Cubic Surfaces with 27 Lines over GF(7)

Orbiter

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1 The field of order 7

The field $\mathbb{F}_7$:
$Z_i = \log_\alpha(1+\alpha^i)$

i	γ_i	$-\gamma_i$	γ_i^{-1}	$\log_\alpha(\gamma_i)$	α^i	Z_i
0	$0 = 0$	0	DNE	DNE	1	2
1	$1 = 1$	6	1	0	3	4
2	$2 = \alpha^2$	5	4	2	2	1
3	$3 = \alpha$	4	5	1	6	DNE
4	$4 = \alpha^4$	3	2	4	4	5
5	$5 = \alpha^5$	2	3	5	5	3
6	$6 = \alpha^3$	1	6	3	1	2

2 The groups

2.1 The semilinear group

Group action PGL$(4,7)$ of degree 400
Group order 4635182361600
tl=$(400, 399, 392, 343, 216)$
Base: $(0, 1, 2, 3, 4)$
Strong generators for a group of order 4635182361600:

$$\begin{bmatrix}1&0&0&0\\0&5&0&0\\0&0&5&0\\0&0&0&5\end{bmatrix}, \begin{bmatrix}1&0&0&0\\0&3&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&3&0\\0&0&0&1\end{bmatrix},$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\1&0&0&1\end{bmatrix}, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&1&0&1\end{bmatrix}, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1&1\end{bmatrix},$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}, \begin{bmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{bmatrix}, \begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

i	P_i
0	$(1,0,0,0)$
1	$(0,1,0,0)$
2	$(0,0,1,0)$
3	$(0,0,0,1)$
4	$(1,1,1,1)$
5	$(1,1,0,0)$
6	$(2,1,0,0)$
7	$(3,1,0,0)$
8	$(4,1,0,0)$
9	$(5,1,0,0)$

,

i	P_i
10	$(6,1,0,0)$
11	$(1,0,1,0)$
12	$(2,0,1,0)$
13	$(3,0,1,0)$
14	$(4,0,1,0)$
15	$(5,0,1,0)$
16	$(6,0,1,0)$
17	$(0,1,1,0)$
18	$(1,1,1,0)$
19	$(2,1,1,0)$

,

i	P_i
20	$(3,1,1,0)$
21	$(4,1,1,0)$
22	$(5,1,1,0)$
23	$(6,1,1,0)$
24	$(0,2,1,0)$
25	$(1,2,1,0)$
26	$(2,2,1,0)$
27	$(3,2,1,0)$
28	$(4,2,1,0)$
29	$(5,2,1,0)$

,

i	P_i
30	$(6,2,1,0)$
31	$(0,3,1,0)$
32	$(1,3,1,0)$
33	$(2,3,1,0)$
34	$(3,3,1,0)$
35	$(4,3,1,0)$
36	$(5,3,1,0)$
37	$(6,3,1,0)$
38	$(0,4,1,0)$
39	$(1,4,1,0)$

,

i	P_i
40	$(2,4,1,0)$
41	$(3,4,1,0)$
42	$(4,4,1,0)$
43	$(5,4,1,0)$
44	$(6,4,1,0)$
45	$(0,5,1,0)$
46	$(1,5,1,0)$
47	$(2,5,1,0)$
48	$(3,5,1,0)$
49	$(4,5,1,0)$

i	P_i
50	$(5,5,1,0)$
51	$(6,5,1,0)$
52	$(0,6,1,0)$
53	$(1,6,1,0)$
54	$(2,6,1,0)$
55	$(3,6,1,0)$
56	$(4,6,1,0)$
57	$(5,6,1,0)$
58	$(6,6,1,0)$
59	$(1,0,0,1)$

,

i	P_i
60	$(2,0,0,1)$
61	$(3,0,0,1)$
62	$(4,0,0,1)$
63	$(5,0,0,1)$
64	$(6,0,0,1)$
65	$(0,1,0,1)$
66	$(1,1,0,1)$
67	$(2,1,0,1)$
68	$(3,1,0,1)$
69	$(4,1,0,1)$

,

i	P_i
70	$(5,1,0,1)$
71	$(6,1,0,1)$
72	$(0,2,0,1)$
73	$(1,2,0,1)$
74	$(2,2,0,1)$
75	$(3,2,0,1)$
76	$(4,2,0,1)$
77	$(5,2,0,1)$
78	$(6,2,0,1)$
79	$(0,3,0,1)$

,

i	P_i
80	$(1,3,0,1)$
81	$(2,3,0,1)$
82	$(3,3,0,1)$
83	$(4,3,0,1)$
84	$(5,3,0,1)$
85	$(6,3,0,1)$
86	$(0,4,0,1)$
87	$(1,4,0,1)$
88	$(2,4,0,1)$
89	$(3,4,0,1)$

,

i	P_i
90	$(4,4,0,1)$
91	$(5,4,0,1)$
92	$(6,4,0,1)$
93	$(0,5,0,1)$
94	$(1,5,0,1)$
95	$(2,5,0,1)$
96	$(3,5,0,1)$
97	$(4,5,0,1)$
98	$(5,5,0,1)$
99	$(6,5,0,1)$

i	P_i
100	$(0,6,0,1)$
101	$(1,6,0,1)$
102	$(2,6,0,1)$
103	$(3,6,0,1)$
104	$(4,6,0,1)$
105	$(5,6,0,1)$
106	$(6,6,0,1)$
107	$(0,0,1,1)$
108	$(1,0,1,1)$
109	$(2,0,1,1)$

,

i	P_i
110	$(3,0,1,1)$
111	$(4,0,1,1)$
112	$(5,0,1,1)$
113	$(6,0,1,1)$
114	$(0,1,1,1)$
115	$(2,1,1,1)$
116	$(3,1,1,1)$
117	$(4,1,1,1)$
118	$(5,1,1,1)$
119	$(6,1,1,1)$

,

i	P_i
120	$(0,2,1,1)$
121	$(1,2,1,1)$
122	$(2,2,1,1)$
123	$(3,2,1,1)$
124	$(4,2,1,1)$
125	$(5,2,1,1)$
126	$(6,2,1,1)$
127	$(0,3,1,1)$
128	$(1,3,1,1)$
129	$(2,3,1,1)$

,

i	P_i
130	$(3,3,1,1)$
131	$(4,3,1,1)$
132	$(5,3,1,1)$
133	$(6,3,1,1)$
134	$(0,4,1,1)$
135	$(1,4,1,1)$
136	$(2,4,1,1)$
137	$(3,4,1,1)$
138	$(4,4,1,1)$
139	$(5,4,1,1)$

,

i	P_i
140	$(6,4,1,1)$
141	$(0,5,1,1)$
142	$(1,5,1,1)$
143	$(2,5,1,1)$
144	$(3,5,1,1)$
145	$(4,5,1,1)$
146	$(5,5,1,1)$
147	$(6,5,1,1)$
148	$(0,6,1,1)$
149	$(1,6,1,1)$

i	P_i	i	P_i	i	P_i	i	P_i	i	P_i
150	$(2,6,1,1)$	160	$(5,0,2,1)$	170	$(1,2,2,1)$	180	$(4,3,2,1)$	190	$(0,5,2,1)$
151	$(3,6,1,1)$	161	$(6,0,2,1)$	171	$(2,2,2,1)$	181	$(5,3,2,1)$	191	$(1,5,2,1)$
152	$(4,6,1,1)$	162	$(0,1,2,1)$	172	$(3,2,2,1)$	182	$(6,3,2,1)$	192	$(2,5,2,1)$
153	$(5,6,1,1)$	163	$(1,1,2,1)$	173	$(4,2,2,1)$	183	$(0,4,2,1)$	193	$(3,5,2,1)$
154	$(6,6,1,1)$	164	$(2,1,2,1)$	174	$(5,2,2,1)$	184	$(1,4,2,1)$	194	$(4,5,2,1)$
155	$(0,0,2,1)$	165	$(3,1,2,1)$	175	$(6,2,2,1)$	185	$(2,4,2,1)$	195	$(5,5,2,1)$
156	$(1,0,2,1)$	166	$(4,1,2,1)$	176	$(0,3,2,1)$	186	$(3,4,2,1)$	196	$(6,5,2,1)$
157	$(2,0,2,1)$	167	$(5,1,2,1)$	177	$(1,3,2,1)$	187	$(4,4,2,1)$	197	$(0,6,2,1)$
158	$(3,0,2,1)$	168	$(6,1,2,1)$	178	$(2,3,2,1)$	188	$(5,4,2,1)$	198	$(1,6,2,1)$
159	$(4,0,2,1)$	169	$(0,2,2,1)$	179	$(3,3,2,1)$	189	$(6,4,2,1)$	199	$(2,6,2,1)$

i	P_i	i	P_i	i	P_i	i	P_i	i	P_i
200	$(3,6,2,1)$	210	$(6,0,3,1)$	220	$(2,2,3,1)$	230	$(5,3,3,1)$	240	$(1,5,3,1)$
201	$(4,6,2,1)$	211	$(0,1,3,1)$	221	$(3,2,3,1)$	231	$(6,3,3,1)$	241	$(2,5,3,1)$
202	$(5,6,2,1)$	212	$(1,1,3,1)$	222	$(4,2,3,1)$	232	$(0,4,3,1)$	242	$(3,5,3,1)$
203	$(6,6,2,1)$	213	$(2,1,3,1)$	223	$(5,2,3,1)$	233	$(1,4,3,1)$	243	$(4,5,3,1)$
204	$(0,0,3,1)$	214	$(3,1,3,1)$	224	$(6,2,3,1)$	234	$(2,4,3,1)$	244	$(5,5,3,1)$
205	$(1,0,3,1)$	215	$(4,1,3,1)$	225	$(0,3,3,1)$	235	$(3,4,3,1)$	245	$(6,5,3,1)$
206	$(2,0,3,1)$	216	$(5,1,3,1)$	226	$(1,3,3,1)$	236	$(4,4,3,1)$	246	$(0,6,3,1)$
207	$(3,0,3,1)$	217	$(6,1,3,1)$	227	$(2,3,3,1)$	237	$(5,4,3,1)$	247	$(1,6,3,1)$
208	$(4,0,3,1)$	218	$(0,2,3,1)$	228	$(3,3,3,1)$	238	$(6,4,3,1)$	248	$(2,6,3,1)$
209	$(5,0,3,1)$	219	$(1,2,3,1)$	229	$(4,3,3,1)$	239	$(0,5,3,1)$	249	$(3,6,3,1)$

