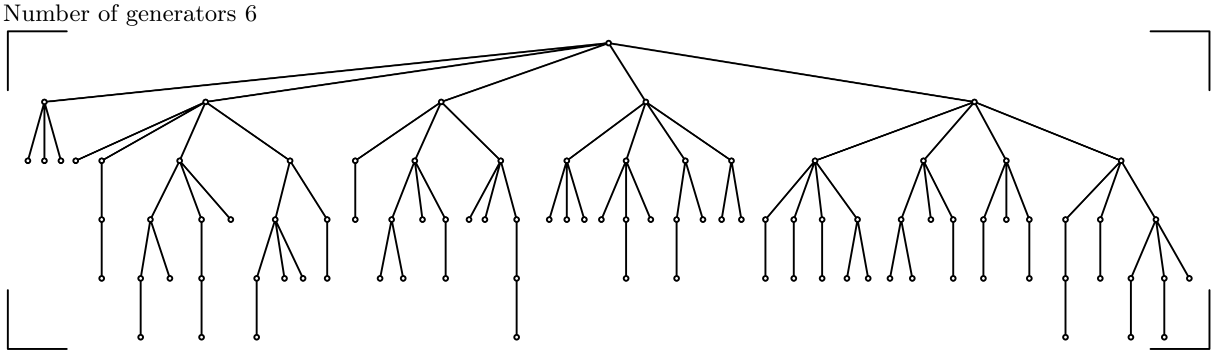


Extension number 3
Orbit representative 355
Flag orbit length 12
Flag orbit is fused to node 5 extension 2
Fusion element:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^5 & \alpha^5 & 0 & 0 \\ \alpha^3 & 0 & 1 & 0 \\ 1 & 1 & \alpha^5 & \alpha^5 \end{bmatrix}_2$$

Node 5 at Level 4 Orbit 1 / 5 Tree 4 / 9

Number of generators 6

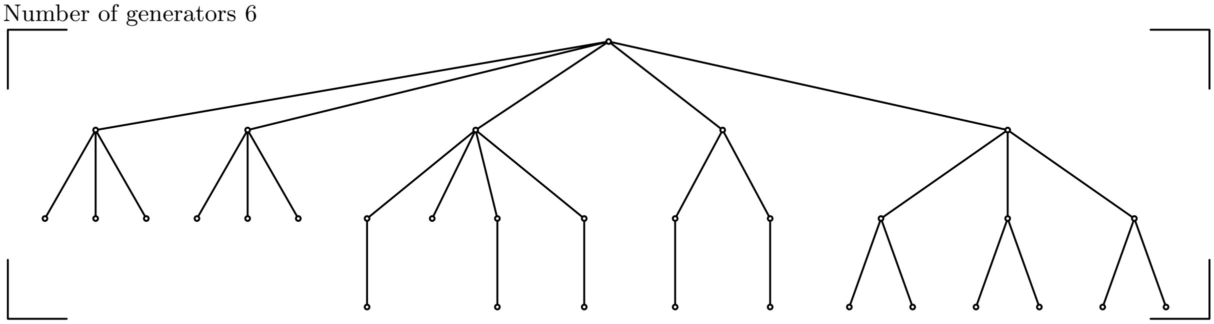


Extension number 4
Orbit representative 360

Flag orbit length 96
Flag orbit is defining new orbit 16 at level 5

Node 5 at Level 4 Orbit 1 / 5 Tree 5 / 9

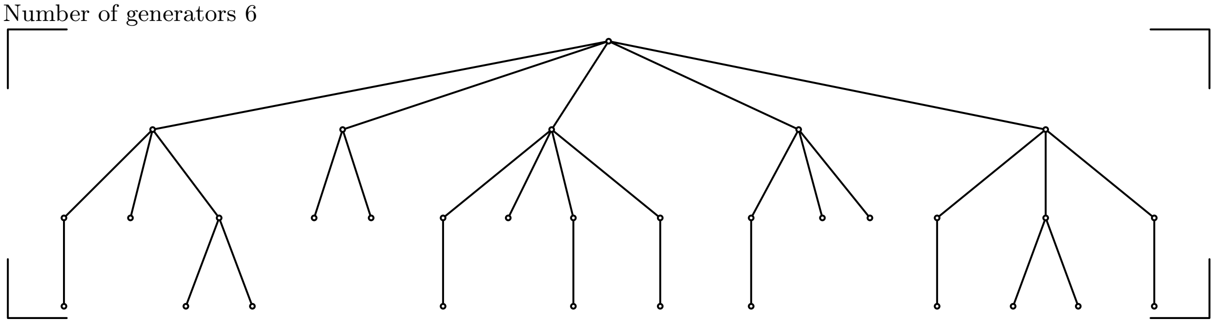
Number of generators 6



Extension number 5
Orbit representative 384
Flag orbit length 32
Flag orbit is defining new orbit 17 at level 5

Node 5 at Level 4 Orbit 1 / 5 Tree 6 / 9

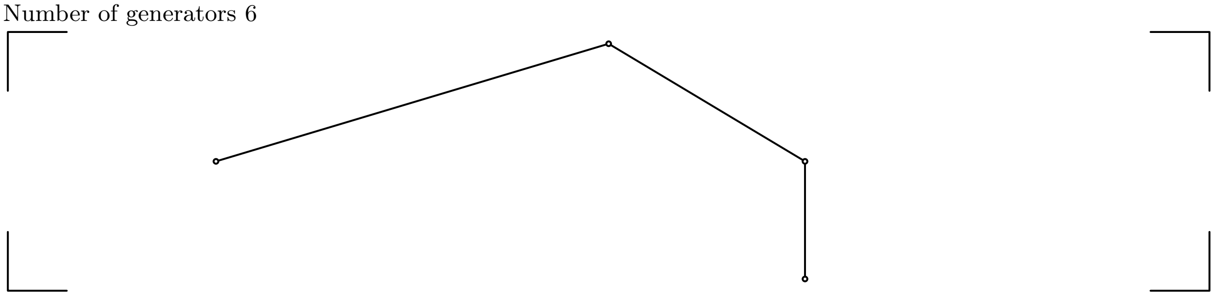
Number of generators 6



Extension number 6
Orbit representative 608
Flag orbit length 32
Flag orbit is defining new orbit 18 at level 5

Node 5 at Level 4 Orbit 1 / 5 Tree 7 / 9

Number of generators 6



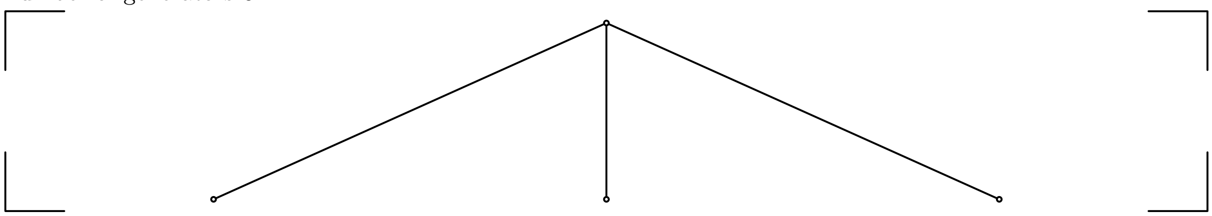
Extension number 7
Orbit representative 640
Flag orbit length 4
Flag orbit is fused to node 4 extension 1

Fusion element:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ \alpha^6 & \alpha^4 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \alpha^6 & \alpha^4 \end{bmatrix}_2$$

Node 5 at Level 4 Orbit 1 / 5 Tree 8 / 9

Number of generators 6



Extension number 8
Orbit representative 641
Flag orbit length 4
Flag orbit is fused to node 5 extension 2
Fusion element:

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & \alpha^4 & 0 & 0 \\ \alpha^4 & \alpha^4 & \alpha^4 & \alpha^4 \\ \alpha^4 & \alpha^5 & \alpha^4 & \alpha \end{bmatrix}_2$$

Node 6 at Level 4 Orbit 2 / 5

$$\{0, 3, 208, 288\}_{28}$$

Strong generators for a group of order 28:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha^6 & 0 \\ 0 & 0 & 0 & \alpha^6 \end{bmatrix}_0, \begin{bmatrix} 1 & \alpha^2 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & \alpha^6 \\ 0 & 0 & 1 & 1 \end{bmatrix}_0, \begin{bmatrix} 1 & 1 & 0 & 0 \\ \alpha^5 & 1 & 0 & 0 \\ 0 & 0 & \alpha^3 & \alpha^3 \\ 0 & 0 & \alpha^4 & \alpha^3 \end{bmatrix}_0$$

There are 14 extensions
Number of generators 3
Generators for the Schreier trees:
Generators for a group of order 28:

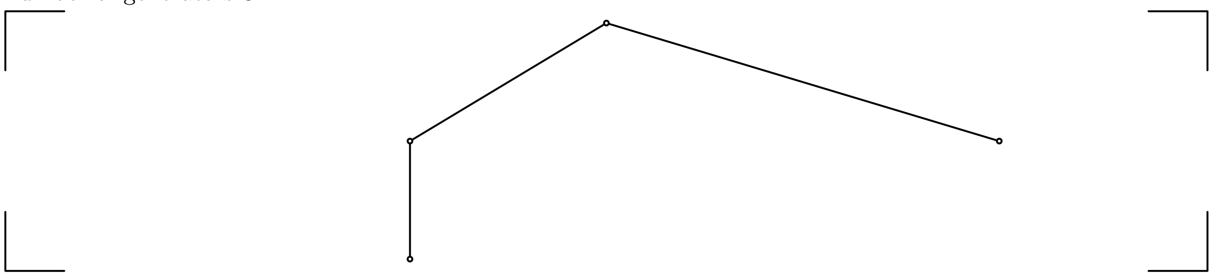
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha^6 & 0 \\ 0 & 0 & 0 & \alpha^6 \end{bmatrix}_0 \begin{bmatrix} 1 & \alpha^2 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & \alpha^6 \\ 0 & 0 & 1 & 1 \end{bmatrix}_0 \begin{bmatrix} 1 & 1 & 0 & 0 \\ \alpha^5 & 1 & 0 & 0 \\ 0 & 0 & \alpha^3 & \alpha^3 \\ 0 & 0 & \alpha^4 & \alpha^3 \end{bmatrix}_0$$

Orbit 0 / 14: Point 344 lies in an orbit of length 4 with average word length 2 $H_3 = 1.89279$
Orbit 1 / 14: Point 345 lies in an orbit of length 28 with average word length 3.85714 $H_3 = 4.26186$
Orbit 2 / 14: Point 360 lies in an orbit of length 4 with average word length 2 $H_3 = 1.89279$
Orbit 3 / 14: Point 361 lies in an orbit of length 28 with average word length 3.85714 $H_3 = 4.26186$
Orbit 4 / 14: Point 368 lies in an orbit of length 4 with average word length 2 $H_3 = 1.89279$
Orbit 5 / 14: Point 369 lies in an orbit of length 28 with average word length 3.85714 $H_3 = 4.26186$
Orbit 6 / 14: Point 376 lies in an orbit of length 4 with average word length 2 $H_3 = 1.89279$
Orbit 7 / 14: Point 377 lies in an orbit of length 28 with average word length 3.85714 $H_3 = 4.26186$
Orbit 8 / 14: Point 384 lies in an orbit of length 4 with average word length 2 $H_3 = 1.89279$
Orbit 9 / 14: Point 385 lies in an orbit of length 28 with average word length 3.85714 $H_3 = 4.26186$
Orbit 10 / 14: Point 600 lies in an orbit of length 4 with average word length 2 $H_3 = 1.89279$
Orbit 11 / 14: Point 601 lies in an orbit of length 28 with average word length 3.85714 $H_3 = 4.26186$

Orbit 12 / 14: Point 632 lies in an orbit of length 1 with average word length 1 $H_3 = 0$
Orbit 13 / 14: Point 633 lies in an orbit of length 7 with average word length 2.28571 $H_3 = 2.52372$

Node 6 at Level 4 Orbit 2 / 5 Tree 0 / 14

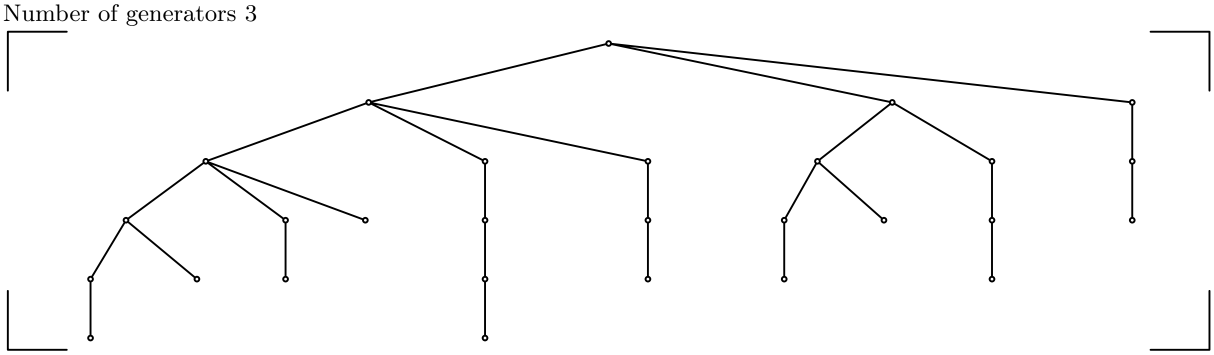
Number of generators 3



Extension number 0
Orbit representative 344
Flag orbit length 4
Flag orbit is defining new orbit 19 at level 5

Node 6 at Level 4 Orbit 2 / 5 Tree 1 / 14

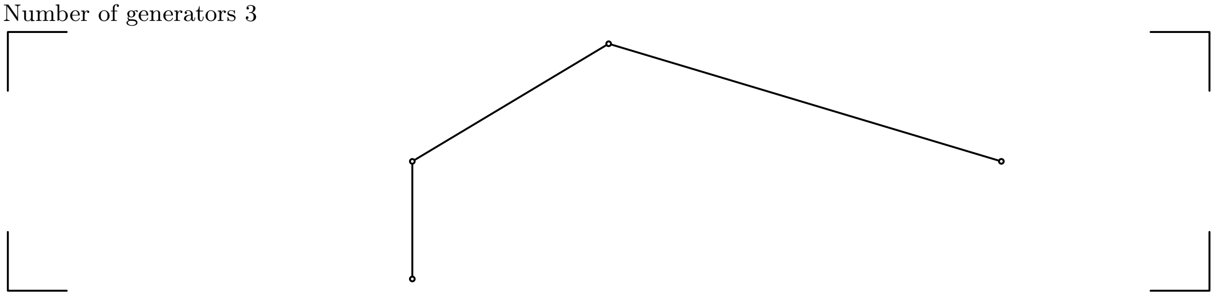
Number of generators 3



Extension number 1
Orbit representative 345
Flag orbit length 28
Flag orbit is defining new orbit 20 at level 5

Node 6 at Level 4 Orbit 2 / 5 Tree 2 / 14

Number of generators 3



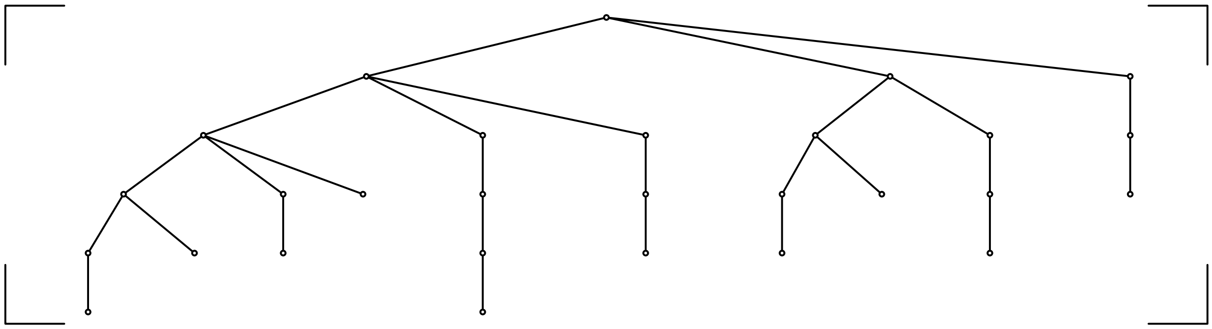
Extension number 2
Orbit representative 360
Flag orbit length 4
Flag orbit is fused to node 4 extension 3

Fusion element:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^2 & \alpha^2 & 0 & 0 \\ 0 & 0 & \alpha^2 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_2$$

Node 6 at Level 4 Orbit 2 / 5 Tree 3 / 14

Number of generators 3

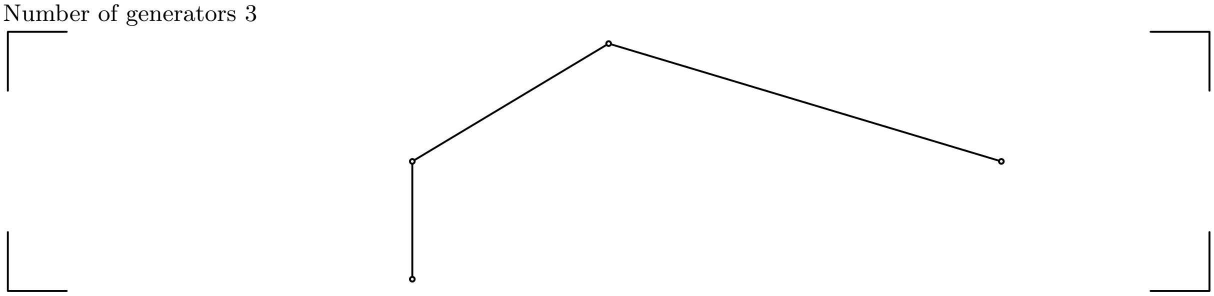


Extension number 3
Orbit representative 361
Flag orbit length 28
Flag orbit is fused to node 5 extension 5
Fusion element:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^2 & \alpha^2 & 0 & 0 \\ \alpha^3 & 0 & \alpha^5 & 0 \\ \alpha^5 & \alpha^5 & \alpha^3 & \alpha^3 \end{bmatrix}_2$$

Node 6 at Level 4 Orbit 2 / 5 Tree 4 / 14

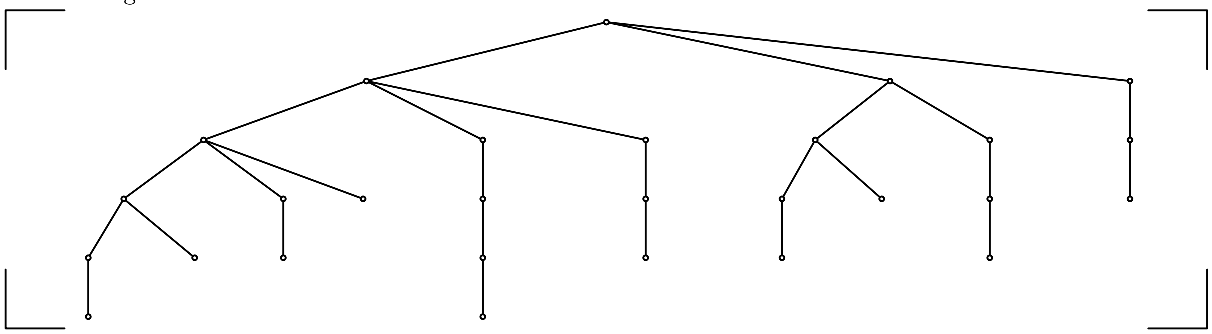
Number of generators 3



Extension number 4
Orbit representative 368
Flag orbit length 4
Flag orbit is defining new orbit 21 at level 5