i	P_i	i	P_i	i	P_i	i	P_i	i	P_i
250	$(4,6,3,1)$	260	$(0,1,4,1)$	270	$(3,2,4,1)$	280	$(6,3,4,1)$	290	$(2,5,4,1)$
251	$(5,6,3,1)$	261	$(1,1,4,1)$	271	$(4,2,4,1)$	281	$(0,4,4,1)$	291	$(3,5,4,1)$
252	$(6,6,3,1)$	262	$(2,1,4,1)$	272	$(5,2,4,1)$	282	$(1,4,4,1)$	292	$(4,5,4,1)$
253	$(0,0,4,1)$	263	$(3,1,4,1)$	273	$(6,2,4,1)$	283	$(2,4,4,1)$	293	$(5,5,4,1)$
254	$(1,0,4,1)$	264	$(4,1,4,1)$	274	$(0,3,4,1)$	284	$(3,4,4,1)$	294	$(6,5,4,1)$
255	$(2,0,4,1)$	265	$(5,1,4,1)$	275	$(1,3,4,1)$	285	$(4,4,4,1)$	295	$(0,6,4,1)$
256	$(3,0,4,1)$	266	$(6,1,4,1)$	276	$(2,3,4,1)$	286	$(5,4,4,1)$	296	$(1,6,4,1)$
257	$(4,0,4,1)$	267	$(0,2,4,1)$	277	$(3,3,4,1)$	287	$(6,4,4,1)$	297	$(2,6,4,1)$
258	$(5,0,4,1)$	268	$(1,2,4,1)$	278	$(4,3,4,1)$	288	$(0,5,4,1)$	298	$(3,6,4,1)$
259	$(6,0,4,1)$	269	$(2,2,4,1)$	279	$(5,3,4,1)$	289	$(1,5,4,1)$	299	$(4,6,4,1)$

i	P_i	i	P_i	i	P_i	i	P_i	i	P_i
300	$(5,6,4,1)$	310	$(1,1,5,1)$	320	$(4,2,5,1)$	330	$(0,4,5,1)$	340	$(3,5,5,1)$
301	$(6,6,4,1)$	311	$(2,1,5,1)$	321	$(5,2,5,1)$	331	$(1,4,5,1)$	341	$(4,5,5,1)$
302	$(0,0,5,1)$	312	$(3,1,5,1)$	322	$(6,2,5,1)$	332	$(2,4,5,1)$	342	$(5,5,5,1)$
303	$(1,0,5,1)$	313	$(4,1,5,1)$	323	$(0,3,5,1)$	333	$(3,4,5,1)$	343	$(6,5,5,1)$
304	$(2,0,5,1)$	314	$(5,1,5,1)$	324	$(1,3,5,1)$	334	$(4,4,5,1)$	344	$(0,6,5,1)$
305	$(3,0,5,1)$	315	$(6,1,5,1)$	325	$(2,3,5,1)$	335	$(5,4,5,1)$	345	$(1,6,5,1)$
306	$(4,0,5,1)$	316	$(0,2,5,1)$	326	$(3,3,5,1)$	336	$(6,4,5,1)$	346	$(2,6,5,1)$
307	$(5,0,5,1)$	317	$(1,2,5,1)$	327	$(4,3,5,1)$	337	$(0,5,5,1)$	347	$(3,6,5,1)$
308	$(6,0,5,1)$	318	$(2,2,5,1)$	328	$(5,3,5,1)$	338	$(1,5,5,1)$	348	$(4,6,5,1)$
309	$(0,1,5,1)$	319	$(3,2,5,1)$	329	$(6,3,5,1)$	339	$(2,5,5,1)$	349	$(5,6,5,1)$

i	P_i	i	P_i	i	P_i	i	P_i	i	P_i
350	$(6,6,5,1)$	360	$(2,1,6,1)$	370	$(5,2,6,1)$	380	$(1,4,6,1)$	390	$(4,5,6,1)$
351	$(0,0,6,1)$	361	$(3,1,6,1)$	371	$(6,2,6,1)$	381	$(2,4,6,1)$	391	$(5,5,6,1)$
352	$(1,0,6,1)$	362	$(4,1,6,1)$	372	$(0,3,6,1)$	382	$(3,4,6,1)$	392	$(6,5,6,1)$
353	$(2,0,6,1)$	363	$(5,1,6,1)$	373	$(1,3,6,1)$	383	$(4,4,6,1)$	393	$(0,6,6,1)$
354	$(3,0,6,1)$	364	$(6,1,6,1)$	374	$(2,3,6,1)$	384	$(5,4,6,1)$	394	$(1,6,6,1)$
355	$(4,0,6,1)$	365	$(0,2,6,1)$	375	$(3,3,6,1)$	385	$(6,4,6,1)$	395	$(2,6,6,1)$
356	$(5,0,6,1)$	366	$(1,2,6,1)$	376	$(4,3,6,1)$	386	$(0,5,6,1)$	396	$(3,6,6,1)$
357	$(6,0,6,1)$	367	$(2,2,6,1)$	377	$(5,3,6,1)$	387	$(1,5,6,1)$	397	$(4,6,6,1)$
358	$(0,1,6,1)$	368	$(3,2,6,1)$	378	$(6,3,6,1)$	388	$(2,5,6,1)$	398	$(5,6,6,1)$
359	$(1,1,6,1)$	369	$(4,2,6,1)$	379	$(0,4,6,1)$	389	$(3,5,6,1)$	399	$(6,6,6,1)$

i	P_i

2.2 The orthogonal group

Group action PGL$(4,7)Wedge$ of degree 19608
Does not have strong generators.

2.3 The group stabilizing the fixed line

Group action PGL$(4,7)Wedgeres448$ of degree 448
Does not have strong generators.

i	P_i	i	P_i	i	P_i	i	P_i	i	P_i
0	$(0,1,0,0,0,0)$	10	$(1,0,1,0,0,0)$	20	$(4,1,1,0,0,0)$	30	$(0,3,1,0,0,0)$	40	$(3,4,1,0,0,0)$
1	$(0,0,1,0,0,0)$	11	$(2,0,1,0,0,0)$	21	$(5,1,1,0,0,0)$	31	$(1,3,1,0,0,0)$	41	$(4,4,1,0,0,0)$
2	$(0,0,0,1,0,0)$	12	$(3,0,1,0,0,0)$	22	$(6,1,1,0,0,0)$	32	$(2,3,1,0,0,0)$	42	$(5,4,1,0,0,0)$
3	$(0,0,0,0,1,0)$	13	$(4,0,1,0,0,0)$	23	$(0,2,1,0,0,0)$	33	$(3,3,1,0,0,0)$	43	$(6,4,1,0,0,0)$
4	$(1,1,0,0,0,0)$	14	$(5,0,1,0,0,0)$	24	$(1,2,1,0,0,0)$	34	$(4,3,1,0,0,0)$	44	$(0,5,1,0,0,0)$
5	$(2,1,0,0,0,0)$	15	$(6,0,1,0,0,0)$	25	$(2,2,1,0,0,0)$	35	$(5,3,1,0,0,0)$	45	$(1,5,1,0,0,0)$
6	$(3,1,0,0,0,0)$	16	$(0,1,1,0,0,0)$	26	$(3,2,1,0,0,0)$	36	$(6,3,1,0,0,0)$	46	$(2,5,1,0,0,0)$
7	$(4,1,0,0,0,0)$	17	$(1,1,1,0,0,0)$	27	$(4,2,1,0,0,0)$	37	$(0,4,1,0,0,0)$	47	$(3,5,1,0,0,0)$
8	$(5,1,0,0,0,0)$	18	$(2,1,1,0,0,0)$	28	$(5,2,1,0,0,0)$	38	$(1,4,1,0,0,0)$	48	$(4,5,1,0,0,0)$
9	$(6,1,0,0,0,0)$	19	$(3,1,1,0,0,0)$	29	$(6,2,1,0,0,0)$	39	$(2,4,1,0,0,0)$	49	$(5,5,1,0,0,0)$

i	P_i	i	P_i	i	P_i	i	P_i	i	P_i
50	$(6,5,1,0,0,0)$	60	$(3,0,0,1,0,0)$	70	$(6,1,0,1,0,0)$	80	$(2,3,0,1,0,0)$	90	$(5,4,0,1,0,0)$
51	$(0,6,1,0,0,0)$	61	$(4,0,0,1,0,0)$	71	$(0,2,0,1,0,0)$	81	$(3,3,0,1,0,0)$	91	$(6,4,0,1,0,0)$
52	$(1,6,1,0,0,0)$	62	$(5,0,0,1,0,0)$	72	$(1,2,0,1,0,0)$	82	$(4,3,0,1,0,0)$	92	$(0,5,0,1,0,0)$
53	$(2,6,1,0,0,0)$	63	$(6,0,0,1,0,0)$	73	$(2,2,0,1,0,0)$	83	$(5,3,0,1,0,0)$	93	$(1,5,0,1,0,0)$
54	$(3,6,1,0,0,0)$	64	$(0,1,0,1,0,0)$	74	$(3,2,0,1,0,0)$	84	$(6,3,0,1,0,0)$	94	$(2,5,0,1,0,0)$
55	$(4,6,1,0,0,0)$	65	$(1,1,0,1,0,0)$	75	$(4,2,0,1,0,0)$	85	$(0,4,0,1,0,0)$	95	$(3,5,0,1,0,0)$
56	$(5,6,1,0,0,0)$	66	$(2,1,0,1,0,0)$	76	$(5,2,0,1,0,0)$	86	$(1,4,0,1,0,0)$	96	$(4,5,0,1,0,0)$
57	$(6,6,1,0,0,0)$	67	$(3,1,0,1,0,0)$	77	$(6,2,0,1,0,0)$	87	$(2,4,0,1,0,0)$	97	$(5,5,0,1,0,0)$
58	$(1,0,0,1,0,0)$	68	$(4,1,0,1,0,0)$	78	$(0,3,0,1,0,0)$	88	$(3,4,0,1,0,0)$	98	$(6,5,0,1,0,0)$
59	$(2,0,0,1,0,0)$	69	$(5,1,0,1,0,0)$	79	$(1,3,0,1,0,0)$	89	$(4,4,0,1,0,0)$	99	$(0,6,0,1,0,0)$

i	P_i	i	P_i	i	P_i	i	P_i	i	P_i
100	$(1,6,0,1,0,0)$	110	$(5,0,0,0,1,0)$	120	$(1,0,2,0,1,0)$	130	$(4,0,3,0,1,0)$	140	$(0,0,5,0,1,0)$
101	$(2,6,0,1,0,0)$	111	$(6,0,0,0,1,0)$	121	$(2,0,2,0,1,0)$	131	$(5,0,3,0,1,0)$	141	$(1,0,5,0,1,0)$
102	$(3,6,0,1,0,0)$	112	$(0,0,1,0,1,0)$	122	$(3,0,2,0,1,0)$	132	$(6,0,3,0,1,0)$	142	$(2,0,5,0,1,0)$
103	$(4,6,0,1,0,0)$	113	$(1,0,1,0,1,0)$	123	$(4,0,2,0,1,0)$	133	$(0,0,4,0,1,0)$	143	$(3,0,5,0,1,0)$
104	$(5,6,0,1,0,0)$	114	$(2,0,1,0,1,0)$	124	$(5,0,2,0,1,0)$	134	$(1,0,4,0,1,0)$	144	$(4,0,5,0,1,0)$
105	$(6,6,0,1,0,0)$	115	$(3,0,1,0,1,0)$	125	$(6,0,2,0,1,0)$	135	$(2,0,4,0,1,0)$	145	$(5,0,5,0,1,0)$
106	$(1,0,0,0,1,0)$	116	$(4,0,1,0,1,0)$	126	$(0,0,3,0,1,0)$	136	$(3,0,4,0,1,0)$	146	$(6,0,5,0,1,0)$
107	$(2,0,0,0,1,0)$	117	$(5,0,1,0,1,0)$	127	$(1,0,3,0,1,0)$	137	$(4,0,4,0,1,0)$	147	$(0,0,6,0,1,0)$
108	$(3,0,0,0,1,0)$	118	$(6,0,1,0,1,0)$	128	$(2,0,3,0,1,0)$	138	$(5,0,4,0,1,0)$	148	$(1,0,6,0,1,0)$
109	$(4,0,0,0,1,0)$	119	$(0,0,2,0,1,0)$	129	$(3,0,3,0,1,0)$	139	$(6,0,4,0,1,0)$	149	$(2,0,6,0,1,0)$

i	P_i	i	P_i	i	P_i	i	P_i	i	P_i
150	$(3,0,6,0,1,0)$	160	$(6,0,0,1,1,0)$	170	$(2,2,2,1,1,0)$	180	$(5,3,3,1,1,0)$	190	$(1,5,5,1,1,0)$
151	$(4,0,6,0,1,0)$	161	$(0,1,1,1,1,0)$	171	$(3,2,2,1,1,0)$	181	$(6,3,3,1,1,0)$	191	$(2,5,5,1,1,0)$
152	$(5,0,6,0,1,0)$	162	$(1,1,1,1,1,0)$	172	$(4,2,2,1,1,0)$	182	$(0,4,4,1,1,0)$	192	$(3,5,5,1,1,0)$
153	$(6,0,6,0,1,0)$	163	$(2,1,1,1,1,0)$	173	$(5,2,2,1,1,0)$	183	$(1,4,4,1,1,0)$	193	$(4,5,5,1,1,0)$
154	$(0,0,0,1,1,0)$	164	$(3,1,1,1,1,0)$	174	$(6,2,2,1,1,0)$	184	$(2,4,4,1,1,0)$	194	$(5,5,5,1,1,0)$
155	$(1,0,0,1,1,0)$	165	$(4,1,1,1,1,0)$	175	$(0,3,3,1,1,0)$	185	$(3,4,4,1,1,0)$	195	$(6,5,5,1,1,0)$
156	$(2,0,0,1,1,0)$	166	$(5,1,1,1,1,0)$	176	$(1,3,3,1,1,0)$	186	$(4,4,4,1,1,0)$	196	$(0,6,6,1,1,0)$
157	$(3,0,0,1,1,0)$	167	$(6,1,1,1,1,0)$	177	$(2,3,3,1,1,0)$	187	$(5,4,4,1,1,0)$	197	$(1,6,6,1,1,0)$
158	$(4,0,0,1,1,0)$	168	$(0,2,2,1,1,0)$	178	$(3,3,3,1,1,0)$	188	$(6,4,4,1,1,0)$	198	$(2,6,6,1,1,0)$
159	$(5,0,0,1,1,0)$	169	$(1,2,2,1,1,0)$	179	$(4,3,3,1,1,0)$	189	$(0,5,5,1,1,0)$	199	$(3,6,6,1,1,0)$

i	P_i	i	P_i	i	P_i	i	P_i	i	P_i
200	$(4,6,6,1,1,0)$	210	$(0,2,1,2,1,0)$	220	$(3,4,2,2,1,0)$	230	$(6,6,3,2,1,0)$	240	$(2,3,5,2,1,0)$
201	$(5,6,6,1,1,0)$	211	$(1,2,1,2,1,0)$	221	$(4,4,2,2,1,0)$	231	$(0,1,4,2,1,0)$	241	$(3,3,5,2,1,0)$
202	$(6,6,6,1,1,0)$	212	$(2,2,1,2,1,0)$	222	$(5,4,2,2,1,0)$	232	$(1,1,4,2,1,0)$	242	$(4,3,5,2,1,0)$
203	$(0,0,0,2,1,0)$	213	$(3,2,1,2,1,0)$	223	$(6,4,2,2,1,0)$	233	$(2,1,4,2,1,0)$	243	$(5,3,5,2,1,0)$
204	$(1,0,0,2,1,0)$	214	$(4,2,1,2,1,0)$	224	$(0,6,3,2,1,0)$	234	$(3,1,4,2,1,0)$	244	$(6,3,5,2,1,0)$
205	$(2,0,0,2,1,0)$	215	$(5,2,1,2,1,0)$	225	$(1,6,3,2,1,0)$	235	$(4,1,4,2,1,0)$	245	$(0,5,6,2,1,0)$
206	$(3,0,0,2,1,0)$	216	$(6,2,1,2,1,0)$	226	$(2,6,3,2,1,0)$	236	$(5,1,4,2,1,0)$	246	$(1,5,6,2,1,0)$
207	$(4,0,0,2,1,0)$	217	$(0,4,2,2,1,0)$	227	$(3,6,3,2,1,0)$	237	$(6,1,4,2,1,0)$	247	$(2,5,6,2,1,0)$
208	$(5,0,0,2,1,0)$	218	$(1,4,2,2,1,0)$	228	$(4,6,3,2,1,0)$	238	$(0,3,5,2,1,0)$	248	$(3,5,6,2,1,0)$
209	$(6,0,0,2,1,0)$	219	$(2,4,2,2,1,0)$	229	$(5,6,3,2,1,0)$	239	$(1,3,5,2,1,0)$	249	$(4,5,6,2,1,0)$

i	P_i	i	P_i	i	P_i	i	P_i	i	P_i
250	$(5,5,6,2,1,0)$	260	$(1,3,1,3,1,0)$	270	$(4,6,2,3,1,0)$	280	$(0,5,4,3,1,0)$	290	$(3,1,5,3,1,0)$
251	$(6,5,6,2,1,0)$	261	$(2,3,1,3,1,0)$	271	$(5,6,2,3,1,0)$	281	$(1,5,4,3,1,0)$	291	$(4,1,5,3,1,0)$
252	$(0,0,0,3,1,0)$	262	$(3,3,1,3,1,0)$	272	$(6,6,2,3,1,0)$	282	$(2,5,4,3,1,0)$	292	$(5,1,5,3,1,0)$
253	$(1,0,0,3,1,0)$	263	$(4,3,1,3,1,0)$	273	$(0,2,3,3,1,0)$	283	$(3,5,4,3,1,0)$	293	$(6,1,5,3,1,0)$
254	$(2,0,0,3,1,0)$	264	$(5,3,1,3,1,0)$	274	$(1,2,3,3,1,0)$	284	$(4,5,4,3,1,0)$	294	$(0,4,6,3,1,0)$
255	$(3,0,0,3,1,0)$	265	$(6,3,1,3,1,0)$	275	$(2,2,3,3,1,0)$	285	$(5,5,4,3,1,0)$	295	$(1,4,6,3,1,0)$
256	$(4,0,0,3,1,0)$	266	$(0,6,2,3,1,0)$	276	$(3,2,3,3,1,0)$	286	$(6,5,4,3,1,0)$	296	$(2,4,6,3,1,0)$
257	$(5,0,0,3,1,0)$	267	$(1,6,2,3,1,0)$	277	$(4,2,3,3,1,0)$	287	$(0,1,5,3,1,0)$	297	$(3,4,6,3,1,0)$
258	$(6,0,0,3,1,0)$	268	$(2,6,2,3,1,0)$	278	$(5,2,3,3,1,0)$	288	$(1,1,5,3,1,0)$	298	$(4,4,6,3,1,0)$
259	$(0,3,1,3,1,0)$	269	$(3,6,2,3,1,0)$	279	$(6,2,3,3,1,0)$	289	$(2,1,5,3,1,0)$	299	$(5,4,6,3,1,0)$

i	P_i
300	$(6,4,6,3,1,0)$
301	$(0,0,0,4,1,0)$
302	$(1,0,0,4,1,0)$
303	$(2,0,0,4,1,0)$
304	$(3,0,0,4,1,0)$
305	$(4,0,0,4,1,0)$
306	$(5,0,0,4,1,0)$
307	$(6,0,0,4,1,0)$
308	$(0,4,1,4,1,0)$
309	$(1,4,1,4,1,0)$

,

i	P_i
310	$(2,4,1,4,1,0)$
311	$(3,4,1,4,1,0)$
312	$(4,4,1,4,1,0)$
313	$(5,4,1,4,1,0)$
314	$(6,4,1,4,1,0)$
315	$(0,1,2,4,1,0)$
316	$(1,1,2,4,1,0)$
317	$(2,1,2,4,1,0)$
318	$(3,1,2,4,1,0)$
319	$(4,1,2,4,1,0)$

,

i	P_i
320	$(5,1,2,4,1,0)$
321	$(6,1,2,4,1,0)$
322	$(0,5,3,4,1,0)$
323	$(1,5,3,4,1,0)$
324	$(2,5,3,4,1,0)$
325	$(3,5,3,4,1,0)$
326	$(4,5,3,4,1,0)$
327	$(5,5,3,4,1,0)$
328	$(6,5,3,4,1,0)$
329	$(0,2,4,4,1,0)$

,

i	P_i
330	$(1,2,4,4,1,0)$
331	$(2,2,4,4,1,0)$
332	$(3,2,4,4,1,0)$
333	$(4,2,4,4,1,0)$
334	$(5,2,4,4,1,0)$
335	$(6,2,4,4,1,0)$
336	$(0,6,5,4,1,0)$
337	$(1,6,5,4,1,0)$
338	$(2,6,5,4,1,0)$
339	$(3,6,5,4,1,0)$