Node 6 at Level 4 Orbit 2 / 5 Tree 5 / 14

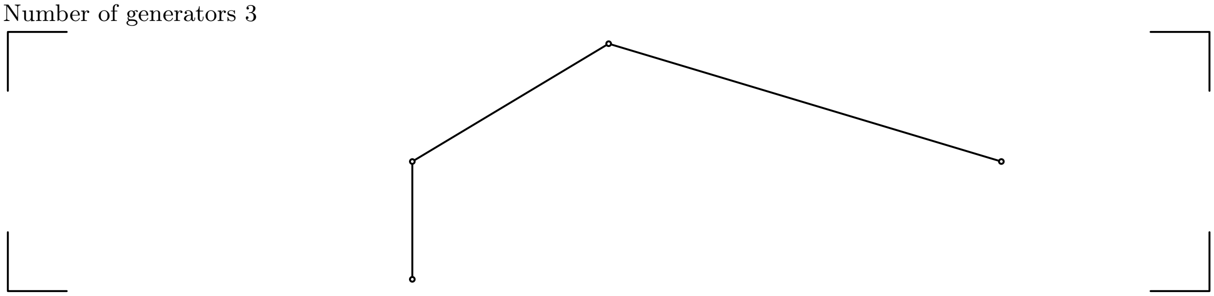
Number of generators 3



Extension number 5
Orbit representative 369
Flag orbit length 28
Flag orbit is defining new orbit 22 at level 5

Node 6 at Level 4 Orbit 2 / 5 Tree 6 / 14

Number of generators 3

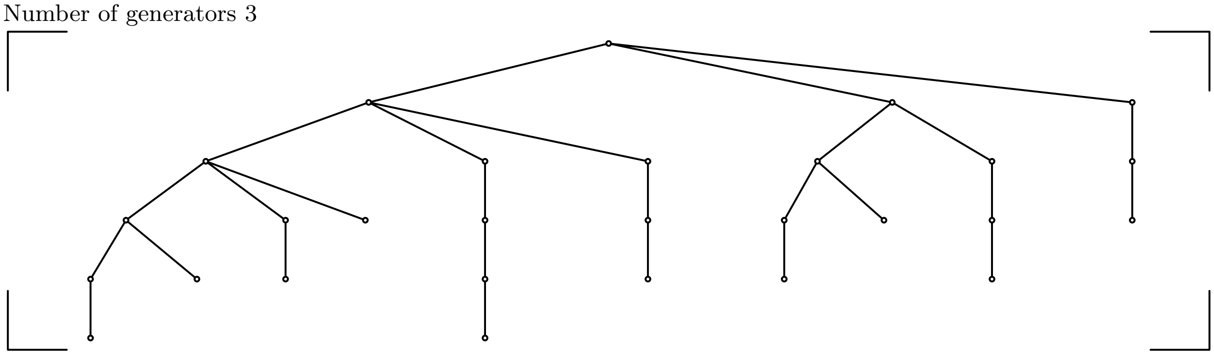


Extension number 6
Orbit representative 376
Flag orbit length 4
Flag orbit is fused to node 4 extension 2
Fusion element:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^5 & \alpha & 0 & 0 \\ 0 & 0 & \alpha^5 & 0 \\ 0 & 0 & \alpha^6 & \alpha^3 \end{bmatrix}_2$$

Node 6 at Level 4 Orbit 2 / 5 Tree 7 / 14

Number of generators 3



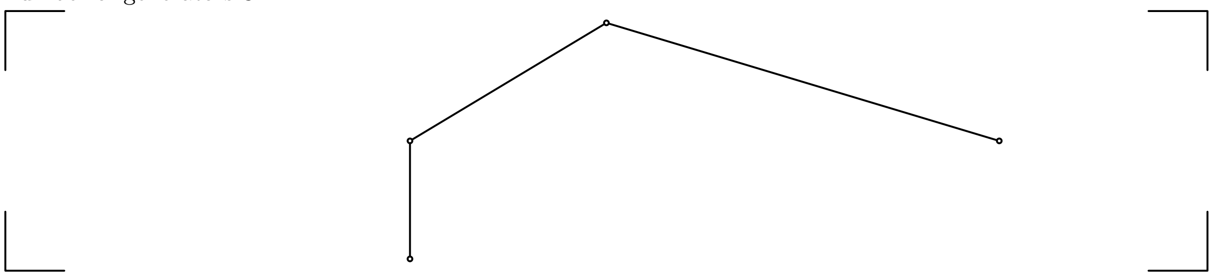
Extension number 7
Orbit representative 377

Flag orbit length 28
Flag orbit is fused to node 5 extension 4
Fusion element:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^5 & \alpha & 0 & 0 \\ 0 & 0 & \alpha^2 & 0 \\ 0 & 0 & \alpha^3 & 1 \end{bmatrix}_2$$

Node 6 at Level 4 Orbit 2 / 5 Tree 8 / 14

Number of generators 3

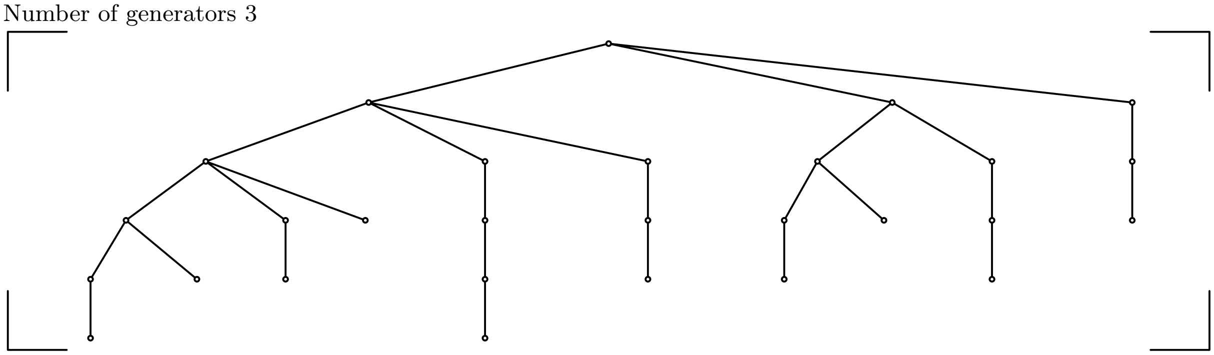


Extension number 8
Orbit representative 384
Flag orbit length 4
Flag orbit is fused to node 4 extension 4
Fusion element:

$$\begin{bmatrix} 1 & \alpha^4 & 0 & 0 \\ \alpha^4 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 1 \\ 0 & 0 & \alpha^6 & 0 \end{bmatrix}_2$$

Node 6 at Level 4 Orbit 2 / 5 Tree 9 / 14

Number of generators 3

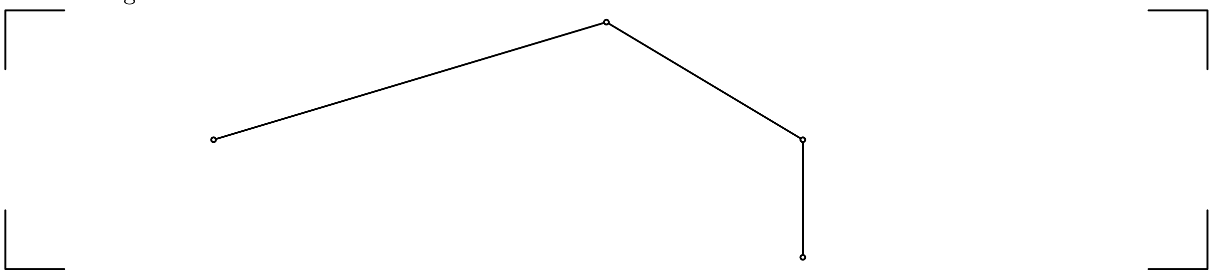


Extension number 9
Orbit representative 385
Flag orbit length 28
Flag orbit is fused to node 5 extension 6
Fusion element:

$$\begin{bmatrix} 1 & \alpha^4 & 0 & 0 \\ \alpha^4 & 0 & 0 & 0 \\ \alpha^6 & \alpha^3 & \alpha^6 & \alpha^5 \\ \alpha^6 & 0 & \alpha^4 & 0 \end{bmatrix}_2$$

Node 6 at Level 4 Orbit 2 / 5 Tree 10 / 14

Number of generators 3

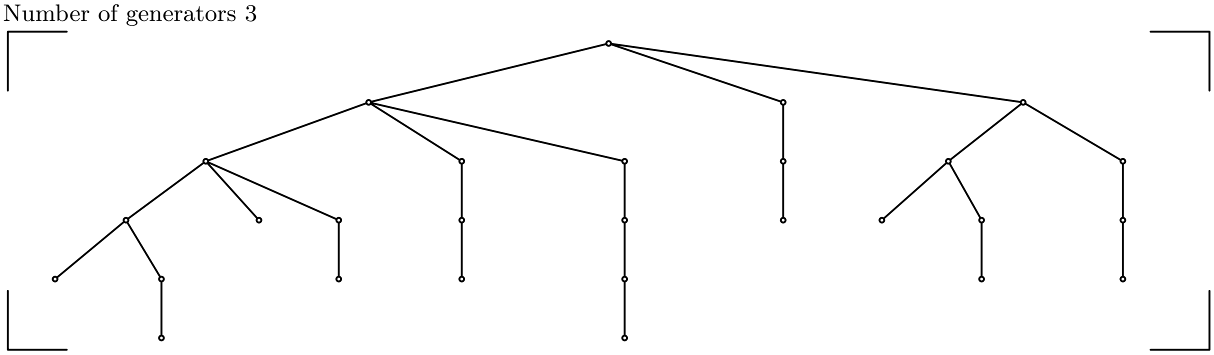


Extension number 10
Orbit representative 600
Flag orbit length 4
Flag orbit is fused to node 4 extension 2
Fusion element:

$$\begin{bmatrix} 1 & \alpha^5 & 0 & 0 \\ 0 & \alpha^6 & 0 & 0 \\ 0 & 0 & \alpha^2 & 1 \\ 0 & 0 & 0 & \alpha^4 \end{bmatrix}_2$$

Node 6 at Level 4 Orbit 2 / 5 Tree 11 / 14

Number of generators 3



Extension number 11
Orbit representative 601
Flag orbit length 28
Flag orbit is fused to node 5 extension 4
Fusion element:

$$\begin{bmatrix} 1 & \alpha^5 & 0 & 0 \\ 0 & \alpha^6 & 0 & 0 \\ \alpha^6 & \alpha^5 & \alpha^6 & \alpha^4 \\ 0 & \alpha^4 & 0 & \alpha \end{bmatrix}_2$$

Node 6 at Level 4 Orbit 2 / 5 Tree 12 / 14

Number of generators 3

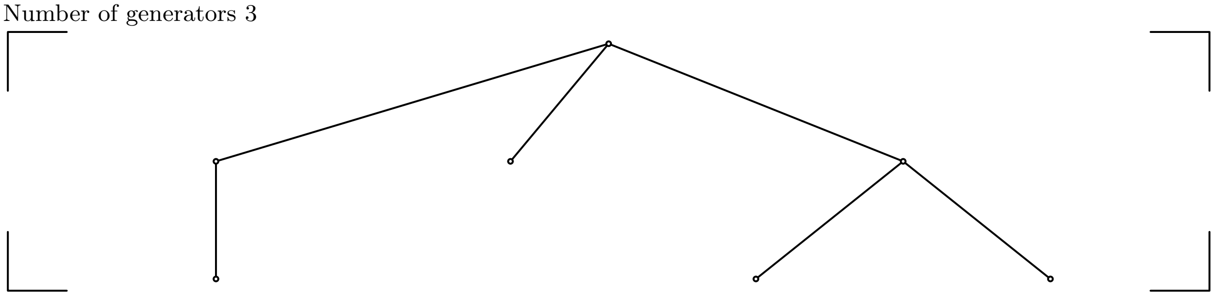
Extension number 12
Orbit representative 632
Flag orbit length 1
Flag orbit is fused to node 6 extension 0

Fusion element:

$$\begin{bmatrix} 1 & \alpha^3 & 0 & 0 \\ 0 & \alpha^2 & 0 & 0 \\ 0 & 0 & \alpha & \alpha^2 \\ 0 & 0 & 0 & \alpha^6 \end{bmatrix}_1$$

Node 6 at Level 4 Orbit 2 / 5 Tree 13 / 14

Number of generators 3



Extension number 13
Orbit representative 633
Flag orbit length 7
Flag orbit is fused to node 6 extension 1
Fusion element:

$$\begin{bmatrix} 1 & \alpha & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & \alpha & \alpha^2 & \alpha^5 \\ 1 & 1 & \alpha^2 & \alpha^2 \end{bmatrix}_1$$

Node 7 at Level 4 Orbit 3 / 5

$$\{0, 3, 208, 296\}_{84}$$

Strong generators for a group of order 84:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & \alpha \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & \alpha^3 & 0 & 0 \\ 0 & 0 & \alpha^5 & 0 \\ 0 & 0 & \alpha^5 & \alpha^3 \end{bmatrix}_2, \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & \alpha^5 \\ 0 & 0 & \alpha^5 & \alpha^5 \end{bmatrix}_2$$

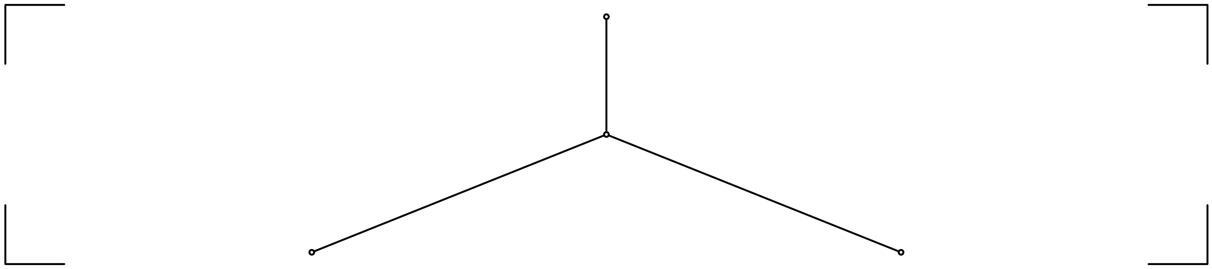
There are 10 extensions
Number of generators 3
Generators for the Schreier trees:
Generators for a group of order 84:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & \alpha \end{bmatrix}_0 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & \alpha^3 & 0 & 0 \\ 0 & 0 & \alpha^5 & 0 \\ 0 & 0 & \alpha^5 & \alpha^3 \end{bmatrix}_2 \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & \alpha^5 \\ 0 & 0 & \alpha^5 & \alpha^5 \end{bmatrix}_2$$

Orbit 0 / 10: Point 344 lies in an orbit of length 4 with average word length 2.25 $H_3 = 2$
Orbit 1 / 10: Point 345 lies in an orbit of length 28 with average word length 3.53571 $H_3 = 4.18266$
Orbit 2 / 10: Point 352 lies in an orbit of length 12 with average word length 3.25 $H_3 = 3.33472$
Orbit 3 / 10: Point 353 lies in an orbit of length 84 with average word length 4.52381 $H_3 = 5.40698$
Orbit 4 / 10: Point 368 lies in an orbit of length 4 with average word length 2.25 $H_3 = 2$
Orbit 5 / 10: Point 369 lies in an orbit of length 28 with average word length 3.53571 $H_3 = 4.18266$
Orbit 6 / 10: Point 600 lies in an orbit of length 1 with average word length 1 $H_3 = 0$
Orbit 7 / 10: Point 601 lies in an orbit of length 7 with average word length 2.57143 $H_3 = 2.63093$
Orbit 8 / 10: Point 608 lies in an orbit of length 4 with average word length 2 $H_3 = 1.89279$
Orbit 9 / 10: Point 609 lies in an orbit of length 28 with average word length 3.53571 $H_3 = 4.18266$

Node 7 at Level 4 Orbit 3 / 5 Tree 0 / 10

Number of generators 3

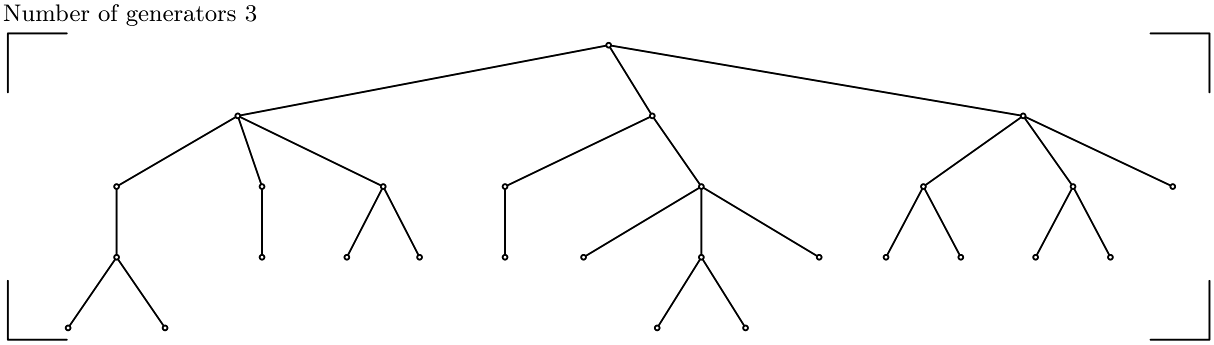


Extension number 0
Orbit representative 344
Flag orbit length 4
Flag orbit is fused to node 6 extension 4
Fusion element:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_0$$

Node 7 at Level 4 Orbit 3 / 5 Tree 1 / 10

Number of generators 3

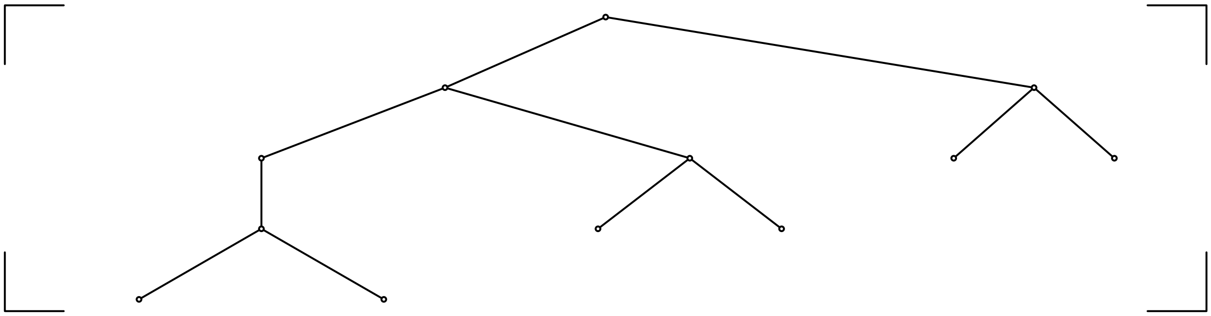


Extension number 1
Orbit representative 345
Flag orbit length 28
Flag orbit is fused to node 6 extension 5
Fusion element:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^2 & \alpha^3 & 0 & 0 \\ \alpha^2 & 0 & 1 & 0 \\ \alpha^3 & 1 & \alpha & \alpha^5 \end{bmatrix}_2$$

Node 7 at Level 4 Orbit 3 / 5 Tree 2 / 10

Number of generators 3

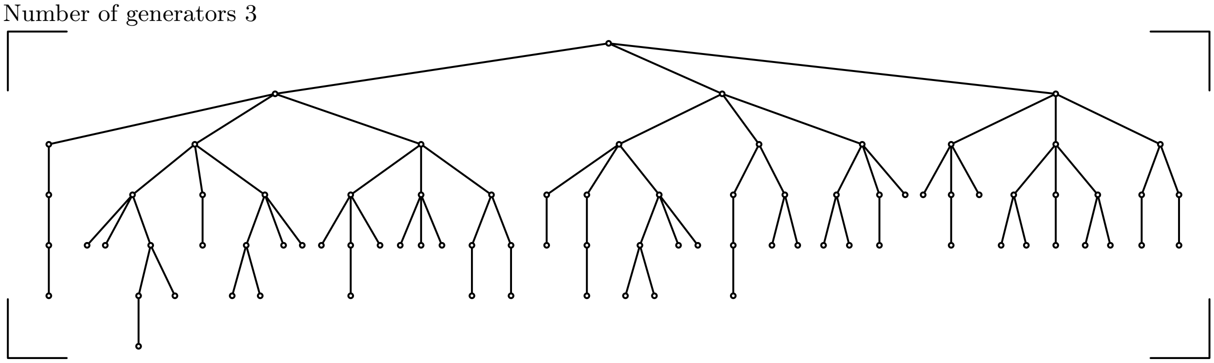


Extension number 2
Orbit representative 352
Flag orbit length 12
Flag orbit is fused to node 4 extension 2
Fusion element:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha^5 & 0 & 0 \\ 0 & 0 & \alpha^6 & 0 \\ 0 & 0 & 0 & \alpha^4 \end{bmatrix}_2$$

Node 7 at Level 4 Orbit 3 / 5 Tree 3 / 10

Number of generators 3

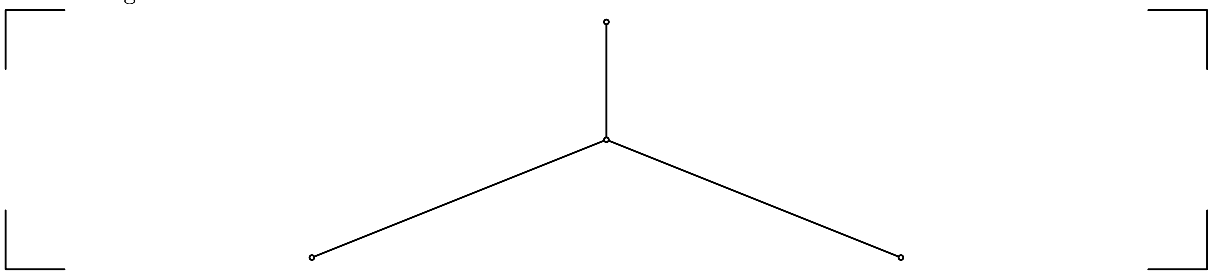


Extension number 3
Orbit representative 353
Flag orbit length 84
Flag orbit is fused to node 5 extension 4
Fusion element:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha^5 & 0 & 0 \\ \alpha^5 & 0 & \alpha^4 & 0 \\ 0 & \alpha^2 & 0 & \alpha^2 \end{bmatrix}_2$$

Node 7 at Level 4 Orbit 3 / 5 Tree 4 / 10

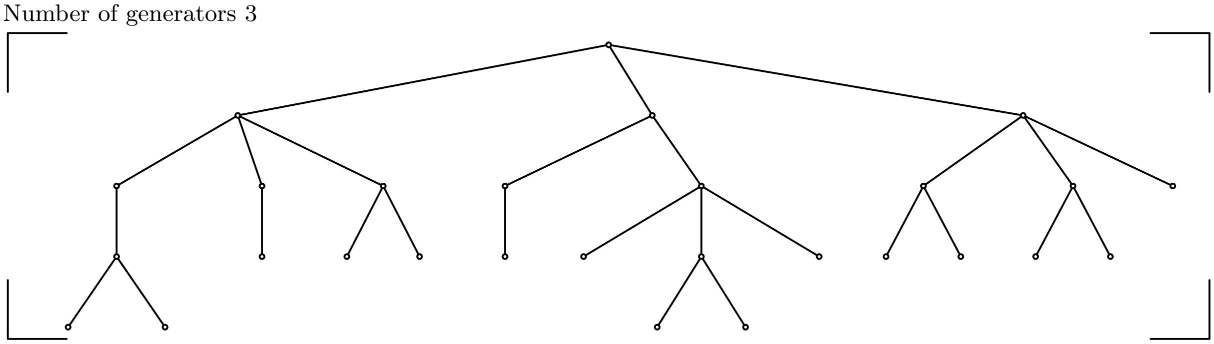
Number of generators 3



Extension number 4
Orbit representative 368
Flag orbit length 4
Flag orbit is defining new orbit 23 at level 5

Node 7 at Level 4 Orbit 3 / 5 Tree 5 / 10

Number of generators 3



Extension number 5
Orbit representative 369
Flag orbit length 28
Flag orbit is defining new orbit 24 at level 5

Node 7 at Level 4 Orbit 3 / 5 Tree 6 / 10

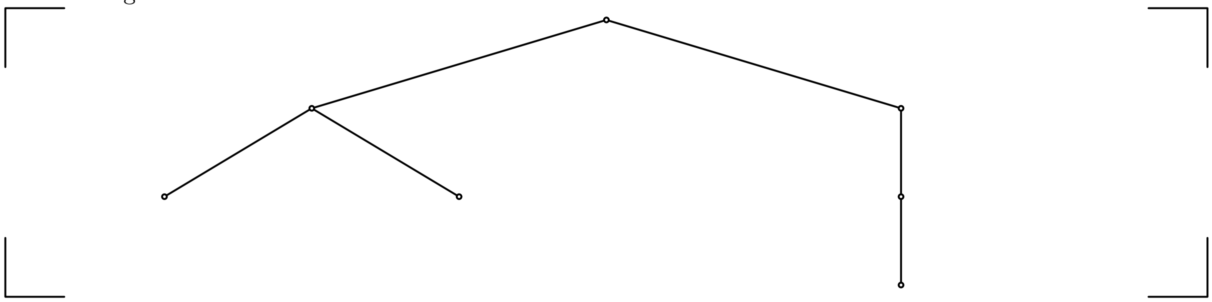
Number of generators 3

Extension number 6
Orbit representative 600
Flag orbit length 1
Flag orbit is fused to node 7 extension 4
Fusion element:

$$\begin{bmatrix} 1 & \alpha^2 & 0 & 0 \\ 0 & \alpha^3 & 0 & 0 \\ 0 & 0 & \alpha^3 & \alpha^4 \\ 0 & 0 & 0 & \alpha \end{bmatrix}_1$$

Node 7 at Level 4 Orbit 3 / 5 Tree 7 / 10

Number of generators 3

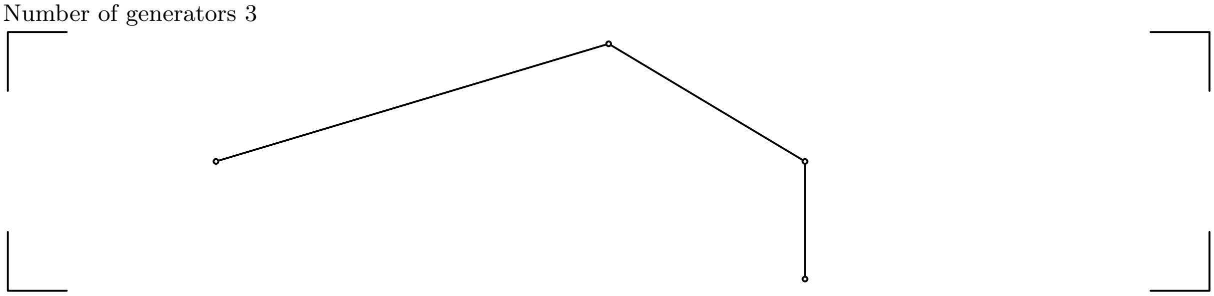


Extension number 7
Orbit representative 601
Flag orbit length 7
Flag orbit is fused to node 7 extension 5
Fusion element:

$$\begin{bmatrix} 1 & \alpha^2 & 0 & 0 \\ 0 & \alpha^3 & 0 & 0 \\ \alpha^2 & 0 & \alpha & \alpha^2 \\ 0 & \alpha^2 & 0 & \alpha^6 \end{bmatrix}_1$$

Node 7 at Level 4 Orbit 3 / 5 Tree 8 / 10

Number of generators 3

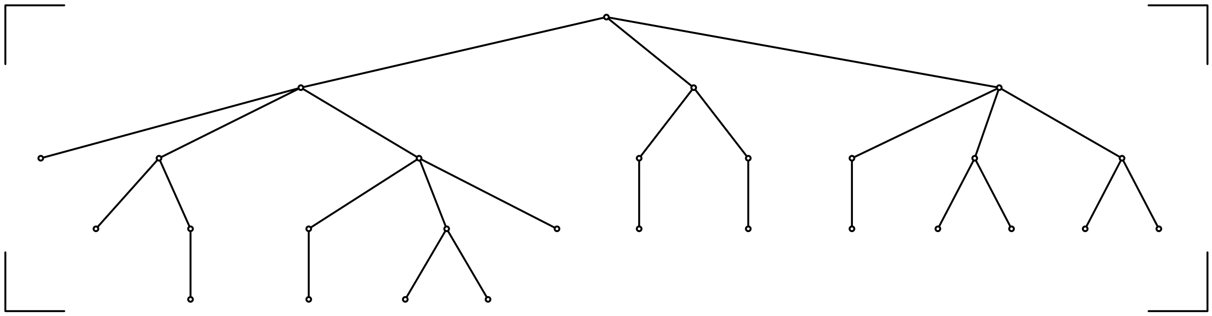


Extension number 8
Orbit representative 608
Flag orbit length 4
Flag orbit is fused to node 4 extension 3
Fusion element:

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ \alpha^2 & \alpha^5 & 0 & 0 \\ 0 & 0 & \alpha^6 & \alpha^6 \\ 0 & 0 & 1 & \alpha^3 \end{bmatrix}_1$$

Node 7 at Level 4 Orbit 3 / 5 Tree 9 / 10

Number of generators 3



Extension number 9
Orbit representative 609
Flag orbit length 28
Flag orbit is fused to node 5 extension 5
Fusion element:

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ \alpha^2 & \alpha^5 & 0 & 0 \\ \alpha^2 & \alpha^2 & 1 & 1 \\ \alpha^3 & \alpha^4 & \alpha & \alpha^4 \end{bmatrix}_1$$

Node 8 at Level 4 Orbit 4 / 5

$$\{0, 3, 208, 320\}_{84}$$

Strong generators for a group of order 84:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha^6 & 0 \\ 0 & 0 & 0 & \alpha^6 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^4 & \alpha^4 & 0 & 0 \\ 0 & 0 & \alpha^2 & 0 \\ 0 & 0 & \alpha^3 & \alpha^3 \end{bmatrix}_1, \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & \alpha & \alpha \\ 0 & 0 & \alpha & 0 \end{bmatrix}_1,$$

$$\begin{bmatrix} 1 & \alpha^6 & 0 & 0 \\ \alpha^4 & 0 & 0 & 0 \\ 0 & 0 & \alpha & \alpha^6 \\ 0 & 0 & \alpha^2 & 0 \end{bmatrix}_2$$

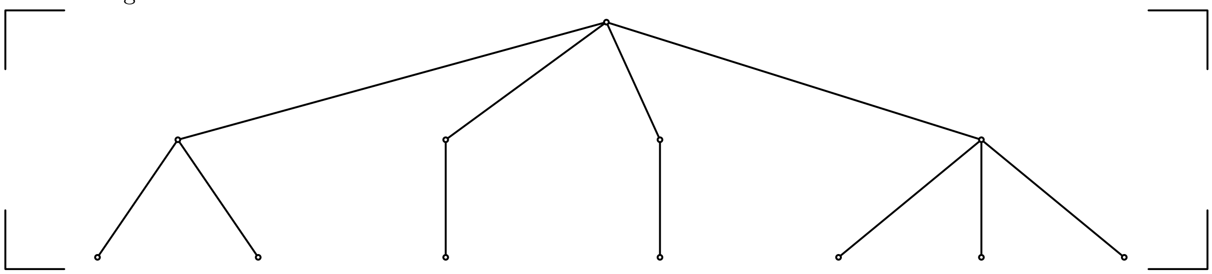
There are 10 extensions
Number of generators 4
Generators for the Schreier trees:
Generators for a group of order 84:

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ \alpha^4 & 1 & 0 & 0 \\ 0 & 0 & \alpha^6 & \alpha^6 \\ 0 & 0 & 1 & \alpha^6 \end{bmatrix}_0 \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^4 & \alpha^4 & 0 & 0 \\ 0 & 0 & \alpha^2 & 0 \\ 0 & 0 & \alpha^3 & \alpha^3 \end{bmatrix}_1 \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & \alpha^5 \\ 0 & 0 & \alpha^5 & \alpha^5 \end{bmatrix}_2 \begin{bmatrix} 0 & 1 & 0 & 0 \\ \alpha^4 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha^5 \\ 0 & 0 & \alpha^6 & 0 \end{bmatrix}_0$$

Orbit 0 / 10: Point 344 lies in an orbit of length 12 with average word length 2.5 $H_4 = 2.45345$
Orbit 1 / 10: Point 345 lies in an orbit of length 84 with average word length 4.08333 $H_4 = 4.21103$
Orbit 2 / 10: Point 360 lies in an orbit of length 4 with average word length 2 $H_4 = 1.5$
Orbit 3 / 10: Point 361 lies in an orbit of length 28 with average word length 3.39286 $H_4 = 3.28493$
Orbit 4 / 10: Point 376 lies in an orbit of length 4 with average word length 2 $H_4 = 1.5$
Orbit 5 / 10: Point 377 lies in an orbit of length 28 with average word length 3.39286 $H_4 = 3.28493$
Orbit 6 / 10: Point 600 lies in an orbit of length 4 with average word length 2 $H_4 = 1.5$
Orbit 7 / 10: Point 601 lies in an orbit of length 28 with average word length 3.17857 $H_4 = 3.23787$
Orbit 8 / 10: Point 616 lies in an orbit of length 1 with average word length 1 $H_4 = 0$
Orbit 9 / 10: Point 617 lies in an orbit of length 7 with average word length 2.57143 $H_4 = 2.08496$

Node 8 at Level 4 Orbit 4 / 5 Tree 0 / 10

Number of generators 4

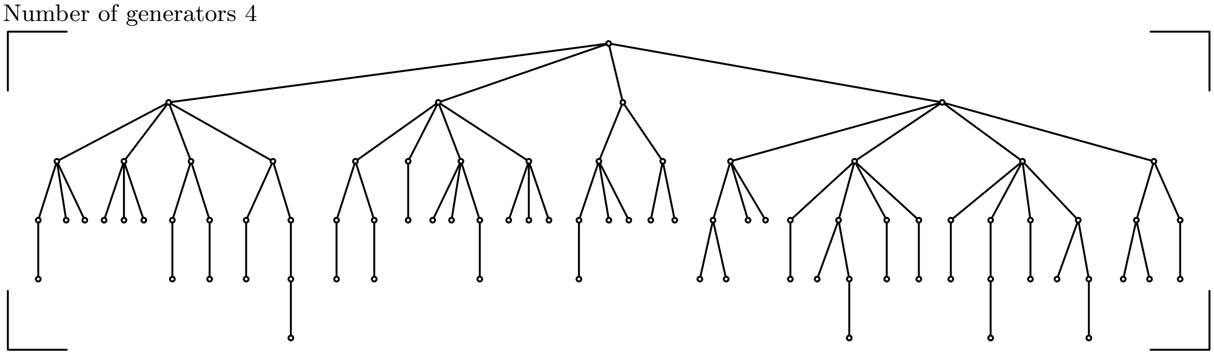


Extension number 0
Orbit representative 344
Flag orbit length 12
Flag orbit is fused to node 4 extension 2
Fusion element:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha & \alpha & 0 & 0 \\ 0 & 0 & \alpha^5 & 0 \\ 0 & 0 & \alpha^3 & \alpha^3 \end{bmatrix}_2$$

Node 8 at Level 4 Orbit 4 / 5 Tree 1 / 10

Number of generators 4

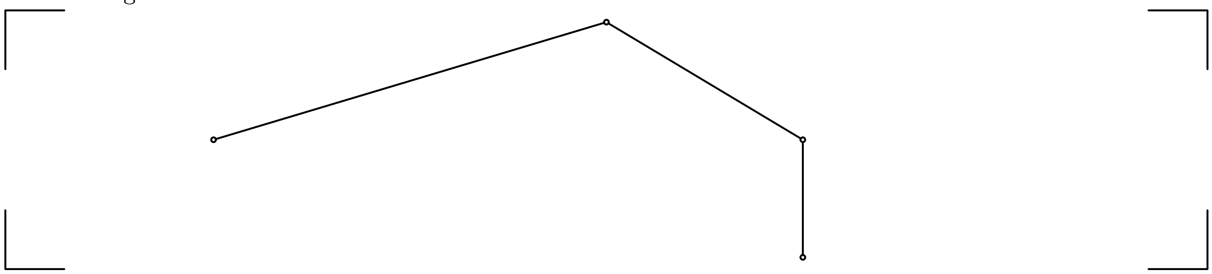


Extension number 1
Orbit representative 345
Flag orbit length 84
Flag orbit is fused to node 5 extension 4
Fusion element:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha & \alpha & 0 & 0 \\ 1 & 0 & \alpha^5 & 0 \\ \alpha & \alpha & \alpha^3 & \alpha^3 \end{bmatrix}_2$$

Node 8 at Level 4 Orbit 4 / 5 Tree 2 / 10

Number of generators 4

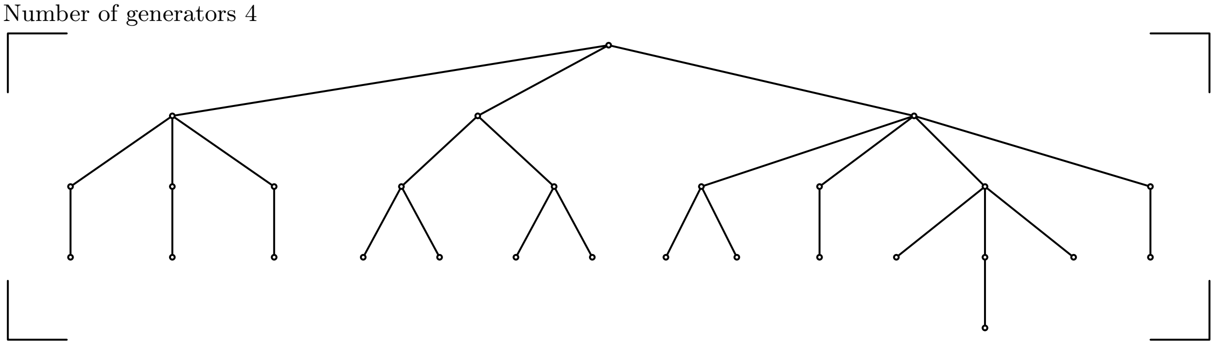


Extension number 2
Orbit representative 360
Flag orbit length 4
Flag orbit is fused to node 4 extension 4
Fusion element:

$$\begin{bmatrix} 1 & \alpha^4 & 0 & 0 \\ 0 & \alpha^6 & 0 & 0 \\ 0 & 0 & 1 & \alpha^6 \\ 0 & 0 & 0 & \alpha^4 \end{bmatrix}_2$$

Node 8 at Level 4 Orbit 4 / 5 Tree 3 / 10

Number of generators 4

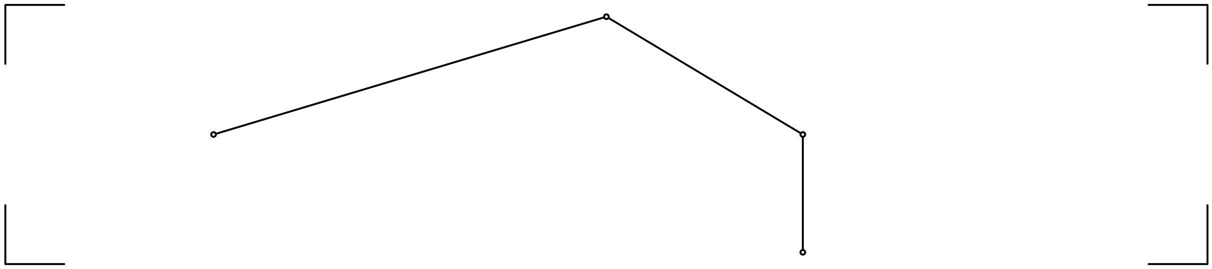


Extension number 3
Orbit representative 361
Flag orbit length 28
Flag orbit is fused to node 5 extension 6
Fusion element:

$$\begin{bmatrix} 1 & \alpha^2 & 0 & 0 \\ \alpha^4 & \alpha^4 & 0 & 0 \\ \alpha^4 & \alpha^6 & \alpha & \alpha^4 \\ \alpha^5 & \alpha^5 & \alpha^2 & \alpha^2 \end{bmatrix}_0$$