,

i	P_i
340	$(4,6,5,4,1,0)$
341	$(5,6,5,4,1,0)$
342	$(6,6,5,4,1,0)$
343	$(0,3,6,4,1,0)$
344	$(1,3,6,4,1,0)$
345	$(2,3,6,4,1,0)$
346	$(3,3,6,4,1,0)$
347	$(4,3,6,4,1,0)$
348	$(5,3,6,4,1,0)$
349	$(6,3,6,4,1,0)$

i	P_i
350	$(0,0,0,5,1,0)$
351	$(1,0,0,5,1,0)$
352	$(2,0,0,5,1,0)$
353	$(3,0,0,5,1,0)$
354	$(4,0,0,5,1,0)$
355	$(5,0,0,5,1,0)$
356	$(6,0,0,5,1,0)$
357	$(0,5,1,5,1,0)$
358	$(1,5,1,5,1,0)$
359	$(2,5,1,5,1,0)$

,

i	P_i
360	$(3,5,1,5,1,0)$
361	$(4,5,1,5,1,0)$
362	$(5,5,1,5,1,0)$
363	$(6,5,1,5,1,0)$
364	$(0,3,2,5,1,0)$
365	$(1,3,2,5,1,0)$
366	$(2,3,2,5,1,0)$
367	$(3,3,2,5,1,0)$
368	$(4,3,2,5,1,0)$
369	$(5,3,2,5,1,0)$

,

i	P_i
370	$(6,3,2,5,1,0)$
371	$(0,1,3,5,1,0)$
372	$(1,1,3,5,1,0)$
373	$(2,1,3,5,1,0)$
374	$(3,1,3,5,1,0)$
375	$(4,1,3,5,1,0)$
376	$(5,1,3,5,1,0)$
377	$(6,1,3,5,1,0)$
378	$(0,6,4,5,1,0)$
379	$(1,6,4,5,1,0)$

,

i	P_i
380	$(2,6,4,5,1,0)$
381	$(3,6,4,5,1,0)$
382	$(4,6,4,5,1,0)$
383	$(5,6,4,5,1,0)$
384	$(6,6,4,5,1,0)$
385	$(0,4,5,5,1,0)$
386	$(1,4,5,5,1,0)$
387	$(2,4,5,5,1,0)$
388	$(3,4,5,5,1,0)$
389	$(4,4,5,5,1,0)$

,

i	P_i
390	$(5,4,5,5,1,0)$
391	$(6,4,5,5,1,0)$
392	$(0,2,6,5,1,0)$
393	$(1,2,6,5,1,0)$
394	$(2,2,6,5,1,0)$
395	$(3,2,6,5,1,0)$
396	$(4,2,6,5,1,0)$
397	$(5,2,6,5,1,0)$
398	$(6,2,6,5,1,0)$
399	$(0,0,0,6,1,0)$

i	P_i
400	$(1,0,0,6,1,0)$
401	$(2,0,0,6,1,0)$
402	$(3,0,0,6,1,0)$
403	$(4,0,0,6,1,0)$
404	$(5,0,0,6,1,0)$
405	$(6,0,0,6,1,0)$
406	$(0,6,1,6,1,0)$
407	$(1,6,1,6,1,0)$
408	$(2,6,1,6,1,0)$
409	$(3,6,1,6,1,0)$

,

i	P_i
410	$(4,6,1,6,1,0)$
411	$(5,6,1,6,1,0)$
412	$(6,6,1,6,1,0)$
413	$(0,5,2,6,1,0)$
414	$(1,5,2,6,1,0)$
415	$(2,5,2,6,1,0)$
416	$(3,5,2,6,1,0)$
417	$(4,5,2,6,1,0)$
418	$(5,5,2,6,1,0)$
419	$(6,5,2,6,1,0)$

,

i	P_i
420	$(0,4,3,6,1,0)$
421	$(1,4,3,6,1,0)$
422	$(2,4,3,6,1,0)$
423	$(3,4,3,6,1,0)$
424	$(4,4,3,6,1,0)$
425	$(5,4,3,6,1,0)$
426	$(6,4,3,6,1,0)$
427	$(0,3,4,6,1,0)$
428	$(1,3,4,6,1,0)$
429	$(2,3,4,6,1,0)$

,

i	P_i
430	$(3,3,4,6,1,0)$
431	$(4,3,4,6,1,0)$
432	$(5,3,4,6,1,0)$
433	$(6,3,4,6,1,0)$
434	$(0,2,5,6,1,0)$
435	$(1,2,5,6,1,0)$
436	$(2,2,5,6,1,0)$
437	$(3,2,5,6,1,0)$
438	$(4,2,5,6,1,0)$
439	$(5,2,5,6,1,0)$

,

i	P_i
440	$(6,2,5,6,1,0)$
441	$(0,1,6,6,1,0)$
442	$(1,1,6,6,1,0)$
443	$(2,1,6,6,1,0)$
444	$(3,1,6,6,1,0)$
445	$(4,1,6,6,1,0)$
446	$(5,1,6,6,1,0)$
447	$(6,1,6,6,1,0)$

Strong generators for a group of order 1626379776:

$$\begin{bmatrix}1&0&0&0\\0&5&0&0\\0&0&5&0\\0&0&0&5\end{bmatrix}, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&5&0\\0&0&0&3\end{bmatrix}, \begin{bmatrix}1&0&0&0\\0&3&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix},$$

$$\begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&3&0\\ 0&0&0&1 \end{bmatrix}, \begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 1&0&0&1 \end{bmatrix}, \begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&0&1&1 \end{bmatrix},$$

$$\begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&5&0\\ 2&6&6&3 \end{bmatrix}, \begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&1&1&0\\ 0&0&0&1 \end{bmatrix}, \begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&0&1\\ 0&0&1&0 \end{bmatrix},$$

$$\begin{bmatrix} 1&0&0&0\\ 1&1&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{bmatrix}, \begin{bmatrix} 0&1&0&0\\ 1&0&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{bmatrix}$$

Poset classification up to depth 5

3 The orbits

3.1 Number of orbits at depth

Depth	Nb of orbits
0	1
1	1
2	1
3	1
4	8
5	14

3.2 Orbit representatives: overview

N = node
D = depth or level
O = orbit with a level
Rep = orbit representative
SO = (order of stabilizer, orbit length)
L = number of live points
F = number of flags
Gen = number of generators for the stabilizer of the orbit rep.

Table 1: Orbit Representatives

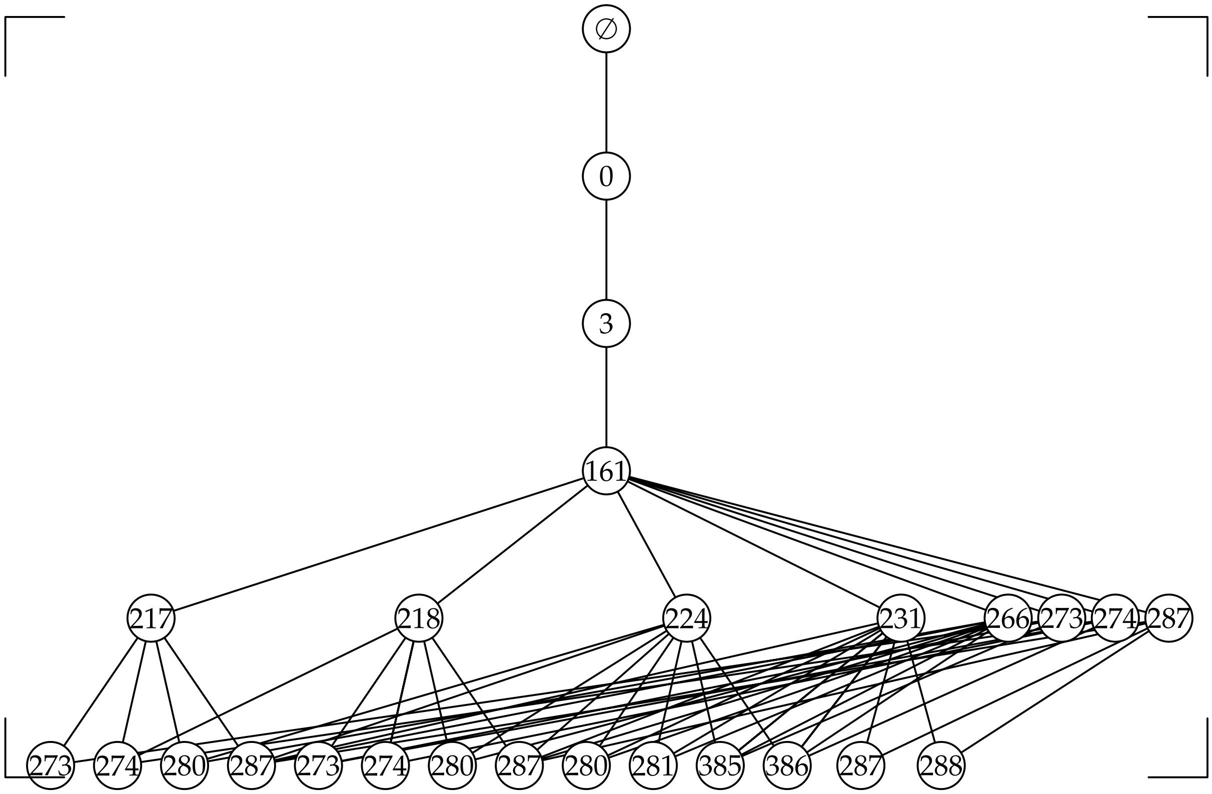
N	D	O	Rep	SO	L	F	Gen
0	0	0	{ }	(1626379776, 1)	448	1	11
1	1	0	{ 0 }	(3630312, 448)	343	1	9
2	2	0	{ 0, 3 }	(21168, 76832)	252	1	8
3	3	0	{ 0, 3, 161 }	(252, 6453888)	175	8	5
4	4	0	{ 0, 3, 161, 217 }	(336, 4840416)	112	4	5
5	4	1	{ 0, 3, 161, 218 }	(56, 29042496)	112	6	4
6	4	2	{ 0, 3, 161, 224 }	(24, 67765824)	112	8	4
7	4	3	{ 0, 3, 161, 231 }	(24, 67765824)	112	10	3
8	4	4	{ 0, 3, 161, 266 }	(24, 67765824)	112	8	4
9	4	5	{ 0, 3, 161, 273 }	(504, 3226944)	112	3	5
10	4	6	{ 0, 3, 161, 274 }	(84, 19361664)	112	4	4
11	4	7	{ 0, 3, 161, 287 }	(72, 22588608)	112	4	5

Continued on next page

Table 1 – continued from previous page

N	D	O	Rep	SO	L	F	Gen
12	5	0	{ 0, 3, 161, 217, 273 }	(252, 6453888)			5
13	5	1	{ 0, 3, 161, 217, 274 }	(14, 116169984)			2
14	5	2	{ 0, 3, 161, 217, 280 }	(12, 135531648)			2
15	5	3	{ 0, 3, 161, 217, 287 }	(6, 271063296)			1
16	5	4	{ 0, 3, 161, 218, 273 }	(21, 77446656)			2
17	5	5	{ 0, 3, 161, 218, 274 }	(14, 116169984)			2
18	5	6	{ 0, 3, 161, 218, 280 }	(2, 813189888)			1
19	5	7	{ 0, 3, 161, 218, 287 }	(1, 1626379776)			0
20	5	8	{ 0, 3, 161, 224, 280 }	(12, 135531648)			2
21	5	9	{ 0, 3, 161, 224, 281 }	(2, 813189888)			1
22	5	10	{ 0, 3, 161, 224, 385 }	(6, 271063296)			2
23	5	11	{ 0, 3, 161, 224, 386 }	(1, 1626379776)			0
24	5	12	{ 0, 3, 161, 231, 287 }	(36, 45177216)			5
25	5	13	{ 0, 3, 161, 231, 288 }	(6, 271063296)			3

4 The poset of orbits



5 Stabilizers and Schreier trees

5.1 Stabilizers and Schreier trees at level 0

Node 0 at Level 0 Orbit 0 / 1

$$\{\}_{1626379776}$$

Strong generators for a group of order 1626379776:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix},$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 2 & 6 & 6 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

There are 1 extensions
Number of generators 11

Generators for the Schreier trees:
Generators for a group of order 1626379776:

$$\begin{bmatrix}1&3&0&0\\1&0&0&0\\0&0&3&1\\0&0&3&0\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&5&0\\0&0&0&3\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&3&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&3&0\\0&0&0&1\end{bmatrix}$$

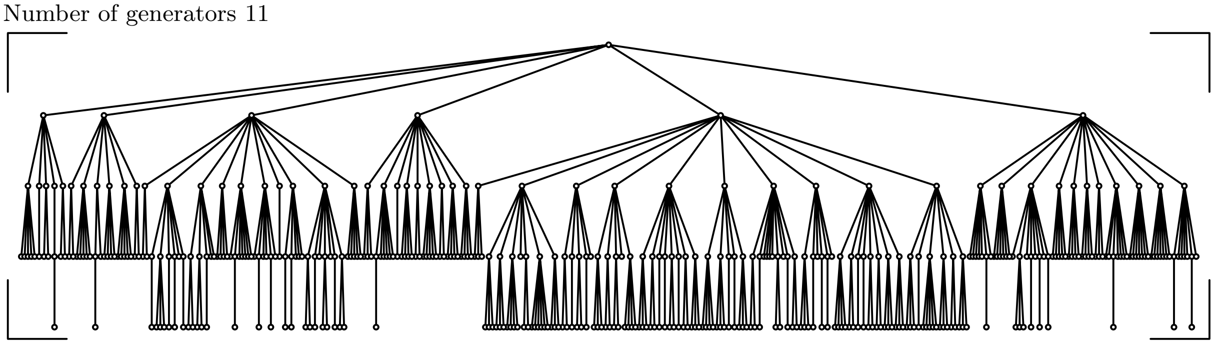
$$\begin{bmatrix}1&6&0&0\\2&0&0&0\\6&6&2&0\\0&0&0&2\end{bmatrix}\begin{bmatrix}1&1&0&0\\1&0&0&0\\1&2&3&6\\0&0&0&5\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&5&0\\2&6&6&3\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&1&1&0\\0&0&0&1\end{bmatrix}$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}\begin{bmatrix}1&0&0&0\\1&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&2&0&0\\1&0&0&0\\1&4&2&3\\0&0&5&0\end{bmatrix}$$

Orbit 0 / 1: Point 0 lies in an orbit of length 442 with average word length 4.22172 $H_{11} = 3.1409$

Node 0 at Level 0 Orbit 0 / 1 Tree 0 / 1

Number of generators 11



Extension number 0
Orbit representative 0
Flag orbit length 448
Flag orbit is defining new orbit 1 at level 1

5.2 Stabilizers and Schreier trees at level 1

Node 1 at Level 1 Orbit 0 / 1

$$\{0\}_{3630312}$$

Strong generators for a group of order 3630312:

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&0&2\end{bmatrix},\begin{bmatrix}1&0&0&0\\0&4&0&0\\0&0&4&0\\0&0&0&1\end{bmatrix},\begin{bmatrix}1&0&0&0\\0&5&0&0\\0&0&5&0\\0&0&0&2\end{bmatrix},$$

$$\begin{bmatrix}1&0&0&0\\0&5&0&0\\0&0&1&0\\0&0&0&4\end{bmatrix},\begin{bmatrix}1&0&0&0\\0&4&0&0\\0&0&3&0\\0&0&0&3\end{bmatrix},\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\1&0&3&6\end{bmatrix},$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&1&0&1\end{bmatrix},\begin{bmatrix}1&0&0&0\\0&1&0&0\\4&0&6&0\\6&0&4&1\end{bmatrix},\begin{bmatrix}1&0&0&0\\1&6&0&0\\1&0&1&0\\0&0&0&1\end{bmatrix}$$

There are 1 extensions
Number of generators 9
Generators for the Schreier trees:
Generators for a group of order 3630312:

$$\begin{bmatrix} 1&0&0&0\\ 5&5&0&0\\ 1&0&3&0\\ 4&0&2&1 \end{bmatrix} \begin{bmatrix} 1&0&0&0\\ 1&1&0&0\\ 1&0&3&0\\ 1&0&2&2 \end{bmatrix} \begin{bmatrix} 1&0&0&0\\ 0&4&0&0\\ 0&0&4&0\\ 0&0&2&1 \end{bmatrix} \begin{bmatrix} 1&0&0&0\\ 0&5&0&0\\ 0&0&1&0\\ 0&0&0&4 \end{bmatrix}$$

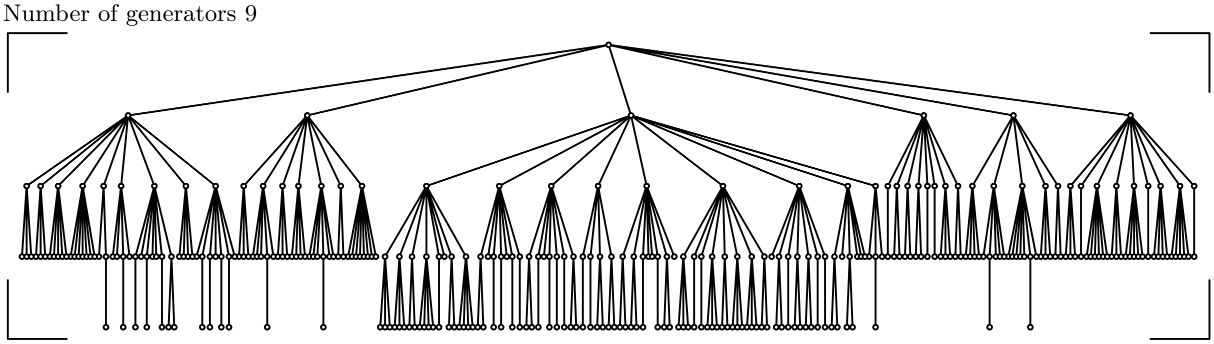
$$\begin{bmatrix} 1&0&0&0\\ 0&4&0&0\\ 0&0&3&0\\ 0&0&0&3 \end{bmatrix} \begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 1&0&3&6 \end{bmatrix} \begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&1&0&1 \end{bmatrix} \begin{bmatrix} 1&0&0&0\\ 6&1&0&0\\ 6&0&6&0\\ 5&0&4&5 \end{bmatrix}$$

$$\begin{bmatrix} 1&0&0&0\\ 1&1&0&0\\ 1&0&1&0\\ 4&0&3&1 \end{bmatrix}$$

Orbit 0 / 1: Point 3 lies in an orbit of length 343 with average word length 4.09329 $H_9 = 3.29829$

Node 1 at Level 1 Orbit 0 / 1 Tree 0 / 1

Number of generators 9



Extension number 0
Orbit representative 3
Flag orbit length 343
Flag orbit is defining new orbit 2 at level 2

5.3 Stabilizers and Schreier trees at level 2

Node 2 at Level 2 Orbit 0 / 1

$$\{0,3\}_{21168}$$

Strong generators for a group of order 21168:

$$\begin{bmatrix} 1&0&0&0\\ 0&2&0&0\\ 0&0&2&0\\ 0&0&0&5 \end{bmatrix}, \begin{bmatrix} 1&0&0&0\\ 0&6&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{bmatrix}, \begin{bmatrix} 1&0&0&0\\ 0&5&0&0\\ 0&0&1&0\\ 0&0&0&3 \end{bmatrix},$$

$$\begin{bmatrix} 1&0&0&0\\ 0&4&0&0\\ 0&0&2&0\\ 0&0&0&3 \end{bmatrix}, \begin{bmatrix} 1&0&0&0\\ 0&6&0&0\\ 0&0&3&0\\ 0&0&0&3 \end{bmatrix}, \begin{bmatrix} 1&0&0&0\\ 0&4&0&0\\ 0&0&3&0\\ 0&5&0&2 \end{bmatrix},$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 3 & 0 & 3 & 0 \\ 0 & 5 & 0 & 6 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

There are 1 extensions
Number of generators 8
Generators for the Schreier trees:
Generators for a group of order 21168:

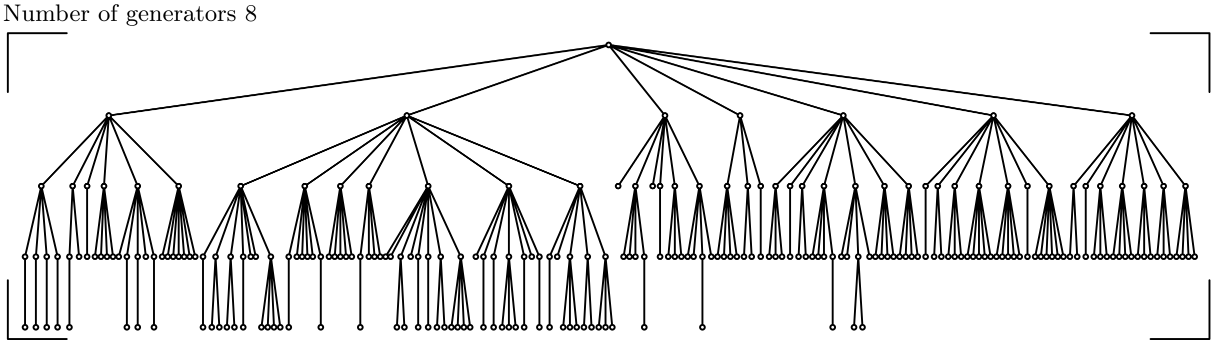
$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 6 & 0 & 0 & 0 \\ 0 & 4 & 0 & 4 \\ 3 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 6 & 0 & 0 & 0 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 5 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 2 & 0 & 6 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 0 & 4 & 0 & 3 \\ 5 & 0 & 2 & 0 \end{bmatrix}$$