Node 8 at Level 4 Orbit 4 / 5 Tree 4 / 10

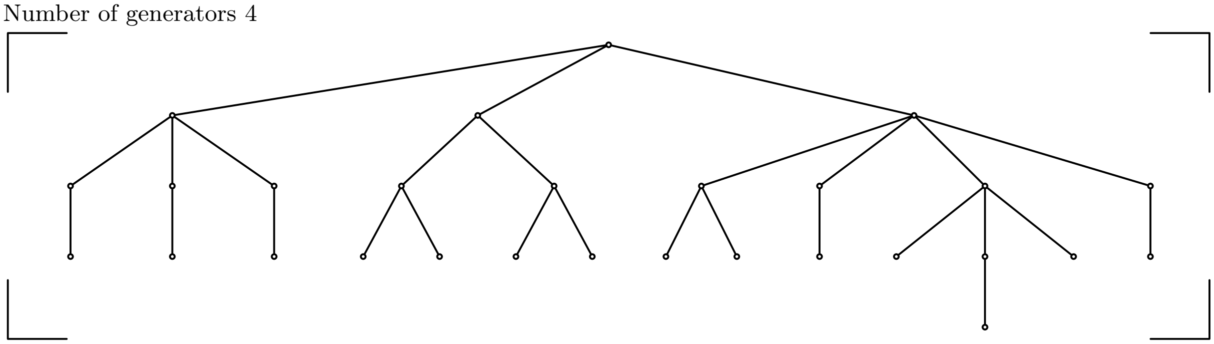
Number of generators 4



Extension number 4
Orbit representative 376
Flag orbit length 4
Flag orbit is defining new orbit 25 at level 5

Node 8 at Level 4 Orbit 4 / 5 Tree 5 / 10

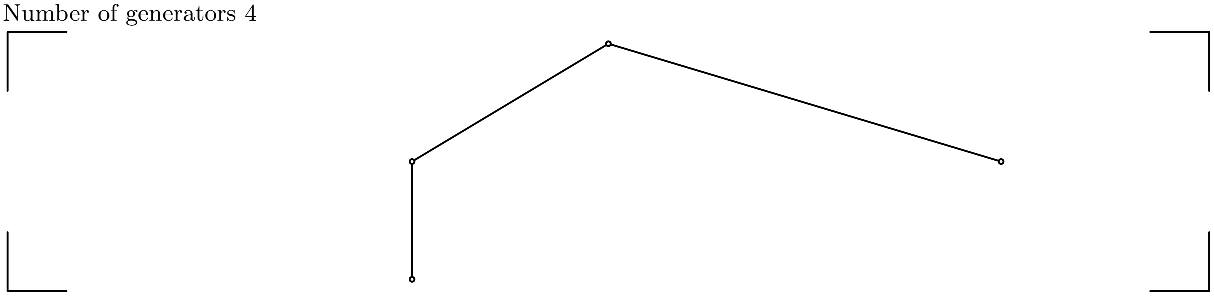
Number of generators 4



Extension number 5
Orbit representative 377
Flag orbit length 28
Flag orbit is defining new orbit 26 at level 5

Node 8 at Level 4 Orbit 4 / 5 Tree 6 / 10

Number of generators 4

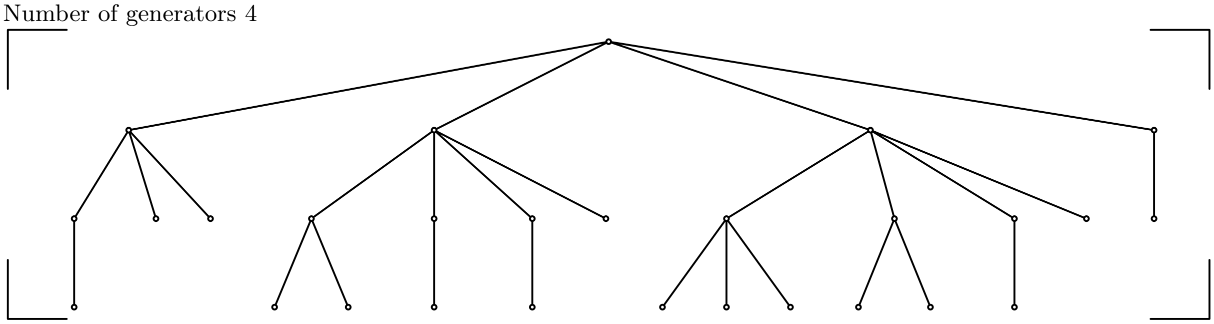


Extension number 6
Orbit representative 600
Flag orbit length 4
Flag orbit is fused to node 6 extension 4
Fusion element:

$$
\begin{bmatrix} 1 & \alpha^2 & 0 & 0 \\ 0 & \alpha^3 & 0 & 0 \\ 0 & 0 & \alpha^3 & \alpha^4 \\ 0 & 0 & 0 & \alpha \end{bmatrix}_1
$$

Node 8 at Level 4 Orbit 4 / 5 Tree 7 / 10

Number of generators 4



Extension number 7
Orbit representative 601
Flag orbit length 28
Flag orbit is fused to node 6 extension 5
Fusion element:

$$
\begin{bmatrix} 1 & \alpha^2 & 0 & 0 \\ 0 & \alpha^3 & 0 & 0 \\ \alpha^3 & \alpha & \alpha^4 & \alpha^5 \\ 0 & \alpha^4 & 0 & \alpha^2 \end{bmatrix}_1
$$

Node 8 at Level 4 Orbit 4 / 5 Tree 8 / 10

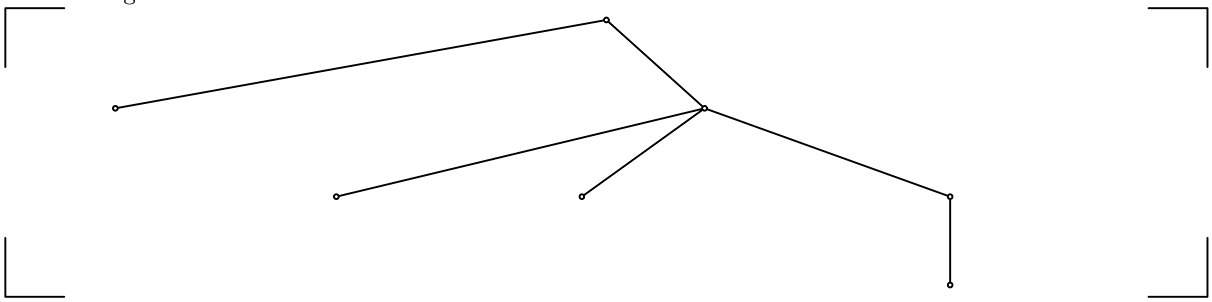
Number of generators 4

Extension number 8
Orbit representative 616
Flag orbit length 1
Flag orbit is fused to node 8 extension 4
Fusion element:

$$
\begin{bmatrix} 1 & \alpha^4 & 0 & 0 \\ 0 & \alpha^6 & 0 & 0 \\ 0 & 0 & \alpha^5 & \alpha^6 \\ 0 & 0 & 0 & \alpha^3 \end{bmatrix}_1
$$

Node 8 at Level 4 Orbit 4 / 5 Tree 9 / 10

Number of generators 4



Extension number 9
Orbit representative 617
Flag orbit length 7
Flag orbit is fused to node 8 extension 5
Fusion element:

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ \alpha^4 & \alpha^2 & 0 & 0 \\ 1 & 1 & \alpha^2 & \alpha^2 \\ \alpha & \alpha^6 & \alpha^3 & \alpha^6 \end{bmatrix}_1$$

5.6 Stabilizers and Schreier trees at level 5

Node 9 at Level 5 Orbit 0 / 18

$$\{0, 3, 208, 280, 352\}_{672}$$

Strong generators for a group of order 672:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha^6 & 0 \\ 0 & 0 & 0 & \alpha^6 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha^5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \alpha^5 \end{bmatrix}_2, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & \alpha^6 & 0 \\ 0 & 1 & 0 & \alpha^6 \end{bmatrix}_0,$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha & \alpha & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & \alpha^2 & \alpha^2 \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & \alpha & 0 & 0 \\ \alpha & 0 & \alpha & 0 \\ \alpha & \alpha^2 & \alpha & \alpha^2 \end{bmatrix}_1$$

There are 0 extensions
Number of generators 5

Node 10 at Level 5 Orbit 1 / 18

$$\{0, 3, 208, 280, 353\}_{24}$$

Strong generators for a group of order 24:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \alpha^4 & 0 & 1 & 0 \\ 0 & \alpha^4 & 0 & 1 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \alpha^2 & 0 & 1 & 0 \\ 0 & \alpha^2 & 0 & 1 \end{bmatrix}_0,$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & \alpha^5 & 0 & 0 \\ \alpha & 0 & \alpha^2 & 0 \\ \alpha & \alpha^6 & \alpha^2 & 1 \end{bmatrix}_2$$

There are 0 extensions
Number of generators 4

Node 11 at Level 5 Orbit 2 / 18

$$\{0, 3, 208, 280, 360\}_{7}$$

Strong generators for a group of order 7:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha^3 & 0 \\ 0 & 0 & 0 & \alpha^3 \end{bmatrix}_0$$

There are 0 extensions
Number of generators 1

Node 12 at Level 5 Orbit 3 / 18

$$\{0, 3, 208, 280, 384\}_{21}$$

Strong generators for a group of order 21:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha^2 & 0 \\ 0 & 0 & 0 & \alpha^2 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & \alpha^5 & 0 & 0 \\ 0 & 0 & \alpha^2 & 0 \\ 0 & 0 & \alpha^2 & 1 \end{bmatrix}_2$$

There are 0 extensions
Number of generators 2

Node 13 at Level 5 Orbit 4 / 18

$$\{0, 3, 208, 280, 608\}_{21}$$

Strong generators for a group of order 21:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha^3 & 0 \\ 0 & 0 & 0 & \alpha^3 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha & \alpha & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & \alpha^2 & \alpha^2 \end{bmatrix}_1$$

There are 0 extensions
Number of generators 2

Node 14 at Level 5 Orbit 5 / 18

$$\{0, 3, 208, 280, 641\}_{96}$$

Strong generators for a group of order 96:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \alpha & 0 & 1 & 0 \\ 0 & \alpha & 0 & 1 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \alpha^2 & 0 & 1 & 0 \\ 0 & \alpha^2 & 0 & 1 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \alpha^3 & 0 & 1 & 0 \\ 0 & \alpha^3 & 0 & 1 \end{bmatrix}_0,$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & \alpha^5 & 0 & 0 \\ \alpha^2 & 0 & \alpha^2 & 0 \\ \alpha^2 & 1 & \alpha^2 & 1 \end{bmatrix}_2, \begin{bmatrix} 0 & 1 & 0 & 0 \\ \alpha & 0 & 0 & 0 \\ 0 & \alpha^3 & 0 & 1 \\ \alpha^4 & 0 & \alpha & 0 \end{bmatrix}_0$$

There are 0 extensions
Number of generators 5

Node 15 at Level 5 Orbit 6 / 18

$$\{0, 3, 208, 281, 354\}_{24}$$

Strong generators for a group of order 24:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \alpha & 0 & 1 & 0 \\ 0 & \alpha & 0 & 1 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \alpha^3 & 0 & 1 & 0 \\ 0 & \alpha^3 & 0 & 1 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \alpha^2 & 0 & 1 & 0 \\ 0 & \alpha^2 & 0 & 1 \end{bmatrix}_0,$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha & \alpha & 0 & 0 \\ \alpha^2 & 0 & \alpha^5 & 0 \\ \alpha^2 & \alpha^2 & \alpha^6 & \alpha^6 \end{bmatrix}_1$$

There are 0 extensions
Number of generators 4

Node 16 at Level 5 Orbit 7 / 18

$$\{0, 3, 208, 281, 360\}_{1}$$

Strong generators for a group of order 1:

There are 0 extensions
Number of generators 0

Node 17 at Level 5 Orbit 8 / 18

$$\{0, 3, 208, 281, 384\}_{3}$$

Strong generators for a group of order 3:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha & \alpha & 0 & 0 \\ \alpha & 0 & \alpha^5 & 0 \\ \alpha^3 & \alpha^3 & \alpha^6 & \alpha^6 \end{bmatrix}_1$$

There are 0 extensions
Number of generators 1

Node 18 at Level 5 Orbit 9 / 18

$$\{0, 3, 208, 281, 608\}_{3}$$

Strong generators for a group of order 3:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha & \alpha & 0 & 0 \\ \alpha^2 & 0 & \alpha^5 & 0 \\ \alpha^2 & \alpha^2 & \alpha^6 & \alpha^6 \end{bmatrix}_1$$

There are 0 extensions
Number of generators 1

Node 19 at Level 5 Orbit 10 / 18

$$\{0, 3, 208, 288, 344\}_{28}$$

Strong generators for a group of order 28:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & \alpha \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha & 1 & 0 & 0 \\ 0 & 0 & \alpha^4 & 0 \\ 0 & 0 & \alpha^2 & \alpha^4 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^5 & 1 & 0 & 0 \\ 0 & 0 & \alpha^6 & 0 \\ 0 & 0 & 1 & \alpha^6 \end{bmatrix}_0$$

There are 0 extensions
Number of generators 3

Node 20 at Level 5 Orbit 11 / 18

$$\{0, 3, 208, 288, 345\}_4$$

Strong generators for a group of order 4:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^5 & 1 & 0 & 0 \\ \alpha^5 & 0 & 1 & 0 \\ \alpha^6 & \alpha & \alpha & 1 \end{bmatrix}_0$$

There are 0 extensions
Number of generators 2

Node 21 at Level 5 Orbit 12 / 18

$$\{0, 3, 208, 288, 368\}_{21}$$

Strong generators for a group of order 21:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha^3 & 0 \\ 0 & 0 & 0 & \alpha^3 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^3 & \alpha^3 & 0 & 0 \\ 0 & 0 & \alpha^3 & 0 \\ 0 & 0 & \alpha & \alpha \end{bmatrix}_2$$

There are 0 extensions
Number of generators 2

Node 22 at Level 5 Orbit 13 / 18

$$\{0, 3, 208, 288, 369\}_3$$

Strong generators for a group of order 3:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^3 & \alpha^3 & 0 & 0 \\ \alpha^4 & 0 & \alpha & 0 \\ \alpha^3 & \alpha^3 & \alpha^6 & \alpha^6 \end{bmatrix}_2$$

There are 0 extensions
Number of generators 1

Node 23 at Level 5 Orbit 14 / 18

$$\{0, 3, 208, 296, 368\}_{84}$$

Strong generators for a group of order 84:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha^6 & 0 \\ 0 & 0 & 0 & \alpha^6 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha^2 & 0 & 0 \\ 0 & 0 & \alpha^6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^2 & \alpha^2 & 0 & 0 \\ 0 & 0 & \alpha^2 & 0 \\ 0 & 0 & \alpha^3 & \alpha^3 \end{bmatrix}_1$$

There are 0 extensions
Number of generators 3

Node 24 at Level 5 Orbit 15 / 18

$$\{0, 3, 208, 296, 369\}_{12}$$

Strong generators for a group of order 12:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha^2 & 0 & 0 \\ \alpha^3 & 0 & \alpha^6 & 0 \\ 0 & \alpha^4 & 0 & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^2 & \alpha^2 & 0 & 0 \\ 0 & 0 & \alpha^6 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^3 & 1 & 0 & 0 \\ \alpha^4 & 0 & 1 & 0 \\ \alpha^3 & \alpha^6 & \alpha^5 & 1 \end{bmatrix}_0$$

There are 0 extensions
Number of generators 3

Node 25 at Level 5 Orbit 16 / 18

$$\{0, 3, 208, 320, 376\}_{84}$$

Strong generators for a group of order 84:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha^4 & 0 \\ 0 & 0 & 0 & \alpha^4 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha^4 & 0 & 0 \\ 0 & 0 & \alpha^6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^4 & \alpha^4 & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & \alpha^2 & \alpha^2 \end{bmatrix}_1,$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^4 & \alpha^6 & 0 & 0 \\ 0 & 0 & \alpha^6 & 0 \\ 0 & 0 & 1 & \alpha^4 \end{bmatrix}_2$$

There are 0 extensions
Number of generators 4

Node 26 at Level 5 Orbit 17 / 18

$$\{0, 3, 208, 320, 377\}_{12}$$

Strong generators for a group of order 12:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha^4 & 0 & 0 \\ \alpha & 0 & \alpha^6 & 0 \\ 0 & \alpha^2 & 0 & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^4 & \alpha^4 & 0 & 0 \\ 0 & 0 & \alpha^6 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & \alpha^4 & 0 & 0 \\ \alpha^3 & 0 & \alpha^6 & 0 \\ \alpha^3 & \alpha^6 & \alpha^6 & 1 \end{bmatrix}_1$$

There are 0 extensions
Number of generators 3

Classification of $5+1$ Configurations in $\mathrm{PG}(3,8)$

The order of the group is 103675594014720
The group has 18 orbits on five plus one configurations in $\mathrm{PG}(3,8)$.

Of these, 7 impose 19 conditions.
Of these, 4 are associated with double sixes. They are:
0/4 is orbit 11/18 $\{0,3,208,288,345\}_4$ orbit length 5462360064
1/4 is orbit 13/18 $\{0,3,208,288,369\}_3$ orbit length 7283146752
2/4 is orbit 15/18 $\{0,3,208,296,369\}_{12}$ orbit length 1820786688
3/4 is orbit 17/18 $\{0,3,208,320,377\}_{12}$ orbit length 1820786688
The overall number of five plus one configurations associated with double sixes in $\mathrm{PG}(3,8)$ is: 16387080192

Flag orbits for double sixes

The number of primary orbits below is 18
The number of primary orbits above is 2
The number of flag orbits is 4
The flag orbits are:

(1) Flag orbit 0 / 4 down=(11,0) up=(0,-1) is (0, 3, 208, 288, 345, 64, 4680, 138, 362, 825, 0, 64, 4680, 138, 362, 825, 2914, 2871, 472, 340, 81, 4744, 0) with a stabilizer of order 4
Strong generators for a group of order 4:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^5 & 1 & 0 & 0 \\ \alpha^5 & 0 & 1 & 0 \\ \alpha^6 & \alpha & \alpha & 1 \end{bmatrix}_0$$

(2) Flag orbit 1 / 4 down=(13,0) up=(1,-1) is (0, 3, 208, 288, 369, 64, 4680, 138, 362, 845, 0, 64, 4680, 138, 362, 845, 2841, 916, 323, 373, 486, 4744, 0) with a stabilizer of order 3
Strong generators for a group of order 3:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^3 & \alpha^3 & 0 & 0 \\ \alpha^4 & 0 & \alpha & 0 \\ \alpha^3 & \alpha^3 & \alpha^6 & \alpha^6 \end{bmatrix}_2$$

(3) Flag orbit 2 / 4 down=(15,0) up=(1,-1) fuse to 1 is (0, 3, 208, 296, 369, 64, 4680, 138, 289, 845, 0, 64, 4680, 138, 289, 845, 1090, 819, 388, 535, 486, 4744, 0) with a stabilizer of order 12
Strong generators for a group of order 12:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha^2 & 0 & 0 \\ \alpha^3 & 0 & \alpha^6 & 0 \\ 0 & \alpha^4 & 0 & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^2 & \alpha^2 & 0 & 0 \\ 0 & 0 & \alpha^6 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^3 & 1 & 0 & 0 \\ \alpha^4 & 0 & 1 & 0 \\ \alpha^3 & \alpha^6 & \alpha^5 & 1 \end{bmatrix}_0$$

Fusion element:

$$\begin{bmatrix} 1 & \alpha^2 & \alpha^2 & \alpha \\ \alpha^2 & \alpha^4 & \alpha^5 & \alpha^4 \\ \alpha^3 & \alpha^2 & \alpha^5 & \alpha^6 \\ \alpha^4 & \alpha^3 & 1 & \alpha^3 \end{bmatrix}_0$$

(4) Flag orbit 3 / 4 down=(17,0) up=(1,-1) fuse to 1 is (0, 3, 208, 320, 377, 64, 4680, 138, 435, 858, 0, 64, 4680, 138, 435, 858, 799, 1143, 539, 470, 405, 4744, 0) with a stabilizer of order 12
Strong generators for a group of order 12:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha^4 & 0 & 0 \\ \alpha & 0 & \alpha^6 & 0 \\ 0 & \alpha^2 & 0 & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^4 & \alpha^4 & 0 & 0 \\ 0 & 0 & \alpha^6 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & \alpha^4 & 0 & 0 \\ \alpha^3 & 0 & \alpha^6 & 0 \\ \alpha^3 & \alpha^6 & \alpha^6 & 1 \end{bmatrix}_1$$