Orbit 0 / 1: Point 161 lies in an orbit of length 252 with average word length 3.96825 $H_8 = 3.32193$

Node 2 at Level 2 Orbit 0 / 1 Tree 0 / 1

Number of generators 8



Extension number 0
Orbit representative 161
Flag orbit length 252
Flag orbit is defining new orbit 3 at level 3

5.4 Stabilizers and Schreier trees at level 3

Node 3 at Level 3 Orbit 0 / 1

$$\{0, 3, 161\}_{252}$$

Strong generators for a group of order 252:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 5 & 0 \\ 0 & 1 & 0 & 5 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 6 & 6 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 6 & 6 & 6 & 6 \end{bmatrix},$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 6 \\ 5 & 0 & 6 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 5 & 5 & 6 & 6 \\ 0 & 2 & 0 & 1 \end{bmatrix}$$

There are 8 extensions
Number of generators 5

Generators for the Schreier trees:
Generators for a group of order 252:

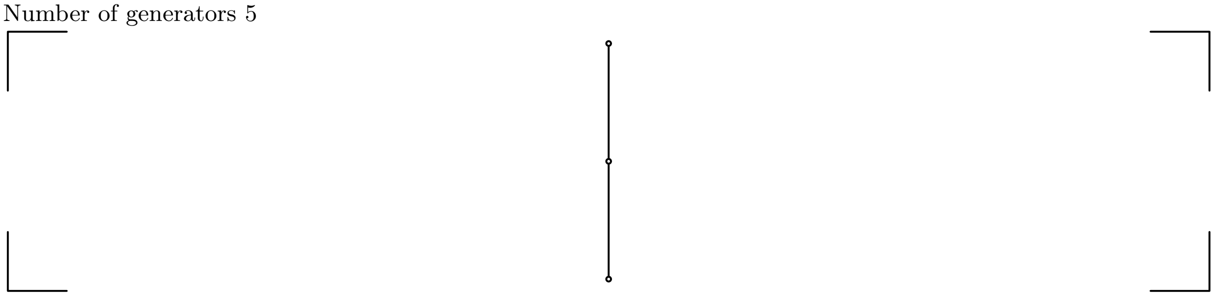
$$\begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&3&0 \\ 0&0&0&3 \end{bmatrix} \begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 1&0&5&0 \\ 0&1&0&5 \end{bmatrix} \begin{bmatrix} 1&1&0&0 \\ 6&0&0&0 \\ 4&4&6&6 \\ 3&0&1&0 \end{bmatrix} \begin{bmatrix} 0&1&0&0 \\ 1&0&0&0 \\ 0&5&0&3 \\ 5&0&3&0 \end{bmatrix}$$

$$\begin{bmatrix} 1&1&0&0 \\ 0&6&0&0 \\ 5&5&6&6 \\ 0&2&0&1 \end{bmatrix}$$

Orbit 0 / 8: Point 217 lies in an orbit of length 3 with average word length 2 $H_5 = 1.11328$
Orbit 1 / 8: Point 218 lies in an orbit of length 18 with average word length 2.94444 $H_5 = 2.46688$
Orbit 2 / 8: Point 224 lies in an orbit of length 42 with average word length 3.45238 $H_5 = 3.09222$
Orbit 3 / 8: Point 231 lies in an orbit of length 42 with average word length 3.45238 $H_5 = 3.09222$
Orbit 4 / 8: Point 266 lies in an orbit of length 42 with average word length 3.45238 $H_5 = 3.09222$
Orbit 5 / 8: Point 273 lies in an orbit of length 2 with average word length 1.5 $H_5 = 0.682606$
Orbit 6 / 8: Point 274 lies in an orbit of length 12 with average word length 2.66667 $H_5 = 2.15338$
Orbit 7 / 8: Point 287 lies in an orbit of length 14 with average word length 2.92857 $H_5 = 2.30737$

Node 3 at Level 3 Orbit 0 / 1 Tree 0 / 8

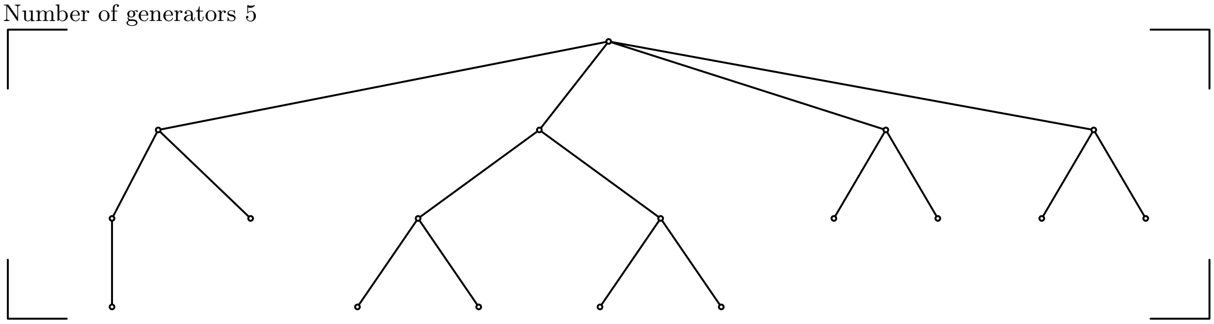
Number of generators 5



Extension number 0
Orbit representative 217
Flag orbit length 3
Flag orbit is defining new orbit 4 at level 4

Node 3 at Level 3 Orbit 0 / 1 Tree 1 / 8

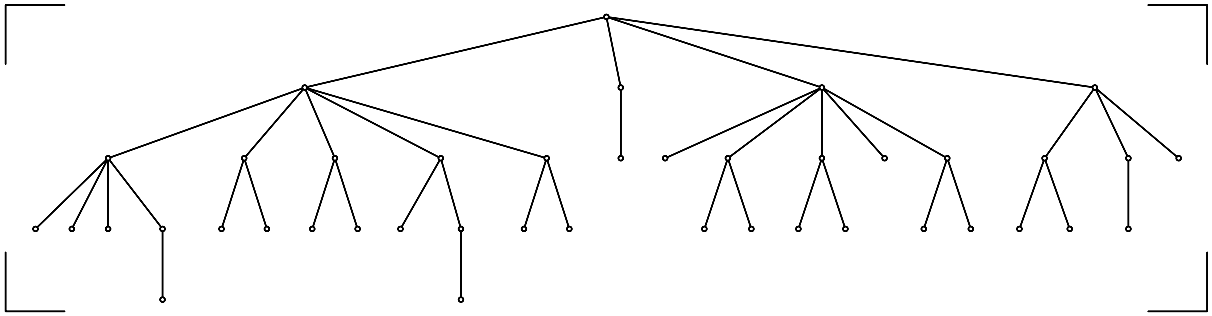
Number of generators 5



Extension number 1
Orbit representative 218
Flag orbit length 18
Flag orbit is defining new orbit 5 at level 4

Node 3 at Level 3 Orbit 0 / 1 Tree 2 / 8

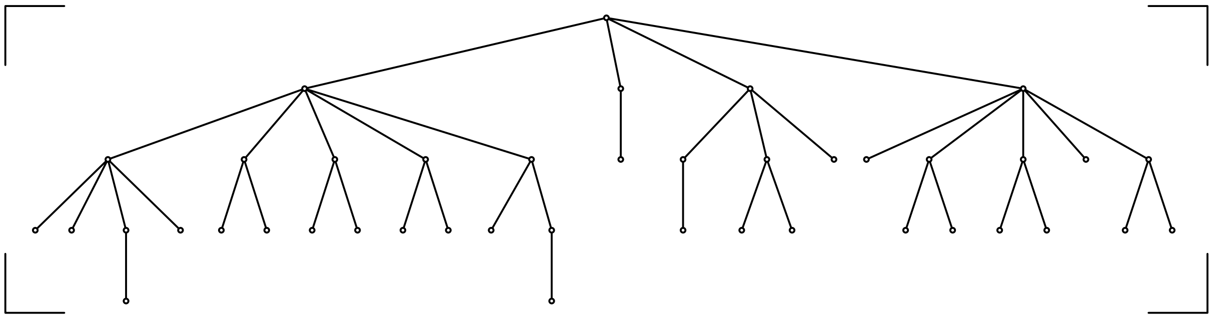
Number of generators 5



Extension number 2
Orbit representative 224
Flag orbit length 42
Flag orbit is defining new orbit 6 at level 4

Node 3 at Level 3 Orbit 0 / 1 Tree 3 / 8

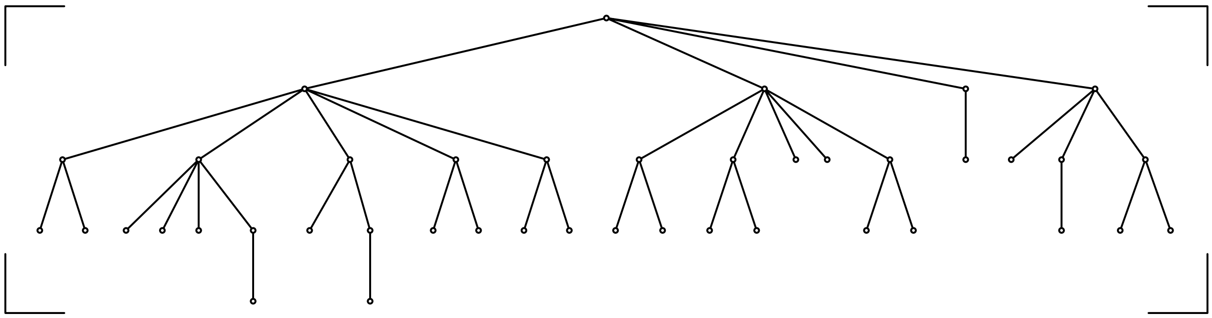
Number of generators 5



Extension number 3
Orbit representative 231
Flag orbit length 42
Flag orbit is defining new orbit 7 at level 4