Fusion element:

$$\begin{bmatrix} 1 & 0 & \alpha^6 & 0 \\ 0 & 0 & \alpha^5 & 0 \\ \alpha^4 & \alpha^3 & \alpha^3 & \alpha^2 \\ 0 & 0 & \alpha^2 & 1 \end{bmatrix}_0$$

Double Sixes

The order of the group is 103675594014720
The group has 2 orbits:

The orbits are:

(1) 0/2 {64, 4680, 138, 362, 825, 2914, 2871, 472, 340, 81, 4744, 0}$_{48}$ orbit length 2159908208640
Strong generators for a group of order 48:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha & 1 & 0 & 0 \\ \alpha & 0 & 1 & 0 \\ \alpha^4 & \alpha^5 & \alpha^5 & 1 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^5 & 1 & 0 & 0 \\ \alpha^5 & 0 & 1 & 0 \\ \alpha^6 & \alpha & \alpha & 1 \end{bmatrix}_0,$$

$$\begin{bmatrix} 0 & 0 & 1 & \alpha^6 \\ 0 & 0 & \alpha^5 & 0 \\ \alpha^3 & \alpha^5 & \alpha^5 & \alpha^4 \\ \alpha^4 & 0 & \alpha^6 & 0 \end{bmatrix}_0, \begin{bmatrix} 1 & \alpha^4 & \alpha^4 & 0 \\ \alpha & \alpha^3 & \alpha^5 & \alpha^4 \\ \alpha^5 & \alpha^2 & \alpha & \alpha^4 \\ \alpha^2 & \alpha^4 & 1 & \alpha^6 \end{bmatrix}_0$$

(2) 1/2 {64, 4680, 138, 362, 845, 2841, 916, 323, 373, 486, 4744, 0}$_{24}$ orbit length 4319816417280
Strong generators for a group of order 24:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & \alpha^2 & 0 & 0 \\ \alpha^5 & 0 & \alpha^3 & 0 \\ \alpha^5 & \alpha^4 & \alpha^3 & \alpha^4 \end{bmatrix}_1, \begin{bmatrix} 1 & \alpha^2 & \alpha^6 & \alpha \\ \alpha^6 & \alpha^4 & \alpha^2 & \alpha^4 \\ \alpha^4 & \alpha^3 & \alpha^3 & \alpha^2 \\ \alpha^6 & \alpha^5 & \alpha^2 & \alpha^5 \end{bmatrix}_0, \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & \alpha^3 & \alpha^2 \\ \alpha^4 & 0 & \alpha^5 & 0 \\ \alpha^2 & \alpha^5 & \alpha^2 & \alpha^4 \end{bmatrix}_1$$

The overall number of objects is: 6479724625920

Flag orbits for surfaces

The number of primary orbits below is 2
The number of primary orbits above is 1
The number of flag orbits is 2
The flag orbits are:

(1) Flag orbit 0 / 2 down=(0,0) up=(0,-1) is (64, 4680, 138, 362, 825, 2914, 2871, 472, 340, 81, 4744, 0, 543, 272, 158, 4154, 1, 1508, 3033, 4742, 4104, 1899, 3062, 3969, 1530, 1507, 579) with a stabilizer of order 48
Strong generators for a group of order 48:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha & 1 & 0 & 0 \\ \alpha & 0 & 1 & 0 \\ \alpha^4 & \alpha^5 & \alpha^5 & 1 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^5 & 1 & 0 & 0 \\ \alpha^5 & 0 & 1 & 0 \\ \alpha^6 & \alpha & \alpha & 1 \end{bmatrix}_0,$$

$$\begin{bmatrix} 0 & 0 & 1 & \alpha^6 \\ 0 & 0 & \alpha^5 & 0 \\ \alpha^3 & \alpha^5 & \alpha^5 & \alpha^4 \\ \alpha^4 & 0 & \alpha^6 & 0 \end{bmatrix}_0, \begin{bmatrix} 1 & \alpha^4 & \alpha^4 & 0 \\ \alpha & \alpha^3 & \alpha^5 & \alpha^4 \\ \alpha^5 & \alpha^2 & \alpha & \alpha^4 \\ \alpha^2 & \alpha^4 & 1 & \alpha^6 \end{bmatrix}_0$$

(2) Flag orbit 1 / 2 down=(1,0) up=(0,-1) fuse to 0 is (64, 4680, 138, 362, 845, 2841, 916, 323, 373, 486, 4744, 0, 316, 377, 492, 648, 3, 981, 1070, 4695, 640, 1152, 3643, 711, 4451, 767, 506) with a stabilizer of order 24
Strong generators for a group of order 24:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & \alpha^2 & 0 & 0 \\ \alpha^5 & 0 & \alpha^3 & 0 \\ \alpha^5 & \alpha^4 & \alpha^3 & \alpha^4 \end{bmatrix}_1, \begin{bmatrix} 1 & \alpha^2 & \alpha^6 & \alpha \\ \alpha^6 & \alpha^4 & \alpha^2 & \alpha^4 \\ \alpha^4 & \alpha^3 & \alpha^3 & \alpha^2 \\ \alpha^6 & \alpha^5 & \alpha^2 & \alpha^5 \end{bmatrix}_0, \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & \alpha^3 & \alpha^2 \\ \alpha^4 & 0 & \alpha^5 & 0 \\ \alpha^2 & \alpha^5 & \alpha^2 & \alpha^4 \end{bmatrix}_1$$

Fusion element:

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & \alpha^6 & 1 & \alpha^6 \\ \alpha^2 & \alpha^4 & \alpha^4 & 0 \\ \alpha^2 & \alpha^6 & \alpha^4 & \alpha^6 \end{bmatrix}_1$$

Surfaces

The order of the group is 103675594014720
The group has 1 orbits:

The orbits are:

(1) 0/1 $\{64, 4680, 138, 362, 825, 2914, 2871, 472, 340, 81, 4744, 0, 543, 272, 158, 4154, 1, 1508, 3033, 4742, 4104, 1899, 3062, 3969, 1530, 1507, 579\}_{576}$ orbit length 179992350720
Strong generators for a group of order 576:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha^3 & 0 & 0 \\ \alpha^3 & 0 & \alpha^3 & 0 \\ 0 & \alpha & 0 & \alpha \end{bmatrix}_2, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha & 1 & 0 & 0 \\ \alpha & 0 & 1 & 0 \\ \alpha^4 & \alpha^5 & \alpha^5 & 1 \end{bmatrix}_0,$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^4 & 0 & 1 & 0 \\ \alpha^4 & 1 & 0 & 0 \\ \alpha^3 & 0 & 0 & 1 \end{bmatrix}_0, \begin{bmatrix} 1 & 1 & \alpha^2 & \alpha^2 \\ 0 & \alpha^2 & \alpha^3 & \alpha^4 \\ \alpha^3 & \alpha^3 & \alpha^6 & \alpha^6 \\ 0 & 1 & \alpha^5 & \alpha^5 \end{bmatrix}_0$$

The overall number of objects is: 179992350720

The Group $\mathrm{P\Gamma L}(4, 8)$

The order of the group is 103675594014720

Cubic Surfaces with 27 Lines in $\mathrm{PG}(3, 8)$

The order of the group is 103675594014720
The group has 1 orbits:

The orbits are:

(1) 0/1 $\{64, 4680, 138, 362, 825, 2914, 2871, 472, 340, 81, 4744, 0, 543, 272, 158, 4154, 1, 1508, 3033, 4742, 4104, 1899, 3062, 3969, 1530, 1507, 579\}_{576}$ orbit length 179992350720
Strong generators for a group of order 576:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha^3 & 0 & 0 \\ \alpha^3 & 0 & \alpha^3 & 0 \\ 0 & \alpha & 0 & \alpha \end{bmatrix}_2, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha & 1 & 0 & 0 \\ \alpha & 0 & 1 & 0 \\ \alpha^4 & \alpha^5 & \alpha^5 & 1 \end{bmatrix}_0,$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^4 & 0 & 1 & 0 \\ \alpha^4 & 1 & 0 & 0 \\ \alpha^3 & 0 & 0 & 1 \end{bmatrix}_0, \begin{bmatrix} 1 & 1 & \alpha^2 & \alpha^2 \\ 0 & \alpha^2 & \alpha^3 & \alpha^4 \\ \alpha^3 & \alpha^3 & \alpha^6 & \alpha^6 \\ 0 & 1 & \alpha^5 & \alpha^5 \end{bmatrix}_0$$

The overall number of objects is: 179992350720

Surface 8#0

The equation

The equation of the surface is :

$$X_0^2X_3+\alpha^6X_1^2X_2+\alpha^6X_1X_2^2+\alpha^3X_0X_3^2+\alpha^4X_0X_1X_3+\alpha^4X_0X_2X_3+\alpha^2X_1X_2X_3=0$$

Number of points on the surface 121
The automorphism group of the surface has order 576
The automorphism group is the following group
Strong generators for a group of order 576:

$$\left[\begin{array}{cccc}1&0&0&0\\0&\alpha^3&0&0\\\alpha^3&0&\alpha^3&0\\0&\alpha&0&\alpha\end{array}\right]_2, \left[\begin{array}{cccc}1&0&0&0\\1&0&1&0\\1&1&0&0\\1&1&1&1\end{array}\right]_0, \left[\begin{array}{cccc}1&0&0&0\\\alpha&1&0&0\\\alpha&0&1&0\\\alpha^4&\alpha^5&\alpha^5&1\end{array}\right]_0,$$

$$\left[\begin{array}{cccc}1&0&0&0\\\alpha^4&0&1&0\\\alpha^4&1&0&0\\\alpha^3&0&0&1\end{array}\right]_0, \left[\begin{array}{cccc}1&1&\alpha^2&\alpha^2\\0&\alpha^2&\alpha^3&\alpha^4\\\alpha^3&\alpha^3&\alpha^6&\alpha^6\\0&1&\alpha^5&\alpha^5\end{array}\right]_0$$

General information

Plane types by points:

$$25^{13},\ 24^{32},\ 18^{96},\ 17^{12},\ 14^{288},\ 13^{48},\ 12^{96}$$

Type of pts on lines:

$$9^{27}$$

Type of lines on point:

$$3^{13},\ 2^{96},\ 1^{12}$$

Type iso of tritangent planes:

$$3^{32},\ 2^{13}$$

The 27 lines

$$\ell_0=a_1=\left[\begin{array}{cccc}1&0&\alpha^6&0\\0&1&\alpha&\alpha^2\end{array}\right]_{472}$$

$$\ell_1=a_2=\left[\begin{array}{cccc}1&0&\alpha^2&0\\0&1&0&\alpha^6\end{array}\right]_{340}$$

$$\ell_2=a_3=\left[\begin{array}{cccc}1&0&1&0\\0&1&0&1\end{array}\right]_{81}$$

$$\ell_3=a_4=\left[\begin{array}{cccc}0&0&1&0\\0&0&0&1\end{array}\right]_{4744}$$

$$\ell_4=a_5=\left[\begin{array}{cccc}1&0&0&0\\0&1&0&0\end{array}\right]_{0}$$

$$\ell_5=a_6=\left[\begin{array}{cccc}1&0&\alpha^4&\alpha^2\\0&1&0&\alpha^5\end{array}\right]_{2871}$$

$$\ell_6=b_1=\left[\begin{array}{cccc}0&1&0&0\\0&0&0&1\end{array}\right]_{4680}$$

$$\ell_7=b_2=\left[\begin{array}{cccc}1&1&0&0\\0&0&1&1\end{array}\right]_{138}$$

$$\ell_8=b_3=\left[\begin{array}{cccc}1&\alpha^2&0&0\\0&0&1&\alpha^6\end{array}\right]_{362}$$

$$\ell_9=b_4=\left[\begin{array}{cccc}1&0&\alpha^5&1\\0&1&\alpha^6&\alpha\end{array}\right]_{825}$$

$$\ell_{10} = b_5 = \begin{bmatrix} 1 & \alpha^4 & 0 & \alpha^2 \\ 0 & 0 & 1 & \alpha^5 \end{bmatrix}_{2914}$$

$$\ell_{11} = b_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{64}$$

$$\ell_{12} = c_{12} = \begin{bmatrix} 1 & 0 & \alpha^2 & \alpha \\ 0 & 1 & 0 & \alpha^6 \end{bmatrix}_{1508}$$

$$\ell_{13} = c_{13} = \begin{bmatrix} 1 & 0 & 1 & \alpha^3 \\ 0 & 1 & 0 & \alpha^3 \end{bmatrix}_{3033}$$

$$\ell_{14} = c_{14} = \begin{bmatrix} 0 & 1 & 0 & \alpha^4 \\ 0 & 0 & 1 & \alpha^4 \end{bmatrix}_{4742}$$

$$\ell_{15} = c_{15} = \begin{bmatrix} 1 & 0 & 0 & \alpha^4 \\ 0 & 1 & 0 & \alpha \end{bmatrix}_{4104}$$

$$\ell_{16} = c_{16} = \begin{bmatrix} 1 & 0 & \alpha^4 & 0 \\ 0 & 1 & 0 & \alpha^2 \end{bmatrix}_{543}$$

$$\ell_{17} = c_{23} = \begin{bmatrix} 1 & 0 & \alpha & \alpha^5 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{1899}$$

$$\ell_{18} = c_{24} = \begin{bmatrix} 1 & 1 & 0 & \alpha^3 \\ 0 & 0 & 1 & \alpha^3 \end{bmatrix}_{3062}$$

$$\ell_{19} = c_{25} = \begin{bmatrix} 1 & 0 & \alpha^6 & \alpha^6 \\ 0 & 1 & \alpha^5 & \alpha^5 \end{bmatrix}_{3969}$$

$$\ell_{20} = c_{26} = \begin{bmatrix} 1 & 0 & \alpha^5 & 0 \\ 0 & 1 & \alpha^3 & \alpha^6 \end{bmatrix}_{272}$$

$$\ell_{21} = c_{34} = \begin{bmatrix} 1 & \alpha^2 & 0 & \alpha \\ 0 & 0 & 1 & \alpha^6 \end{bmatrix}_{1530}$$

$$\ell_{22} = c_{35} = \begin{bmatrix} 1 & 0 & \alpha^2 & \alpha \\ 0 & 1 & \alpha^4 & \alpha^3 \end{bmatrix}_{1507}$$

$$\ell_{23} = c_{36} = \begin{bmatrix} 1 & 0 & \alpha & 0 \\ 0 & 1 & \alpha^2 & 1 \end{bmatrix}_{158}$$

$$\ell_{24} = c_{45} = \begin{bmatrix} 1 & \alpha^4 & 0 & 0 \\ 0 & 0 & 1 & \alpha^2 \end{bmatrix}_{579}$$

$$\ell_{25} = c_{46} = \begin{bmatrix} 1 & 0 & 0 & \alpha^4 \\ 0 & 0 & 1 & \alpha \end{bmatrix}_{4154}$$

$$\ell_{26} = c_{56} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{1}$$

Points on surface

All Points

The surface has 121 points:

	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	10	11	12	13	14	15	16	17	18	19
2	20	21	22	23	24	25	26	79	82	84
3	90	96	98	99	106	109	114	118	122	130
4	137	138	140	149	155	156	164	166	169	173
5	182	183	186	191	200	201	207	211	212	220
6	223	230	232	234	243	246	253	256	257	261
7	265	266	276	278	286	288	291	296	300	301
8	311	313	317	322	323	329	332	337	341	346
9	356	357	378	384	393	397	406	407	411	414
10	423	451	453	457	466	471	477	480	481	485
11	490	496	521	528	536	537	541	546	547	563
12	565	0	0	0	0	0	0	0	0	0

The points on the surface are:

$0: P_{0} = P_{0} = (1, 0, 0, 0)$
$1: P_{1} = P_{1} = (0, 1, 0, 0)$
$2: P_{2} = P_{2} = (0, 0, 1, 0)$
$3: P_{3} = P_{3} = (0, 0, 0, 1)$
$4: P_{4} = P_{4} = (1, 1, 1, 1)$
$5: P_{5} = P_{5} = (1, 1, 0, 0)$
$6: P_{6} = P_{6} = (2, 1, 0, 0)$
$7: P_{7} = P_{7} = (3, 1, 0, 0)$
$8: P_{8} = P_{8} = (4, 1, 0, 0)$
$9: P_{9} = P_{9} = (5, 1, 0, 0)$
$10: P_{10} = P_{10} = (6, 1, 0, 0)$
$11: P_{11} = P_{11} = (7, 1, 0, 0)$
$12: P_{12} = P_{12} = (1, 0, 1, 0)$
$13: P_{13} = P_{13} = (2, 0, 1, 0)$
$14: P_{14} = P_{14} = (3, 0, 1, 0)$
$15: P_{15} = P_{15} = (4, 0, 1, 0)$
$16: P_{16} = P_{16} = (5, 0, 1, 0)$
$17: P_{17} = P_{17} = (6, 0, 1, 0)$
$18: P_{18} = P_{18} = (7, 0, 1, 0)$
$19: P_{19} = P_{19} = (0, 1, 1, 0)$
$20: P_{20} = P_{20} = (1, 1, 1, 0)$
$21: P_{21} = P_{21} = (2, 1, 1, 0)$
$22: P_{22} = P_{22} = (3, 1, 1, 0)$
$23: P_{23} = P_{23} = (4, 1, 1, 0)$
$24: P_{24} = P_{24} = (5, 1, 1, 0)$
$25: P_{25} = P_{25} = (6, 1, 1, 0)$
$26: P_{26} = P_{26} = (7, 1, 1, 0)$
$27: P_{27} = P_{79} = (5, 0, 0, 1)$
$28: P_{28} = P_{82} = (0, 1, 0, 1)$
$29: P_{29} = P_{84} = (2, 1, 0, 1)$
$30: P_{30} = P_{90} = (0, 2, 0, 1)$
$31: P_{31} = P_{96} = (6, 2, 0, 1)$
$32: P_{32} = P_{98} = (0, 3, 0, 1)$
$33: P_{33} = P_{99} = (1, 3, 0, 1)$
$34: P_{34} = P_{106} = (0, 4, 0, 1)$
$35: P_{35} = P_{109} = (3, 4, 0, 1)$
$36: P_{36} = P_{114} = (0, 5, 0, 1)$
$37: P_{37} = P_{118} = (4, 5, 0, 1)$
$38: P_{38} = P_{122} = (0, 6, 0, 1)$
$39: P_{39} = P_{130} = (0, 7, 0, 1)$
$40: P_{40} = P_{137} = (7, 7, 0, 1)$
$41: P_{41} = P_{138} = (0, 0, 1, 1)$
$42: P_{42} = P_{140} = (2, 0, 1, 1)$
$43: P_{43} = P_{149} = (4, 1, 1, 1)$
$44: P_{44} = P_{155} = (2, 2, 1, 1)$
$45: P_{45} = P_{156} = (3, 2, 1, 1)$
$46: P_{46} = P_{164} = (3, 3, 1, 1)$
$47: P_{47} = P_{166} = (5, 3, 1, 1)$
$48: P_{48} = P_{169} = (0, 4, 1, 1)$
$49: P_{49} = P_{173} = (4, 4, 1, 1)$
$50: P_{50} = P_{182} = (5, 5, 1, 1)$
$51: P_{51} = P_{183} = (6, 5, 1, 1)$
$52: P_{52} = P_{186} = (1, 6, 1, 1)$
$53: P_{53} = P_{191} = (6, 6, 1, 1)$
$54: P_{54} = P_{200} = (7, 7, 1, 1)$
$55: P_{55} = P_{201} = (0, 0, 2, 1)$
$56: P_{56} = P_{207} = (6, 0, 2, 1)$
$57: P_{57} = P_{211} = (2, 1, 2, 1)$
$58: P_{58} = P_{212} = (3, 1, 2, 1)$
$59: P_{59} = P_{220} = (3, 2, 2, 1)$
$60: P_{60} = P_{223} = (6, 2, 2, 1)$
$61: P_{61} = P_{230} = (5, 3, 2, 1)$
$62: P_{62} = P_{232} = (7, 3, 2, 1)$
$63: P_{63} = P_{234} = (1, 4, 2, 1)$
$64: P_{64} = P_{243} = (2, 5, 2, 1)$
$65: P_{65} = P_{246} = (5, 5, 2, 1)$
$66: P_{66} = P_{253} = (4, 6, 2, 1)$
$67: P_{67} = P_{256} = (7, 6, 2, 1)$
$68: P_{68} = P_{257} = (0, 7, 2, 1)$
$69: P_{69} = P_{261} = (4, 7, 2, 1)$
$70: P_{70} = P_{265} = (0, 0, 3, 1)$
$71: P_{71} = P_{266} = (1, 0, 3, 1)$
$72: P_{72} = P_{276} = (3, 1, 3, 1)$
$73: P_{73} = P_{278} = (5, 1, 3, 1)$
$74: P_{74} = P_{286} = (5, 2, 3, 1)$
$75: P_{75} = P_{288} = (7, 2, 3, 1)$
$76: P_{76} = P_{291} = (2, 3, 3, 1)$
$77: P_{77} = P_{296} = (7, 3, 3, 1)$
$78: P_{78} = P_{300} = (3, 4, 3, 1)$
$79: P_{79} = P_{301} = (4, 4, 3, 1)$
$80: P_{80} = P_{311} = (6, 5, 3, 1)$
$81: P_{81} = P_{313} = (0, 6, 3, 1)$
$82: P_{82} = P_{317} = (4, 6, 3, 1)$
$83: P_{83} = P_{322} = (1, 7, 3, 1)$
$84: P_{84} = P_{323} = (2, 7, 3, 1)$
$85: P_{85} = P_{329} = (0, 0, 4, 1)$
$86: P_{86} = P_{332} = (3, 0, 4, 1)$
$87: P_{87} = P_{337} = (0, 1, 4, 1)$
$88: P_{88} = P_{341} = (4, 1, 4, 1)$
$89: P_{89} = P_{346} = (1, 2, 4, 1)$
$90: P_{90} = P_{356} = (3, 3, 4, 1)$
$91: P_{91} = P_{357} = (4, 3, 4, 1)$
$92: P_{92} = P_{378} = (1, 6, 4, 1)$
$93: P_{93} = P_{384} = (7, 6, 4, 1)$
$94: P_{94} = P_{393} = (0, 0, 5, 1)$
$95: P_{95} = P_{397} = (4, 0, 5, 1)$
$96: P_{96} = P_{406} = (5, 1, 5, 1)$
$97: P_{97} = P_{407} = (6, 1, 5, 1)$
$98: P_{98} = P_{411} = (2, 2, 5, 1)$
$99: P_{99} = P_{414} = (5, 2, 5, 1)$
$100: P_{100} = P_{423} = (6, 3, 5, 1)$
$101: P_{101} = P_{451} = (2, 7, 5, 1)$
$102: P_{102} = P_{453} = (4, 7, 5, 1)$
$103: P_{103} = P_{457} = (0, 0, 6, 1)$
$104: P_{104} = P_{466} = (1, 1, 6, 1)$
$105: P_{105} = P_{471} = (6, 1, 6, 1)$
$106: P_{106} = P_{477} = (4, 2, 6, 1)$
$107: P_{107} = P_{480} = (7, 2, 6, 1)$
$108: P_{108} = P_{481} = (0, 3, 6, 1)$
$109: P_{109} = P_{485} = (4, 3, 6, 1)$
$110: P_{110} = P_{490} = (1, 4, 6, 1)$
$111: P_{111} = P_{496} = (7, 4, 6, 1)$