Node 3 at Level 3 Orbit 0 / 1 Tree 4 / 8

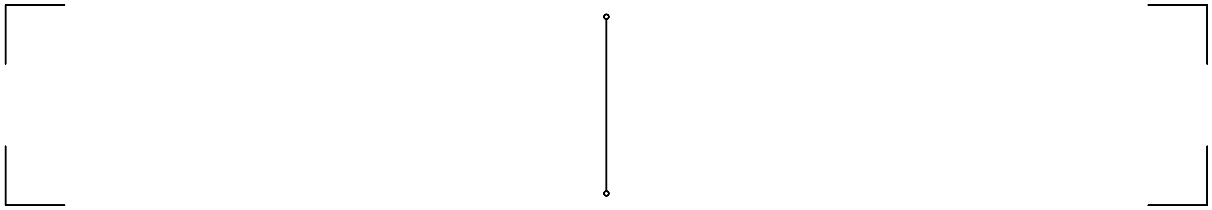
Number of generators 5



Extension number 4
Orbit representative 266
Flag orbit length 42
Flag orbit is defining new orbit 8 at level 4

Node 3 at Level 3 Orbit 0 / 1 Tree 5 / 8

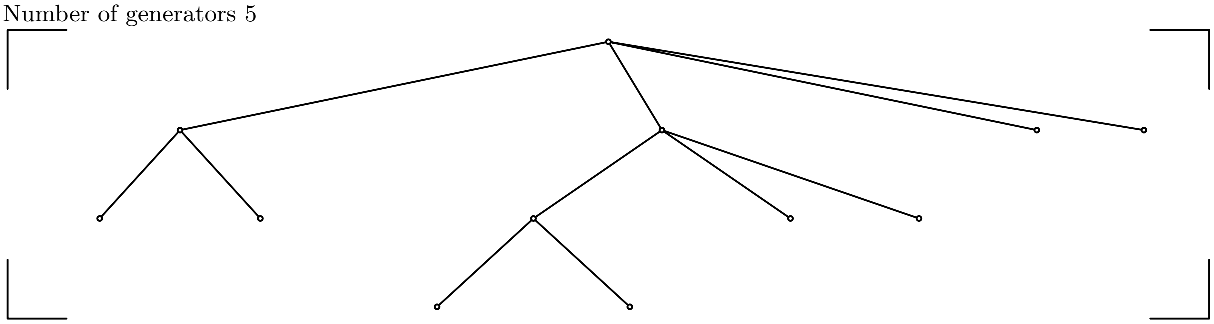
Number of generators 5



Extension number 5
Orbit representative 273
Flag orbit length 2
Flag orbit is defining new orbit 9 at level 4

Node 3 at Level 3 Orbit 0 / 1 Tree 6 / 8

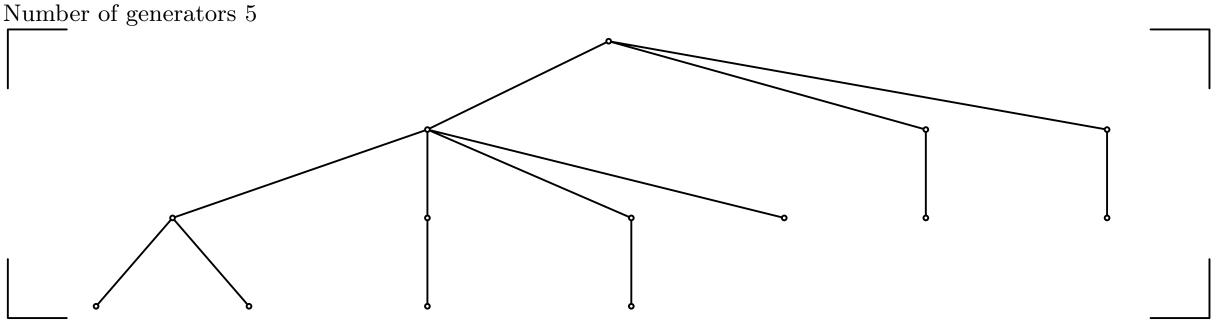
Number of generators 5



Extension number 6
Orbit representative 274
Flag orbit length 12
Flag orbit is defining new orbit 10 at level 4

Node 3 at Level 3 Orbit 0 / 1 Tree 7 / 8

Number of generators 5



Extension number 7
Orbit representative 287
Flag orbit length 14
Flag orbit is defining new orbit 11 at level 4

5.5 Stabilizers and Schreier trees at level 4

Node 4 at Level 4 Orbit 0 / 8

$$\{0, 3, 161, 217\}_{336}$$

Strong generators for a group of order 336:

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&3&0\\0&0&0&3\end{bmatrix}, \begin{bmatrix}1&0&0&0\\0&1&0&0\\2&0&1&0\\0&2&0&1\end{bmatrix}, \begin{bmatrix}1&0&0&0\\5&6&0&0\\6&0&6&0\\2&1&2&1\end{bmatrix}, \begin{bmatrix}1&1&0&0\\0&6&0&0\\0&0&6&6\\0&0&0&1\end{bmatrix}, \begin{bmatrix}1&4&0&0\\6&6&0&0\\2&1&3&5\\5&5&4&4\end{bmatrix}$$

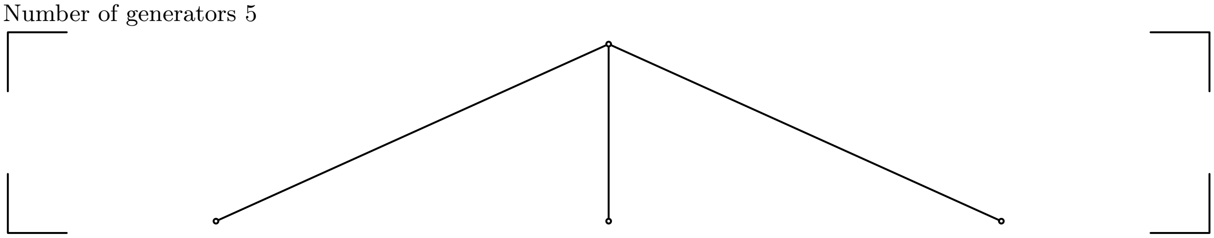
There are 4 extensions
Number of generators 5
Generators for the Schreier trees:
Generators for a group of order 336:

$$\begin{bmatrix}1&1&0&0\\0&6&0&0\\0&0&5&5\\0&0&0&2\end{bmatrix} \begin{bmatrix}1&0&0&0\\5&6&0&0\\6&0&6&0\\2&1&2&1\end{bmatrix} \begin{bmatrix}1&1&0&0\\0&6&0&0\\0&0&4&4\\0&0&0&3\end{bmatrix} \begin{bmatrix}1&1&0&0\\0&6&0&0\\0&0&6&6\\0&0&0&1\end{bmatrix} \begin{bmatrix}1&4&0&0\\6&6&0&0\\2&1&3&5\\5&5&4&4\end{bmatrix}$$

Orbit 0 / 4: Point 273 lies in an orbit of length 4 with average word length 1.75 $H_5 = 1.20906$
Orbit 1 / 4: Point 274 lies in an orbit of length 24 with average word length 3.04167 $H_5 = 2.66581$
Orbit 2 / 4: Point 280 lies in an orbit of length 28 with average word length 3.42857 $H_5 = 2.83599$
Orbit 3 / 4: Point 287 lies in an orbit of length 56 with average word length 4 $H_5 = 3.36244$

Node 4 at Level 4 Orbit 0 / 8 Tree 0 / 4

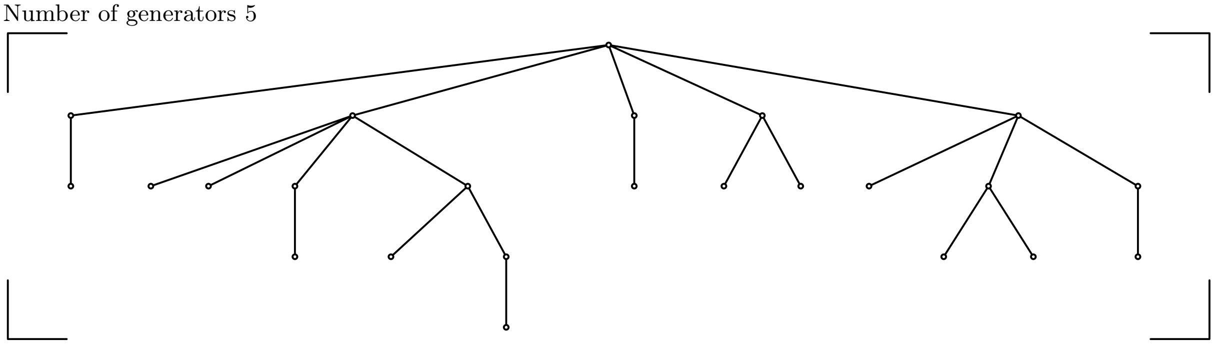
Number of generators 5



Extension number 0
Orbit representative 273
Flag orbit length 4
Flag orbit is defining new orbit 12 at level 5

Node 4 at Level 4 Orbit 0 / 8 Tree 1 / 4

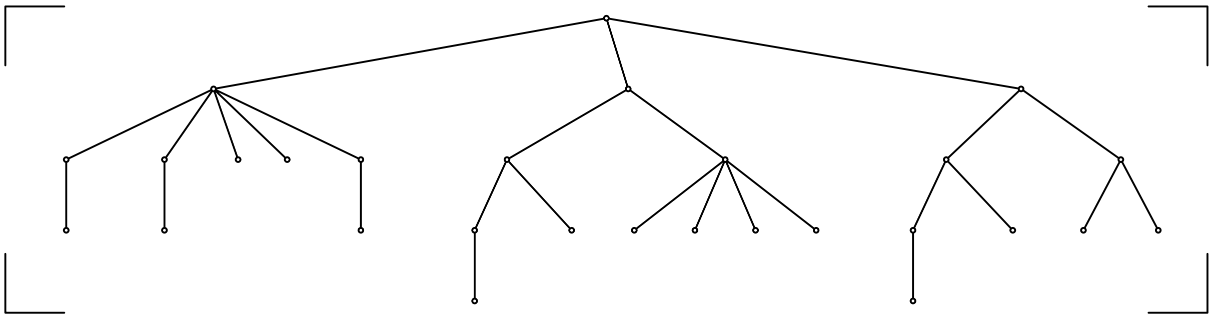
Number of generators 5



Extension number 1
Orbit representative 274
Flag orbit length 24
Flag orbit is defining new orbit 13 at level 5