112 : $P_{112} = P_{521} = (0, 0, 7, 1)$
113 : $P_{113} = P_{528} = (7, 0, 7, 1)$
114 : $P_{114} = P_{536} = (7, 1, 7, 1)$
115 : $P_{115} = P_{537} = (0, 2, 7, 1)$
116 : $P_{116} = P_{541} = (4, 2, 7, 1)$
117 : $P_{117} = P_{546} = (1, 3, 7, 1)$
118 : $P_{118} = P_{547} = (2, 3, 7, 1)$
119 : $P_{119} = P_{563} = (2, 5, 7, 1)$
120 : $P_{120} = P_{565} = (4, 5, 7, 1)$

Eckardt Points

The surface has 13 Eckardt points:

	0	1	2	3	4	5	6	7	8	9
0	0	19	20	22	24	90	96	164	166	201
1	207	276	278	0	0	0	0	0	0	0

	0	1	2	3	4	5	6	7	8	9
0	0	19	20	22	24	30	31	46	47	55
1	56	72	73	0	0	0	0	0	0	0

The Eckardt points on the surface are:

$$
\begin{aligned}
E_0 = P_0 = P_0 = (1, 0, 0, 0) &= \ell_4 \cap \ell_{11} \cap \ell_{26} = a_5 \cap b_6 \cap c_{56}\\
E_1 = P_{19} = P_{19} = (0, 1, 1, 0) &= \ell_{14} \cap \ell_{17} \cap \ell_{26} = c_{14} \cap c_{23} \cap c_{56}\\
E_2 = P_{20} = P_{20} = (1, 1, 1, 0) &= \ell_{13} \cap \ell_{18} \cap \ell_{26} = c_{13} \cap c_{24} \cap c_{56}\\
E_3 = P_{22} = P_{22} = (3, 1, 1, 0) &= \ell_{12} \cap \ell_{21} \cap \ell_{26} = c_{12} \cap c_{34} \cap c_{56}\\
E_4 = P_{24} = P_{24} = (5, 1, 1, 0) &= \ell_5 \cap \ell_{10} \cap \ell_{26} = a_6 \cap b_5 \cap c_{56}\\
E_5 = P_{30} = P_{90} = (0, 2, 0, 1) &= \ell_1 \cap \ell_6 \cap \ell_{12} = a_2 \cap b_1 \cap c_{12}\\
E_6 = P_{31} = P_{96} = (6, 2, 0, 1) &= \ell_{15} \cap \ell_{20} \cap \ell_{21} = c_{15} \cap c_{26} \cap c_{34}\\
E_7 = P_{46} = P_{164} = (3, 3, 1, 1) &= \ell_0 \cap \ell_7 \cap \ell_{12} = a_1 \cap b_2 \cap c_{12}\\
E_8 = P_{47} = P_{166} = (5, 3, 1, 1) &= \ell_{16} \cap \ell_{19} \cap \ell_{21} = c_{16} \cap c_{25} \cap c_{34}\\
E_9 = P_{55} = P_{201} = (0, 0, 2, 1) &= \ell_3 \cap \ell_8 \cap \ell_{21} = a_4 \cap b_3 \cap c_{34}\\
E_{10} = P_{56} = P_{207} = (6, 0, 2, 1) &= \ell_{12} \cap \ell_{22} \cap \ell_{25} = c_{12} \cap c_{35} \cap c_{46}\\
E_{11} = P_{72} = P_{276} = (3, 1, 3, 1) &= \ell_2 \cap \ell_9 \cap \ell_{21} = a_3 \cap b_4 \cap c_{34}\\
E_{12} = P_{73} = P_{278} = (5, 1, 3, 1) &= \ell_{12} \cap \ell_{23} \cap \ell_{24} = c_{12} \cap c_{36} \cap c_{45}
\end{aligned}
$$

The Eckardt points on the surface are:

$$
\begin{aligned}
E_0 = P_0 &= \mathbf{P}(1, 0, 0, 0) = a_5 \cap b_6 \cap c_{56},\\
E_1 = P_{19} &= \mathbf{P}(0, 1, 1, 0) = c_{14} \cap c_{23} \cap c_{56},\\
E_2 = P_{20} &= \mathbf{P}(1, 1, 1, 0) = c_{13} \cap c_{24} \cap c_{56},\\
E_3 = P_{22} &= \mathbf{P}(\alpha^5, 1, 1, 0) = c_{12} \cap c_{34} \cap c_{56},\\
E_4 = P_{24} &= \mathbf{P}(\alpha^3, 1, 1, 0) = a_6 \cap b_5 \cap c_{56},\\
E_5 = P_{90} &= \mathbf{P}(0, \alpha, 0, 1) = a_2 \cap b_1 \cap c_{12},\\
E_6 = P_{96} &= \mathbf{P}(\alpha^6, \alpha, 0, 1) = c_{15} \cap c_{26} \cap c_{34},\\
E_7 = P_{164} &= \mathbf{P}(\alpha^5, \alpha^5, 1, 1) = a_1 \cap b_2 \cap c_{12},\\
E_8 = P_{166} &= \mathbf{P}(\alpha^3, \alpha^5, 1, 1) = c_{16} \cap c_{25} \cap c_{34},\\
E_9 = P_{201} &= \mathbf{P}(0, 0, \alpha, 1) = a_4 \cap b_3 \cap c_{34},\\
E_{10} = P_{207} &= \mathbf{P}(\alpha^6, 0, \alpha, 1) = c_{12} \cap c_{35} \cap c_{46},\\
E_{11} = P_{276} &= \mathbf{P}(\alpha^5, 1, \alpha^5, 1) = a_3 \cap b_4 \cap c_{34},\\
E_{12} = P_{278} &= \mathbf{P}(\alpha^3, 1, \alpha^5, 1) = c_{12} \cap c_{36} \cap c_{45}.
\end{aligned}
$$

Double Points

The surface has 96 Double points:

	0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9	10
1	11	12	13	14	15	16	17	18	79	82
2	84	98	99	106	109	114	118	122	130	137
3	138	140	149	155	156	169	173	182	183	186
4	191	200	211	212	220	223	230	232	234	243
5	246	253	256	257	261	265	266	286	288	291
6	296	300	301	311	313	317	322	323	329	332
7	337	341	346	356	357	393	397	406	407	411
8	414	423	457	466	471	477	480	481	485	521
9	528	536	537	541	546	547	0	0	0	0

	0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9	10
1	11	12	13	14	15	16	17	18	27	28
2	29	32	33	34	35	36	37	38	39	40
3	41	42	43	44	45	48	49	50	51	52
4	53	54	57	58	59	60	61	62	63	64
5	65	66	67	68	69	70	71	74	75	76
6	77	78	79	80	81	82	83	84	85	86
7	87	88	89	90	91	94	95	96	97	98
8	99	100	103	104	105	106	107	108	109	112
9	113	114	115	116	117	118	0	0	0	0

The Double points on the surface are:

$0: P_1 = P_1 = (0,1,0,0)$
$1: P_2 = P_2 = (0,0,1,0)$
$2: P_3 = P_3 = (0,0,0,1)$
$3: P_4 = P_4 = (1,1,1,1)$
$4: P_5 = P_5 = (1,1,0,0)$
$5: P_6 = P_6 = (2,1,0,0)$
$6: P_7 = P_7 = (3,1,0,0)$
$7: P_8 = P_8 = (4,1,0,0)$
$8: P_9 = P_9 = (5,1,0,0)$
$9: P_{10} = P_{10} = (6,1,0,0)$
$10: P_{11} = P_{11} = (7,1,0,0)$
$11: P_{12} = P_{12} = (1,0,1,0)$
$12: P_{13} = P_{13} = (2,0,1,0)$
$13: P_{14} = P_{14} = (3,0,1,0)$
$14: P_{15} = P_{15} = (4,0,1,0)$
$15: P_{16} = P_{16} = (5,0,1,0)$
$16: P_{17} = P_{17} = (6,0,1,0)$
$17: P_{18} = P_{18} = (7,0,1,0)$
$18: P_{27} = P_{79} = (5,0,0,1)$
$19: P_{28} = P_{82} = (0,1,0,1)$
$20: P_{29} = P_{84} = (2,1,0,1)$
$21: P_{32} = P_{98} = (0,3,0,1)$
$22: P_{33} = P_{99} = (1,3,0,1)$
$23: P_{34} = P_{106} = (0,4,0,1)$
$24: P_{35} = P_{109} = (3,4,0,1)$
$25: P_{36} = P_{114} = (0,5,0,1)$
$26: P_{37} = P_{118} = (4,5,0,1)$
$27: P_{38} = P_{122} = (0,6,0,1)$
$28: P_{39} = P_{130} = (0,7,0,1)$
$29: P_{40} = P_{137} = (7,7,0,1)$
$30: P_{41} = P_{138} = (0,0,1,1)$
$31: P_{42} = P_{140} = (2,0,1,1)$
$32: P_{43} = P_{149} = (4,1,1,1)$
$33: P_{44} = P_{155} = (2,2,1,1)$
$34: P_{45} = P_{156} = (3,2,1,1)$
$35: P_{48} = P_{169} = (0,4,1,1)$
$36: P_{49} = P_{173} = (4,4,1,1)$
$37: P_{50} = P_{182} = (5,5,1,1)$
$38: P_{51} = P_{183} = (6,5,1,1)$
$39: P_{52} = P_{186} = (1,6,1,1)$
$40: P_{53} = P_{191} = (6,6,1,1)$
$41: P_{54} = P_{200} = (7,7,1,1)$
$42: P_{57} = P_{211} = (2,1,2,1)$
$43: P_{58} = P_{212} = (3,1,2,1)$
$44: P_{59} = P_{220} = (3,2,2,1)$
$45: P_{60} = P_{223} = (6,2,2,1)$
$46: P_{61} = P_{230} = (5,3,2,1)$
$47: P_{62} = P_{232} = (7,3,2,1)$
$48: P_{63} = P_{234} = (1,4,2,1)$
$49: P_{64} = P_{243} = (2,5,2,1)$
$50: P_{65} = P_{246} = (5,5,2,1)$
$51: P_{66} = P_{253} = (4,6,2,1)$
$52: P_{67} = P_{256} = (7,6,2,1)$
$53: P_{68} = P_{257} = (0,7,2,1)$
$54: P_{69} = P_{261} = (4,7,2,1)$
$55: P_{70} = P_{265} = (0,0,3,1)$
$56: P_{71} = P_{266} = (1,0,3,1)$
$57: P_{74} = P_{286} = (5,2,3,1)$

58 : $P_{75} = P_{288} = (7, 2, 3, 1)$
59 : $P_{76} = P_{291} = (2, 3, 3, 1)$
60 : $P_{77} = P_{296} = (7, 3, 3, 1)$
61 : $P_{78} = P_{300} = (3, 4, 3, 1)$
62 : $P_{79} = P_{301} = (4, 4, 3, 1)$
63 : $P_{80} = P_{311} = (6, 5, 3, 1)$
64 : $P_{81} = P_{313} = (0, 6, 3, 1)$
65 : $P_{82} = P_{317} = (4, 6, 3, 1)$
66 : $P_{83} = P_{322} = (1, 7, 3, 1)$
67 : $P_{84} = P_{323} = (2, 7, 3, 1)$
68 : $P_{85} = P_{329} = (0, 0, 4, 1)$
69 : $P_{86} = P_{332} = (3, 0, 4, 1)$
70 : $P_{87} = P_{337} = (0, 1, 4, 1)$
71 : $P_{88} = P_{341} = (4, 1, 4, 1)$
72 : $P_{89} = P_{346} = (1, 2, 4, 1)$
73 : $P_{90} = P_{356} = (3, 3, 4, 1)$
74 : $P_{91} = P_{357} = (4, 3, 4, 1)$
75 : $P_{94} = P_{393} = (0, 0, 5, 1)$
76 : $P_{95} = P_{397} = (4, 0, 5, 1)$
77 : $P_{96} = P_{406} = (5, 1, 5, 1)$
78 : $P_{97} = P_{407} = (6, 1, 5, 1)$
79 : $P_{98} = P_{411} = (2, 2, 5, 1)$
80 : $P_{99} = P_{414} = (5, 2, 5, 1)$
81 : $P_{100} = P_{423} = (6, 3, 5, 1)$
82 : $P_{103} = P_{457} = (0, 0, 6, 1)$
83 : $P_{104} = P_{466} = (1, 1, 6, 1)$
84 : $P_{105} = P_{471} = (6, 1, 6, 1)$
85 : $P_{106} = P_{477} = (4, 2, 6, 1)$
86 : $P_{107} = P_{480} = (7, 2, 6, 1)$
87 : $P_{108} = P_{481} = (0, 3, 6, 1)$
88 : $P_{109} = P_{485} = (4, 3, 6, 1)$
89 : $P_{112} = P_{521} = (0, 0, 7, 1)$
90 : $P_{113} = P_{528} = (7, 0, 7, 1)$
91 : $P_{114} = P_{536} = (7, 1, 7, 1)$
92 : $P_{115} = P_{537} = (0, 2, 7, 1)$
93 : $P_{116} = P_{541} = (4, 2, 7, 1)$
94 : $P_{117} = P_{546} = (1, 3, 7, 1)$
95 : $P_{118} = P_{547} = (2, 3, 7, 1)$

The double points on the surface are:

$$
\begin{aligned}
P_{61} = P_{230} = (5, 3, 2, 1) &= \ell_0 \cap \ell_8 = a_1 \cap b_3 \\
P_{77} = P_{296} = (7, 3, 3, 1) &= \ell_0 \cap \ell_9 = a_1 \cap b_4 \\
P_{118} = P_{547} = (2, 3, 7, 1) &= \ell_0 \cap \ell_{10} = a_1 \cap b_5 \\
P_{13} = P_{13} = (2, 0, 1, 0) &= \ell_0 \cap \ell_{11} = a_1 \cap b_6 \\
P_{91} = P_{357} = (4, 3, 4, 1) &= \ell_0 \cap \ell_{13} = a_1 \cap c_{13} \\
P_{108} = P_{481} = (0, 3, 6, 1) &= \ell_0 \cap \ell_{14} = a_1 \cap c_{14} \\
P_{33} = P_{99} = (1, 3, 0, 1) &= \ell_0 \cap \ell_{15} = a_1 \cap c_{15} \\
P_{100} = P_{423} = (6, 3, 5, 1) &= \ell_0 \cap \ell_{16} = a_1 \cap c_{16} \\
P_{60} = P_{223} = (6, 2, 2, 1) &= \ell_1 \cap \ell_8 = a_2 \cap b_3 \\
P_{74} = P_{286} = (5, 2, 3, 1) &= \ell_1 \cap \ell_9 = a_2 \cap b_4 \\
P_{107} = P_{480} = (7, 2, 6, 1) &= \ell_1 \cap \ell_{10} = a_2 \cap b_5 \\
P_{14} = P_{14} = (3, 0, 1, 0) &= \ell_1 \cap \ell_{11} = a_2 \cap b_6 \\
P_{116} = P_{541} = (4, 2, 7, 1) &= \ell_1 \cap \ell_{17} = a_2 \cap c_{23} \\
P_{98} = P_{411} = (2, 2, 5, 1) &= \ell_1 \cap \ell_{18} = a_2 \cap c_{24} \\
P_{45} = P_{156} = (3, 2, 1, 1) &= \ell_1 \cap \ell_{19} = a_2 \cap c_{25} \\
P_{89} = P_{346} = (1, 2, 4, 1) &= \ell_1 \cap \ell_{20} = a_2 \cap c_{26} \\
P_{28} = P_{82} = (0, 1, 0, 1) &= \ell_2 \cap \ell_6 = a_3 \cap b_1 \\
P_4 = P_4 = (1, 1, 1, 1) &= \ell_2 \cap \ell_7 = a_3 \cap b_2 \\
P_{96} = P_{406} = (5, 1, 5, 1) &= \ell_2 \cap \ell_{10} = a_3 \cap b_5 \\
P_{12} = P_{12} = (1, 0, 1, 0) &= \ell_2 \cap \ell_{11} = a_3 \cap b_6 \\
P_{105} = P_{471} = (6, 1, 6, 1) &= \ell_2 \cap \ell_{13} = a_3 \cap c_{13} \\
P_{88} = P_{341} = (4, 1, 4, 1) &= \ell_2 \cap \ell_{17} = a_3 \cap c_{23} \\
P_{57} = P_{211} = (2, 1, 2, 1) &= \ell_2 \cap \ell_{22} = a_3 \cap c_{35} \\
P_{114} = P_{536} = (7, 1, 7, 1) &= \ell_2 \cap \ell_{23} = a_3 \cap c_{36} \\
P_3 = P_3 = (0, 0, 0, 1) &= \ell_3 \cap \ell_6 = a_4 \cap b_1 \\
P_{41} = P_{138} = (0, 0, 1, 1) &= \ell_3 \cap \ell_7 = a_4 \cap b_2 \\
P_{85} = P_{329} = (0, 0, 4, 1) &= \ell_3 \cap \ell_{10} = a_4 \cap b_5 \\
P_2 = P_2 = (0, 0, 1, 0) &= \ell_3 \cap \ell_{11} = a_4 \cap b_6
\end{aligned}
$$

$$P_{94} = P_{393} = (0,0,5,1) = \ell_3 \cap \ell_{14} = a_4 \cap c_{14}$$
$$P_{112} = P_{521} = (0,0,7,1) = \ell_3 \cap \ell_{18} = a_4 \cap c_{24}$$
$$P_{70} = P_{265} = (0,0,3,1) = \ell_3 \cap \ell_{24} = a_4 \cap c_{45}$$
$$P_{103} = P_{457} = (0,0,6,1) = \ell_3 \cap \ell_{25} = a_4 \cap c_{46}$$
$$P_1 = P_1 = (0,1,0,0) = \ell_4 \cap \ell_6 = a_5 \cap b_1$$
$$P_5 = P_5 = (1,1,0,0) = \ell_4 \cap \ell_7 = a_5 \cap b_2$$
$$P_7 = P_7 = (3,1,0,0) = \ell_4 \cap \ell_8 = a_5 \cap b_3$$
$$P_6 = P_6 = (2,1,0,0) = \ell_4 \cap \ell_9 = a_5 \cap b_4$$
$$P_{11} = P_{11} = (7,1,0,0) = \ell_4 \cap \ell_{15} = a_5 \cap c_{15}$$
$$P_{10} = P_{10} = (6,1,0,0) = \ell_4 \cap \ell_{19} = a_5 \cap c_{25}$$
$$P_8 = P_8 = (4,1,0,0) = \ell_4 \cap \ell_{22} = a_5 \cap c_{35}$$
$$P_9 = P_9 = (5,1,0,0) = \ell_4 \cap \ell_{24} = a_5 \cap c_{45}$$
$$P_{34} = P_{106} = (0,4,0,1) = \ell_5 \cap \ell_6 = a_6 \cap b_1$$
$$P_{50} = P_{182} = (5,5,1,1) = \ell_5 \cap \ell_7 = a_6 \cap b_2$$
$$P_{67} = P_{256} = (7,6,2,1) = \ell_5 \cap \ell_8 = a_6 \cap b_3$$
$$P_{84} = P_{323} = (2,7,3,1) = \ell_5 \cap \ell_9 = a_6 \cap b_4$$
$$P_{117} = P_{546} = (1,3,7,1) = \ell_5 \cap \ell_{16} = a_6 \cap c_{16}$$
$$P_{106} = P_{477} = (4,2,6,1) = \ell_5 \cap \ell_{20} = a_6 \cap c_{26}$$
$$P_{97} = P_{407} = (6,1,5,1) = \ell_5 \cap \ell_{23} = a_6 \cap c_{36}$$
$$P_{86} = P_{332} = (3,0,4,1) = \ell_5 \cap \ell_{25} = a_6 \cap c_{46}$$
$$P_{39} = P_{130} = (0,7,0,1) = \ell_6 \cap \ell_{13} = b_1 \cap c_{13}$$
$$P_{36} = P_{114} = (0,5,0,1) = \ell_6 \cap \ell_{14} = b_1 \cap c_{14}$$
$$P_{38} = P_{122} = (0,6,0,1) = \ell_6 \cap \ell_{15} = b_1 \cap c_{15}$$
$$P_{32} = P_{98} = (0,3,0,1) = \ell_6 \cap \ell_{16} = b_1 \cap c_{16}$$
$$P_{49} = P_{173} = (4,4,1,1) = \ell_7 \cap \ell_{17} = b_2 \cap c_{23}$$
$$P_{53} = P_{191} = (6,6,1,1) = \ell_7 \cap \ell_{18} = b_2 \cap c_{24}$$
$$P_{54} = P_{200} = (7,7,1,1) = \ell_7 \cap \ell_{19} = b_2 \cap c_{25}$$
$$P_{44} = P_{155} = (2,2,1,1) = \ell_7 \cap \ell_{20} = b_2 \cap c_{26}$$
$$P_{64} = P_{243} = (2,5,2,1) = \ell_8 \cap \ell_{13} = b_3 \cap c_{13}$$
$$P_{69} = P_{261} = (4,7,2,1) = \ell_8 \cap \ell_{17} = b_3 \cap c_{23}$$
$$P_{63} = P_{234} = (1,4,2,1) = \ell_8 \cap \ell_{22} = b_3 \cap c_{35}$$
$$P_{58} = P_{212} = (3,1,2,1) = \ell_8 \cap \ell_{23} = b_3 \cap c_{36}$$
$$P_{81} = P_{313} = (0,6,3,1) = \ell_9 \cap \ell_{14} = b_4 \cap c_{14}$$
$$P_{79} = P_{301} = (4,4,3,1) = \ell_9 \cap \ell_{18} = b_4 \cap c_{24}$$
$$P_{80} = P_{311} = (6,5,3,1) = \ell_9 \cap \ell_{24} = b_4 \cap c_{45}$$
$$P_{71} = P_{266} = (1,0,3,1) = \ell_9 \cap \ell_{25} = b_4 \cap c_{46}$$
$$P_{35} = P_{109} = (3,4,0,1) = \ell_{10} \cap \ell_{15} = b_5 \cap c_{15}$$
$$P_{51} = P_{183} = (6,5,1,1) = \ell_{10} \cap \ell_{19} = b_5 \cap c_{25}$$
$$P_{66} = P_{253} = (4,6,2,1) = \ell_{10} \cap \ell_{22} = b_5 \cap c_{35}$$
$$P_{83} = P_{322} = (1,7,3,1) = \ell_{10} \cap \ell_{24} = b_5 \cap c_{45}$$
$$P_{16} = P_{16} = (5,0,1,0) = \ell_{11} \cap \ell_{16} = b_6 \cap c_{16}$$
$$P_{15} = P_{15} = (4,0,1,0) = \ell_{11} \cap \ell_{20} = b_6 \cap c_{26}$$
$$P_{17} = P_{17} = (6,0,1,0) = \ell_{11} \cap \ell_{23} = b_6 \cap c_{36}$$
$$P_{18} = P_{18} = (7,0,1,0) = \ell_{11} \cap \ell_{25} = b_6 \cap c_{46}$$
$$P_{52} = P_{186} = (1,6,1,1) = \ell_{13} \cap \ell_{19} = c_{13} \cap c_{25}$$
$$P_{99} = P_{414} = (5,2,5,1) = \ell_{13} \cap \ell_{20} = c_{13} \cap c_{26}$$

$$P_{78} = P_{300} = (3,4,3,1) = \ell_{13} \cap \ell_{24} = c_{13} \cap c_{45}$$
$$P_{113} = P_{528} = (7,0,7,1) = \ell_{13} \cap \ell_{25} = c_{13} \cap c_{46}$$
$$P_{48} = P_{169} = (0,4,1,1) = \ell_{14} \cap \ell_{19} = c_{14} \cap c_{25}$$
$$P_{115} = P_{537} = (0,2,7,1) = \ell_{14} \cap \ell_{20} = c_{14} \cap c_{26}$$
$$P_{68} = P_{257} = (0,7,2,1) = \ell_{14} \cap \ell_{22} = c_{14} \cap c_{35}$$
$$P_{87} = P_{337} = (0,1,4,1) = \ell_{14} \cap \ell_{23} = c_{14} \cap c_{36}$$
$$P_{37} = P_{118} = (4,5,0,1) = \ell_{15} \cap \ell_{17} = c_{15} \cap c_{23}$$
$$P_{40} = P_{137} = (7,7,0,1) = \ell_{15} \cap \ell_{18} = c_{15} \cap c_{24}$$
$$P_{29} = P_{84} = (2,1,0,1) = \ell_{15} \cap \ell_{23} = c_{15} \cap c_{36}$$
$$P_{27} = P_{79} = (5,0,0,1) = \ell_{15} \cap \ell_{25} = c_{15} \cap c_{46}$$
$$P_{109} = P_{485} = (4,3,6,1) = \ell_{16} \cap \ell_{17} = c_{16} \cap c_{23}$$
$$P_{90} = P_{356} = (3,3,4,1) = \ell_{16} \cap \ell_{18} = c_{16} \cap c_{24}$$
$$P_{62} = P_{232} = (7,3,2,1) = \ell_{16} \cap \ell_{22} = c_{16} \cap c_{35}$$
$$P_{76} = P_{291} = (2,3,3,1) = \ell_{16} \cap \ell_{24} = c_{16} \cap c_{45}$$
$$P_{82} = P_{317} = (4,6,3,1) = \ell_{17} \cap \ell_{24} = c_{23} \cap c_{45}$$
$$P_{95} = P_{397} = (4,0,5,1) = \ell_{17} \cap \ell_{25} = c_{23} \cap c_{46}$$
$$P_{65} = P_{246} = (5,5,2,1) = \ell_{18} \cap \ell_{22} = c_{24} \cap c_{35}$$
$$P_{104} = P_{466} = (1,1,6,1) = \ell_{18} \cap \ell_{23} = c_{24} \cap c_{36}$$
$$P_{43} = P_{149} = (4,1,1,1) = \ell_{19} \cap \ell_{23} = c_{25} \cap c_{36}$$
$$P_{42} = P_{140} = (2,0,1,1) = \ell_{19} \cap \ell_{25} = c_{25} \cap c_{46}$$
$$P_{59} = P_{220} = (3,2,2,1) = \ell_{20} \cap \ell_{22} = c_{26} \cap c_{35}$$
$$P_{75} = P_{288} = (7,2,3,1) = \ell_{20} \cap \ell_{24} = c_{26} \cap c_{45}$$

Points on lines

Line 0 = a_1 has 9 points: $\{P_i \mid i \in \{13,33,46,61,77,91,100,108,118\}\}$
Line 1 = a_2 has 9 points: $\{P_i \mid i \in \{14,30,45,60,74,89,98,107,116\}\}$
Line 2 = a_3 has 9 points: $\{P_i \mid i \in \{4,12,28,57,72,88,96,105,114\}\}$
Line 3 = a_4 has 9 points: $\{P_i \mid i \in \{2,3,41,55,70,85,94,103,112\}\}$
Line 4 = a_5 has 9 points: $\{P_i \mid i \in \{0,1,5,6,7,8,9,10,11\}\}$
Line 5 = a_6 has 9 points: $\{P_i \mid i \in \{24,34,50,67,84,86,97,106,117\}\}$
Line 6 = b_1 has 9 points: $\{P_i \mid i \in \{1,3,28,30,32,34,36,38,39\}\}$
Line 7 = b_2 has 9 points: $\{P_i \mid i \in \{4,5,41,44,46,49,50,53,54\}\}$
Line 8 = b_3 has 9 points: $\{P_i \mid i \in \{7,55,58,60,61,63,64,67,69\}\}$
Line 9 = b_4 has 9 points: $\{P_i \mid i \in \{6,71,72,74,77,79,80,81,84\}\}$
Line 10 = b_5 has 9 points: $\{P_i \mid i \in \{24,35,51,66,83,85,96,107,118\}\}$
Line 11 = b_6 has 9 points: $\{P_i \mid i \in \{0,2,12,13,14,15,16,17,18\}\}$
Line 12 = c_{12} has 9 points: $\{P_i \mid i \in \{22,30,46,56,73,92,101,111,120\}\}$
Line 13 = c_{13} has 9 points: $\{P_i \mid i \in \{20,39,52,64,78,91,99,105,113\}\}$
Line 14 = c_{14} has 9 points: $\{P_i \mid i \in \{19,36,48,68,81,87,94,108,115\}\}$
Line 15 = c_{15} has 9 points: $\{P_i \mid i \in \{11,27,29,31,33,35,37,38,40\}\}$
Line 16 = c_{16} has 9 points: $\{P_i \mid i \in \{16,32,47,62,76,90,100,109,117\}\}$
Line 17 = c_{23} has 9 points: $\{P_i \mid i \in \{19,37,49,69,82,88,95,109,116\}\}$
Line 18 = c_{24} has 9 points: $\{P_i \mid i \in \{20,40,53,65,79,90,98,104,112\}\}$
Line 19 = c_{25} has 9 points: $\{P_i \mid i \in \{10,42,43,45,47,48,51,52,54\}\}$
Line 20 = c_{26} has 9 points: $\{P_i \mid i \in \{15,31,44,59,75,89,99,106,115\}\}$
Line 21 = c_{34} has 9 points: $\{P_i \mid i \in \{22,31,47,55,72,93,102,110,119\}\}$
Line 22 = c_{35} has 9 points: $\{P_i \mid i \in \{8,56,57,59,62,63,65,66,68\}\}$
Line 23 = c_{36} has 9 points: $\{P_i \mid i \in \{17,29,43,58,73,87,97,104,114\}\}$
Line 24 = c_{45} has 9 points: $\{P_i \mid i \in \{9,70,73,75,76,78,80,82,83\}\}$
Line 25 = c_{46} has 9 points: $\{P_i \mid i \in \{18,27,42,56,71,86,95,103,113\}\}$
Line 26 = c_{56} has 9 points: $\{P_i \mid i \in \{0,19,20,21,22,23,24,25,26\}\}$

Points on surface but on no line

The surface has 0 points not on any line:

	0	1	2	3	4	5	6	7	8	9

The points on the surface but not on lines are:

Tritangent planes

The 45 tritangent planes are:

$$\pi_{12} = \pi_{0} = 493 = \begin{bmatrix} 1 & 0 & 0 & \alpha^{6} \\ 0 & 1 & 0 & \alpha^{6} \\ 0 & 0 & 1 & 1 \end{bmatrix} = V\big(\alpha^{6}X_0 + \alpha^{6}X_1 + X_2 + X_3\big)$$

dual pt rank $= 191 = (6, 6, 1, 1)$.

$$\pi_{21} = \pi_{1} = 300 = \begin{bmatrix} 1 & 0 & \alpha^{2} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(\alpha^{2}X_0 + X_2\big)$$

dual pt rank $= 15 = (4, 0, 1, 0)$.

$$\pi_{13} = \pi_{2} = 270 = \begin{bmatrix} 1 & 0 & 0 & \alpha^{5} \\ 0 & 1 & 0 & \alpha^{3} \\ 0 & 0 & 1 & \alpha^{6} \end{bmatrix} = V\big(\alpha^{5}X_0 + \alpha^{3}X_1 + \alpha^{6}X_2 + X_3\big)$$

dual pt rank $= 500 = (3, 5, 6, 1)$.

$$\pi_{31} = \pi_{3} = 81 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(X_0 + X_2\big)$$

dual pt rank $= 12 = (1, 0, 1, 0)$.

$$\pi_{14} = \pi_{4} = 435 = \begin{bmatrix} 1 & 0 & 0 & \alpha^{3} \\ 0 & 1 & 0 & \alpha^{4} \\ 0 & 0 & 1 & \alpha^{4} \end{bmatrix} = V\big(\alpha^{3}X_0 + \alpha^{4}X_1 + \alpha^{4}X_2 + X_3\big)$$

dual pt rank $= 582 = (5, 7, 7, 1)$.

$$\pi_{41} = \pi_{5} = 584 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(X_0\big)$$

dual pt rank $= 0 = (1, 0, 0, 0)$.

$$\pi_{15} = \pi_{6} = 532 = \begin{bmatrix} 1 & 0 & 0 & \alpha^{4} \\ 0 & 1 & 0 & \alpha \\ 0 & 0 & 1 & \alpha^{5} \end{bmatrix} = V\big(\alpha^{4}X_0 + \alpha X_1 + \alpha^{5}X_2 + X_3\big)$$

dual pt rank $= 288 = (7, 2, 3, 1)$.

$$\pi_{51} = \pi_{7} = 8 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(X_2\big)$$

dual pt rank $= 2 = (0, 0, 1, 0)$.

$$\pi_{16} = \pi_{8} = 36 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \alpha^{2} \\ 0 & 0 & 1 & 0 \end{bmatrix} = V\big(\alpha^{2}X_1 + X_3\big)$$

dual pt rank $= 106 = (0,4,0,1)$.

$$\pi_{61} = \pi_9 = 519 = \begin{bmatrix} 1 & 0 & \alpha^4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(\alpha^4 X_0 + X_2\big)$$

dual pt rank $= 18 = (7,0,1,0)$.

$$\pi_{23} = \pi_{10} = 206 = \begin{bmatrix} 1 & 0 & 0 & \alpha \\ 0 & 1 & 0 & \alpha^6 \\ 0 & 0 & 1 & \alpha^6 \end{bmatrix} = V\big(\alpha X_0 + \alpha^6 X_1 + \alpha^6 X_2 + X_3\big)$$

dual pt rank $= 507 = (2,6,6,1)$.

$$\pi_{32} = \pi_{11} = 83 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = V\big(X_0 + X_1 + X_2 + X_3\big)$$

dual pt rank $= 4 = (1,1,1,1)$.

$$\pi_{24} = \pi_{12} = 278 = \begin{bmatrix} 1 & 0 & 0 & \alpha^5 \\ 0 & 1 & 0 & \alpha^6 \\ 0 & 0 & 1 & \alpha^3 \end{bmatrix} = V\big(\alpha^5 X_0 + \alpha^6 X_1 + \alpha^3 X_2 + X_3\big)$$

dual pt rank $= 444 = (3,6,5,1)$.

$$\pi_{42} = \pi_{13} = 145 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(X_0 + X_1\big)$$

dual pt rank $= 5 = (1,1,0,0)$.

$$\pi_{25} = \pi_{14} = 130 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & \alpha^6 \\ 0 & 0 & 1 & \alpha^5 \end{bmatrix} = V\big(X_0 + \alpha^6 X_1 + \alpha^5 X_2 + X_3\big)$$

dual pt rank $= 314 = (1,6,3,1)$.

$$\pi_{52} = \pi_{15} = 1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} = V\big(X_2 + X_3\big)$$

dual pt rank $= 138 = (0,0,1,1)$.

$$\pi_{26} = \pi_{16} = 54 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \alpha^6 \\ 0 & 0 & 1 & 0 \end{bmatrix} = V\big(\alpha^6 X_1 + X_3\big)$$

dual pt rank $= 122 = (0,6,0,1)$.

$$\pi_{62} = \pi_{17} = 247 = \begin{bmatrix} 1 & 0 & 0 & \alpha^5 \\ 0 & 1 & 0 & \alpha^5 \\ 0 & 0 & 1 & 1 \end{bmatrix} = V\big(\alpha^5 X_0 + \alpha^5 X_1 + X_2 + X_3\big)$$

dual pt rank $= 164 = (3,3,1,1)$.

$$\pi_{34} = \pi_{18} = 453 = \begin{bmatrix} 1 & 0 & 0 & \alpha^6 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & \alpha^6 \end{bmatrix} = V\big(\alpha^6 X_0 + X_1 + \alpha^6 X_2 + X_3\big)$$

dual pt rank $= 471 = (6,1,6,1)$.

$$\pi_{43} = \pi_{19} = 364 = \begin{bmatrix} 1 & \alpha^2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(\alpha^2 X_0 + X_1\big)$$

dual pt rank $= 8 = (4,1,0,0)$.

$$\pi_{35} = \pi_{20} = 231 = \begin{bmatrix} 1 & 0 & 0 & \alpha^5 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & \alpha^5 \end{bmatrix} = V\big(\alpha^5 X_0 + X_1 + \alpha^5 X_2 + X_3\big)$$

dual pt rank $= 276 = (3,1,3,1)$.

$$\pi_{53} = \pi_{21} = 6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \alpha^6 \end{bmatrix} = V\big(\alpha^6 X_2 + X_3\big)$$

dual pt rank $= 457 = (0,0,6,1)$.

$$\pi_{36} = \pi_{22} = 9 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = V\big(X_1 + X_3\big)$$

dual pt rank $= 82 = (0,1,0,1)$.

$$\pi_{63} = \pi_{23} = 106 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & \alpha^5 \\ 0 & 0 & 1 & \alpha^6 \end{bmatrix} = V\big(X_0 + \alpha^5 X_1 + \alpha^6 X_2 + X_3\big)$$

dual pt rank $= 482 = (1,3,6,1)$.

$$\pi_{45} = \pi_{24} = 583 = \begin{bmatrix} 1 & \alpha^4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(\alpha^4 X_0 + X_1\big)$$

dual pt rank $= 11 = (7,1,0,0)$.

$$\pi_{54} = \pi_{25} = 4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \alpha^2 \end{bmatrix} = V\big(\alpha^2 X_2 + X_3\big)$$

dual pt rank $= 329 = (0,0,4,1)$.

$$\pi_{46} = \pi_{26} = 72 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(X_1\big)$$

dual pt rank $= 1 = (0,1,0,0)$.

$$\pi_{64} = \pi_{27} = 540 = \begin{bmatrix} 1 & 0 & 0 & \alpha^4 \\ 0 & 1 & 0 & \alpha^5 \\ 0 & 0 & 1 & \alpha \end{bmatrix} = V\big(\alpha^4 X_0 + \alpha^5 X_1 + \alpha X_2 + X_3\big)$$

dual pt rank $= 232 = (7,3,2,1)$.

$$\pi_{56} = \pi_{28} = 0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = V\big(X_3\big)$$

dual pt rank $= 3 = (0,0,0,1)$.

$$\pi_{65} = \pi_{29} = 30 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \alpha^5 \\ 0 & 0 & 1 & \alpha^5 \end{bmatrix} = V\big(\alpha^5 X_1 + \alpha^5 X_2 + X_3\big)$$

dual pt rank $= 289 = (0,3,3,1)$.

$$\pi_{12,34,56} = \pi_{30} = 60 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \alpha^6 \\ 0 & 0 & 1 & \alpha^6 \end{bmatrix} = V\big(\alpha^6 X_1 + \alpha^6 X_2 + X_3\big)$$

dual pt rank $= 505 = (0,6,6,1)$.

$$\pi_{12,35,46} = \pi_{31} = 567 = \begin{bmatrix} 1 & 0 & 0 & \alpha^4 \\ 0 & 1 & 0 & \alpha^6 \\ 0 & 0 & 1 & \alpha \end{bmatrix} = V\left(\alpha^4 X_0 + \alpha^6 X_1 + \alpha X_2 + X_3\right)$$

dual pt rank $= 256 = (7,6,2,1)$.

$$\pi_{12,36,45} = \pi_{32} = 423 = \begin{bmatrix} 1 & 0 & 0 & \alpha^3 \\ 0 & 1 & 0 & \alpha^6 \\ 0 & 0 & 1 & \alpha^2 \end{bmatrix} = V\left(\alpha^3 X_0 + \alpha^6 X_1 + \alpha^2 X_2 + X_3\right)$$

dual pt rank $= 382 = (5,6,4,1)$.

$$\pi_{13,24,56} = \pi_{33} = 50 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \alpha^3 \\ 0 & 0 & 1 & \alpha^3 \end{bmatrix} = V\left(\alpha^3 X_1 + \alpha^3 X_2 + X_3\right)$$

dual pt rank $= 433 = (0,5,5,1)$.

$$\pi_{13,25,46} = \pi_{34} = 558 = \begin{bmatrix} 1 & 0 & 0 & \alpha^4 \\ 0 & 1 & 0 & \alpha^3 \\ 0 & 0 & 1 & \alpha \end{bmatrix} = V\left(\alpha^4 X_0 + \alpha^3 X_1 + \alpha X_2 + X_3\right)$$

dual pt rank $= 248 = (7,5,2,1)$.

$$\pi_{13,26,45} = \pi_{35} = 122 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & \alpha^3 \\ 0 & 0 & 1 & \alpha^2 \end{bmatrix} = V\left(X_0 + \alpha^3 X_1 + \alpha^2 X_2 + X_3\right)$$

dual pt rank $= 370 = (1,5,4,1)$.

$$\pi_{14,23,56} = \pi_{36} = 70 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \alpha^4 \\ 0 & 0 & 1 & \alpha^4 \end{bmatrix} = V\left(\alpha^4 X_1 + \alpha^4 X_2 + X_3\right)$$

dual pt rank $= 577 = (0,7,7,1)$.

$$\pi_{14,25,36} = \pi_{37} = 289 = \begin{bmatrix} 1 & 0 & 0 & \alpha^5 \\ 0 & 1 & 0 & \alpha^4 \\ 0 & 0 & 1 & \alpha^4 \end{bmatrix} = V\left(\alpha^5 X_0 + \alpha^4 X_1 + \alpha^4 X_2 + X_3\right)$$

dual pt rank $= 580 = (3,7,7,1)$.

$$\pi_{14,26,35} = \pi_{38} = 362 = \begin{bmatrix} 1 & 0 & 0 & \alpha^2 \\ 0 & 1 & 0 & \alpha^4 \\ 0 & 0 & 1 & \alpha^4 \end{bmatrix} = V\left(\alpha^2 X_0 + \alpha^4 X_1 + \alpha^4 X_2 + X_3\right)$$

dual pt rank $= 581 = (4,7,7,1)$.

$$\pi_{15,23,46} = \pi_{39} = 531 = \begin{bmatrix} 1 & 0 & 0 & \alpha^4 \\ 0 & 1 & 0 & \alpha \\ 0 & 0 & 1 & \alpha \end{bmatrix} = V\left(\alpha^4 X_0 + \alpha X_1 + \alpha X_2 + X_3\right)$$

dual pt rank $= 224 = (7,2,2,1)$.

$$\pi_{15,24,36} = \pi_{40} = 534 = \begin{bmatrix} 1 & 0 & 0 & \alpha^4 \\ 0 & 1 & 0 & \alpha \\ 0 & 0 & 1 & \alpha^3 \end{bmatrix} = V\left(\alpha^4 X_0 + \alpha X_1 + \alpha^3 X_2 + X_3\right)$$

dual pt rank $= 416 = (7,2,5,1)$.

$$\pi_{15,26,34} = \pi_{41} = 535 = \begin{bmatrix} 1 & 0 & 0 & \alpha^4 \\ 0 & 1 & 0 & \alpha \\ 0 & 0 & 1 & \alpha^6 \end{bmatrix} = V\left(\alpha^4 X_0 + \alpha X_1 + \alpha^6 X_2 + X_3\right)$$

dual pt rank = 480 = (7, 2, 6, 1).

$$\pi_{16,23,45} = \pi_{42} = 478 = \begin{bmatrix} 1 & 0 & 0 & \alpha^6 \\ 0 & 1 & 0 & \alpha^2 \\ 0 & 0 & 1 & \alpha^2 \end{bmatrix} = V\big(\alpha^6 X_0 + \alpha^2 X_1 + \alpha^2 X_2 + X_3\big)$$

dual pt rank = 367 = (6, 4, 4, 1).

$$\pi_{16,24,35} = \pi_{43} = 114 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & \alpha^2 \\ 0 & 0 & 1 & \alpha^3 \end{bmatrix} = V\big(X_0 + \alpha^2 X_1 + \alpha^3 X_2 + X_3\big)$$

dual pt rank = 426 = (1, 4, 5, 1).

$$\pi_{16,25,34} = \pi_{44} = 407 = \begin{bmatrix} 1 & 0 & 0 & \alpha^3 \\ 0 & 1 & 0 & \alpha^2 \\ 0 & 0 & 1 & \alpha^6 \end{bmatrix} = V\big(\alpha^3 X_0 + \alpha^2 X_1 + \alpha^6 X_2 + X_3\big)$$

dual pt rank = 494 = (5, 4, 6, 1).

The Generalized Quadrangle

The lines in the tritangent planes are:
$\pi_0 = \pi_{12} = \{\ell_i \mid i = 0, 7, 12\} = \{a_1, b_2, c_{12}\}$
$\pi_1 = \pi_{21} = \{\ell_i \mid i = 1, 6, 12\} = \{a_2, b_1, c_{12}\}$
$\pi_2 = \pi_{13} = \{\ell_i \mid i = 0, 8, 13\} = \{a_1, b_3, c_{13}\}$
$\pi_3 = \pi_{31} = \{\ell_i \mid i = 2, 6, 13\} = \{a_3, b_1, c_{13}\}$
$\pi_4 = \pi_{14} = \{\ell_i \mid i = 0, 9, 14\} = \{a_1, b_4, c_{14}\}$
$\pi_5 = \pi_{41} = \{\ell_i \mid i = 3, 6, 14\} = \{a_4, b_1, c_{14}\}$
$\pi_6 = \pi_{15} = \{\ell_i \mid i = 0, 10, 15\} = \{a_1, b_5, c_{15}\}$
$\pi_7 = \pi_{51} = \{\ell_i \mid i = 4, 6, 15\} = \{a_5, b_1, c_{15}\}$
$\pi_8 = \pi_{16} = \{\ell_i \mid i = 0, 11, 16\} = \{a_1, b_6, c_{16}\}$
$\pi_9 = \pi_{61} = \{\ell_i \mid i = 5, 6, 16\} = \{a_6, b_1, c_{16}\}$
$\pi_{10} = \pi_{23} = \{\ell_i \mid i = 1, 8, 17\} = \{a_2, b_3, c_{23}\}$
$\pi_{11} = \pi_{32} = \{\ell_i \mid i = 2, 7, 17\} = \{a_3, b_2, c_{23}\}$
$\pi_{12} = \pi_{24} = \{\ell_i \mid i = 1, 9, 18\} = \{a_2, b_4, c_{24}\}$
$\pi_{13} = \pi_{42} = \{\ell_i \mid i = 3, 7, 18\} = \{a_4, b_2, c_{24}\}$
$\pi_{14} = \pi_{25} = \{\ell_i \mid i = 1, 10, 19\} = \{a_2, b_5, c_{25}\}$
$\pi_{15} = \pi_{52} = \{\ell_i \mid i = 4, 7, 19\} = \{a_5, b_2, c_{25}\}$
$\pi_{16} = \pi_{26} = \{\ell_i \mid i = 1, 11, 20\} = \{a_2, b_6, c_{26}\}$
$\pi_{17} = \pi_{62} = \{\ell_i \mid i = 5, 7, 20\} = \{a_6, b_2, c_{26}\}$
$\pi_{18} = \pi_{34} = \{\ell_i \mid i = 2, 9, 21\} = \{a_3, b_4, c_{34}\}$
$\pi_{19} = \pi_{43} = \{\ell_i \mid i = 3, 8, 21\} = \{a_4, b_3, c_{34}\}$
$\pi_{20} = \pi_{35} = \{\ell_i \mid i = 2, 10, 22\} = \{a_3, b_5, c_{35}\}$
$\pi_{21} = \pi_{53} = \{\ell_i \mid i = 4, 8, 22\} = \{a_5, b_3, c_{35}\}$
$\pi_{22} = \pi_{36} = \{\ell_i \mid i = 2, 11, 23\} = \{a_3, b_6, c_{36}\}$
$\pi_{23} = \pi_{63} = \{\ell_i \mid i = 5, 8, 23\} = \{a_6, b_3, c_{36}\}$
$\pi_{24} = \pi_{45} = \{\ell_i \mid i = 3, 10, 24\} = \{a_4, b_5, c_{45}\}$
$\pi_{25} = \pi_{54} = \{\ell_i \mid i = 4, 9, 24\} = \{a_5, b_4, c_{45}\}$
$\pi_{26} = \pi_{46} = \{\ell_i \mid i = 3, 11, 25\} = \{a_4, b_6, c_{46}\}$
$\pi_{27} = \pi_{64} = \{\ell_i \mid i = 5, 9, 25\} = \{a_6, b_4, c_{46}\}$
$\pi_{28} = \pi_{56} = \{\ell_i \mid i = 4, 11, 26\} = \{a_5, b_6, c_{56}\}$
$\pi_{29} = \pi_{65} = \{\ell_i \mid i = 5, 10, 26\} = \{a_6, b_5, c_{56}\}$
$\pi_{30} = \pi_{12,34,56} = \{\ell_i \mid i = 12, 21, 26\} = \{c_{12}, c_{34}, c_{56}\}$
$\pi_{31} = \pi_{12,35,46} = \{\ell_i \mid i = 12, 22, 25\} = \{c_{12}, c_{35}, c_{46}\}$
$\pi_{32} = \pi_{12,36,45} = \{\ell_i \mid i = 12, 23, 24\} = \{c_{12}, c_{36}, c_{45}\}$
$\pi_{33} = \pi_{13,24,56} = \{\ell_i \mid i = 13, 18, 26\} = \{c_{13}, c_{24}, c_{56}\}$
$\pi_{34} = \pi_{13,25,46} = \{\ell_i \mid i = 13, 19, 25\} = \{c_{13}, c_{25}, c_{46}\}$
$\pi_{35} = \pi_{13,26,45} = \{\ell_i \mid i = 13, 20, 24\} = \{c_{13}, c_{26}, c_{45}\}$
$\pi_{36} = \pi_{14,23,56} = \{\ell_i \mid i = 14, 17, 26\} = \{c_{14}, c_{23}, c_{56}\}$
$\pi_{37} = \pi_{14,25,36} = \{\ell_i \mid i = 14, 19, 23\} = \{c_{14}, c_{25}, c_{36}\}$

$\pi_{38} = \pi_{14,26,35} = \{\ell_i \mid i = 14, 20, 22\} = \{c_{14}, c_{26}, c_{35}\}$
$\pi_{39} = \pi_{15,23,46} = \{\ell_i \mid i = 15, 17, 25\} = \{c_{15}, c_{23}, c_{46}\}$
$\pi_{40} = \pi_{15,24,36} = \{\ell_i \mid i = 15, 18, 23\} = \{c_{15}, c_{24}, c_{36}\}$
$\pi_{41} = \pi_{15,26,34} = \{\ell_i \mid i = 15, 20, 21\} = \{c_{15}, c_{26}, c_{34}\}$
$\pi_{42} = \pi_{16,23,45} = \{\ell_i \mid i = 16, 17, 24\} = \{c_{16}, c_{23}, c_{45}\}$
$\pi_{43} = \pi_{16,24,35} = \{\ell_i \mid i = 16, 18, 22\} = \{c_{16}, c_{24}, c_{35}\}$
$\pi_{44} = \pi_{16,25,34} = \{\ell_i \mid i = 16, 19, 21\} = \{c_{16}, c_{25}, c_{34}\}$
The tritangent planes through the 27 lines are:
$a_1 = \ell_0 \in \{\pi_i \mid i = 0, 4, 8, 2, 6\} = \{\pi_{12}, \pi_{14}, \pi_{16}, \pi_{13}, \pi_{15}\}$
$a_2 = \ell_1 \in \{\pi_i \mid i = 16, 10, 1, 12, 14\} = \{\pi_{26}, \pi_{23}, \pi_{21}, \pi_{24}, \pi_{25}\}$
$a_3 = \ell_2 \in \{\pi_i \mid i = 20, 18, 11, 3, 22\} = \{\pi_{35}, \pi_{34}, \pi_{32}, \pi_{31}, \pi_{36}\}$
$a_4 = \ell_3 \in \{\pi_i \mid i = 19, 5, 24, 13, 26\} = \{\pi_{43}, \pi_{41}, \pi_{45}, \pi_{42}, \pi_{46}\}$
$a_5 = \ell_4 \in \{\pi_i \mid i = 21, 25, 7, 15, 28\} = \{\pi_{53}, \pi_{54}, \pi_{51}, \pi_{52}, \pi_{56}\}$
$a_6 = \ell_5 \in \{\pi_i \mid i = 17, 29, 23, 27, 9\} = \{\pi_{62}, \pi_{65}, \pi_{63}, \pi_{64}, \pi_{61}\}$
$b_1 = \ell_6 \in \{\pi_i \mid i = 3, 1, 5, 7, 9\} = \{\pi_{31}, \pi_{21}, \pi_{41}, \pi_{51}, \pi_{61}\}$
$b_2 = \ell_7 \in \{\pi_i \mid i = 17, 0, 11, 13, 15\} = \{\pi_{62}, \pi_{12}, \pi_{32}, \pi_{42}, \pi_{52}\}$
$b_3 = \ell_8 \in \{\pi_i \mid i = 21, 23, 10, 19, 2\} = \{\pi_{53}, \pi_{63}, \pi_{23}, \pi_{43}, \pi_{13}\}$
$b_4 = \ell_9 \in \{\pi_i \mid i = 18, 4, 25, 12, 27\} = \{\pi_{34}, \pi_{14}, \pi_{54}, \pi_{24}, \pi_{64}\}$
$b_5 = \ell_{10} \in \{\pi_i \mid i = 29, 20, 24, 6, 14\} = \{\pi_{65}, \pi_{35}, \pi_{45}, \pi_{15}, \pi_{25}\}$
$b_6 = \ell_{11} \in \{\pi_i \mid i = 16, 22, 8, 26, 28\} = \{\pi_{26}, \pi_{36}, \pi_{16}, \pi_{46}, \pi_{56}\}$
$c_{12} = \ell_{12} \in \{\pi_i \mid i = 0, 30, 32, 1, 31\} = \{\pi_{12}, \pi_{12,34,56}, \pi_{12,36,45}, \pi_{21}, \pi_{12,35,46}\}$
$c_{13} = \ell_{13} \in \{\pi_i \mid i = 35, 33, 3, 34, 2\} = \{\pi_{13,26,45}, \pi_{13,24,56}, \pi_{31}, \pi_{13,25,46}, \pi_{13}\}$
$c_{14} = \ell_{14} \in \{\pi_i \mid i = 4, 38, 5, 37, 36\} = \{\pi_{14}, \pi_{14,26,35}, \pi_{41}, \pi_{14,25,36}, \pi_{14,23,56}\}$
$c_{15} = \ell_{15} \in \{\pi_i \mid i = 7, 41, 40, 6, 39\} = \{\pi_{51}, \pi_{15,26,34}, \pi_{15,24,36}, \pi_{15}, \pi_{15,23,46}\}$
$c_{16} = \ell_{16} \in \{\pi_i \mid i = 42, 43, 44, 8, 9\} = \{\pi_{16,23,45}, \pi_{16,24,35}, \pi_{16,25,34}, \pi_{16}, \pi_{61}\}$
$c_{23} = \ell_{17} \in \{\pi_i \mid i = 42, 10, 11, 36, 39\} = \{\pi_{16,23,45}, \pi_{23}, \pi_{32}, \pi_{14,23,56}, \pi_{15,23,46}\}$
$c_{24} = \ell_{18} \in \{\pi_i \mid i = 43, 33, 13, 12, 40\} = \{\pi_{16,24,35}, \pi_{13,24,56}, \pi_{42}, \pi_{24}, \pi_{15,24,36}\}$
$c_{25} = \ell_{19} \in \{\pi_i \mid i = 44, 37, 34, 14, 15\} = \{\pi_{16,25,34}, \pi_{14,25,36}, \pi_{13,25,46}, \pi_{25}, \pi_{52}\}$
$c_{26} = \ell_{20} \in \{\pi_i \mid i = 17, 35, 16, 38, 41\} = \{\pi_{62}, \pi_{13,26,45}, \pi_{26}, \pi_{14,26,35}, \pi_{15,26,34}\}$
$c_{34} = \ell_{21} \in \{\pi_i \mid i = 30, 18, 44, 19, 41\} = \{\pi_{12,34,56}, \pi_{34}, \pi_{16,25,34}, \pi_{43}, \pi_{15,26,34}\}$
$c_{35} = \ell_{22} \in \{\pi_i \mid i = 20, 43, 21, 38, 31\} = \{\pi_{35}, \pi_{16,24,35}, \pi_{53}, \pi_{14,26,35}, \pi_{12,35,46}\}$
$c_{36} = \ell_{23} \in \{\pi_i \mid i = 23, 32, 22, 37, 40\} = \{\pi_{63}, \pi_{12,36,45}, \pi_{36}, \pi_{14,25,36}, \pi_{15,24,36}\}$
$c_{45} = \ell_{24} \in \{\pi_i \mid i = 35, 42, 32, 25, 24\} = \{\pi_{13,26,45}, \pi_{16,23,45}, \pi_{12,36,45}, \pi_{54}, \pi_{45}\}$
$c_{46} = \ell_{25} \in \{\pi_i \mid i = 26, 31, 34, 27, 39\} = \{\pi_{46}, \pi_{12,35,46}, \pi_{13,25,46}, \pi_{64}, \pi_{15,23,46}\}$
$c_{56} = \ell_{26} \in \{\pi_i \mid i = 29, 30, 33, 36, 28\} = \{\pi_{65}, \pi_{12,34,56}, \pi_{13,24,56}, \pi_{14,23,56}, \pi_{56}\}$