Node 4 at Level 4 Orbit 0 / 8 Tree 2 / 4

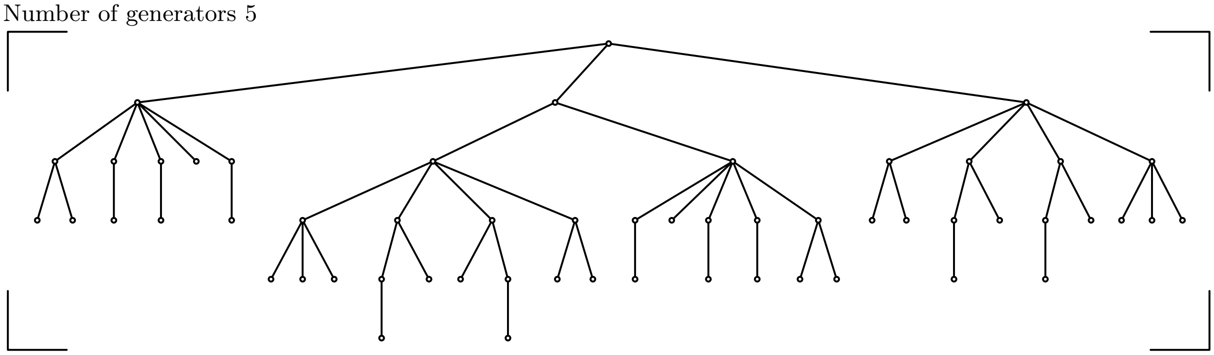
Number of generators 5



Extension number 2
Orbit representative 280
Flag orbit length 28
Flag orbit is defining new orbit 14 at level 5

Node 4 at Level 4 Orbit 0 / 8 Tree 3 / 4

Number of generators 5



Extension number 3
Orbit representative 287
Flag orbit length 56
Flag orbit is defining new orbit 15 at level 5

Node 5 at Level 4 Orbit 1 / 8

$$\{0,3,161,218\}_{56}$$

Strong generators for a group of order 56:

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\2&0&1&0\\0&2&0&1\end{bmatrix},\begin{bmatrix}1&0&0&0\\5&6&0&0\\3&0&6&0\\2&5&2&1\end{bmatrix},\begin{bmatrix}1&1&0&0\\0&6&0&0\\4&4&6&6\\0&3&0&1\end{bmatrix},$$

$$\begin{bmatrix}1&4&0&0\\6&6&0&0\\5&1&1&4\\2&2&6&6\end{bmatrix}$$

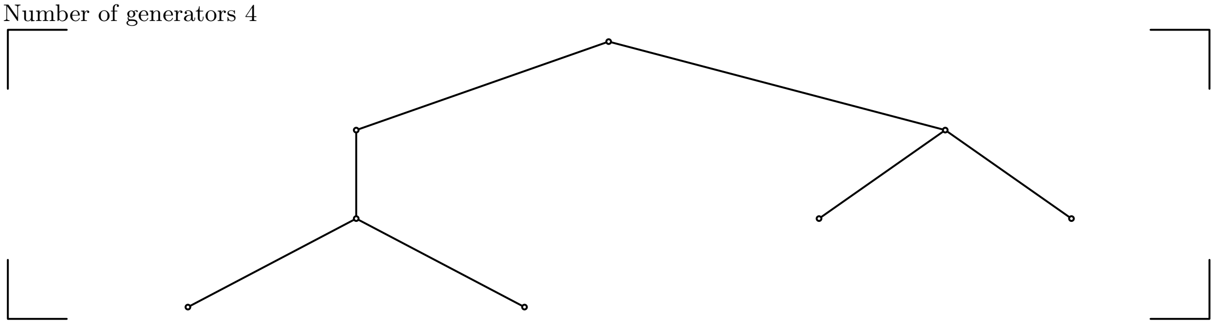
There are 6 extensions
Number of generators 4
Generators for the Schreier trees:
Generators for a group of order 56:

$$\begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 2&0&1&0\\ 0&2&0&1 \end{bmatrix} \begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{bmatrix} \begin{bmatrix} 1&1&0&0\\ 0&6&0&0\\ 4&4&6&6\\ 0&3&0&1 \end{bmatrix} \begin{bmatrix} 0&1&0&0\\ 5&5&0&0\\ 0&3&0&6\\ 2&2&2&2 \end{bmatrix}$$

Orbit 0 / 6: Point 273 lies in an orbit of length 8 with average word length 2.75 $H_4 = 2.22972$
Orbit 1 / 6: Point 274 lies in an orbit of length 8 with average word length 2.75 $H_4 = 2.22972$
Orbit 2 / 6: Point 275 lies in an orbit of length 8 with average word length 2.75 $H_4 = 2.22972$
Orbit 3 / 6: Point 279 lies in an orbit of length 4 with average word length 2.5 $H_4 = 1.66096$
Orbit 4 / 6: Point 280 lies in an orbit of length 28 with average word length 3.75 $H_4 = 3.35712$
Orbit 5 / 6: Point 287 lies in an orbit of length 56 with average word length 4.375 $H_4 = 3.96832$

Node 5 at Level 4 Orbit 1 / 8 Tree 0 / 6

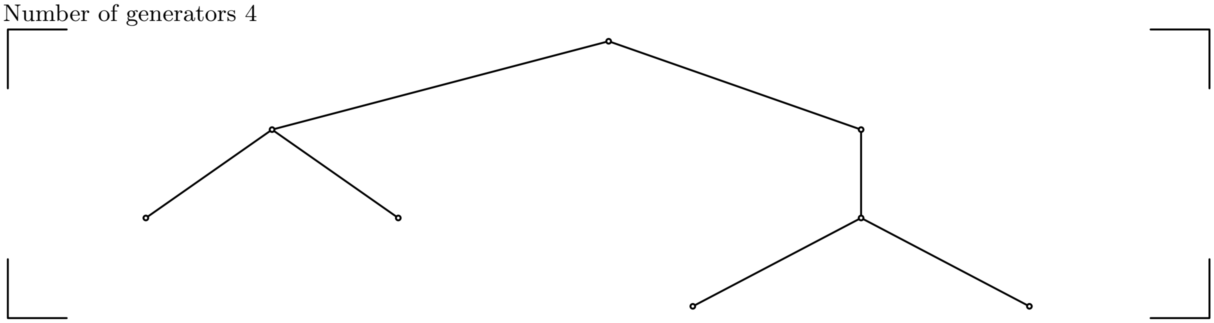
Number of generators 4



Extension number 0
Orbit representative 273
Flag orbit length 8
Flag orbit is defining new orbit 16 at level 5

Node 5 at Level 4 Orbit 1 / 8 Tree 1 / 6

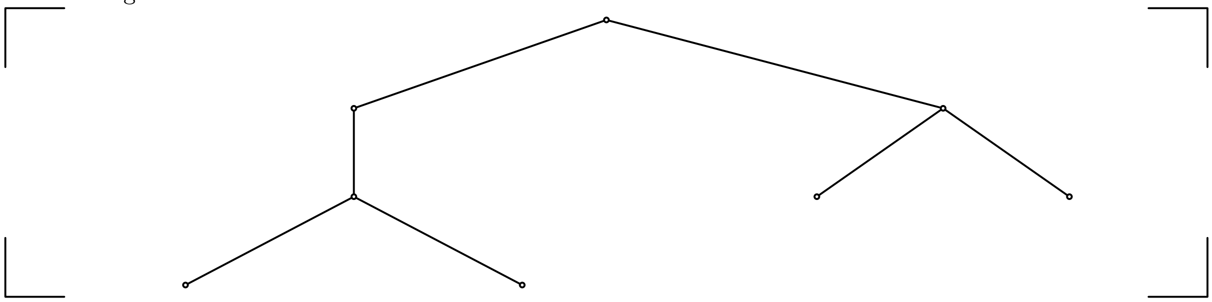
Number of generators 4



Extension number 1
Orbit representative 274
Flag orbit length 8
Flag orbit is defining new orbit 17 at level 5

Node 5 at Level 4 Orbit 1 / 8 Tree 2 / 6

Number of generators 4

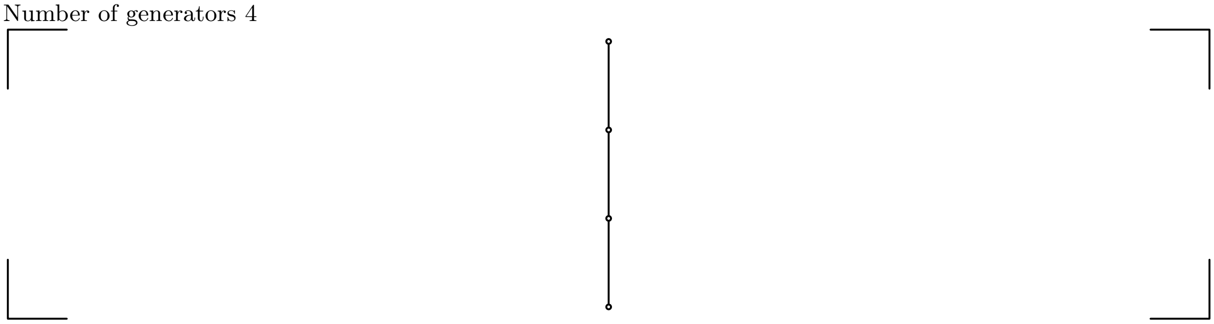


Extension number 2
Orbit representative 275
Flag orbit length 8
Flag orbit is fused to node 4 extension 1
Fusion element:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 5 & 4 & 0 & 0 \\ 0 & 6 & 0 & 1 \\ 4 & 5 & 5 & 4 \end{bmatrix}$$

Node 5 at Level 4 Orbit 1 / 8 Tree 3 / 6

Number of generators 4

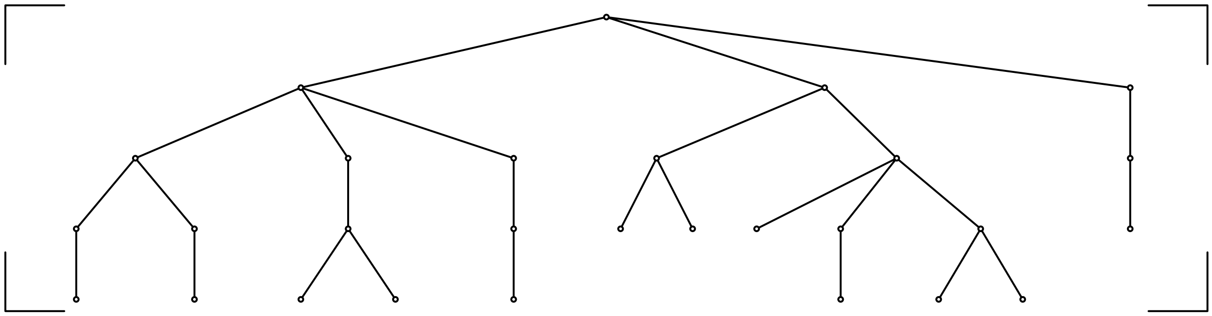


Extension number 3
Orbit representative 279
Flag orbit length 4
Flag orbit is fused to node 5 extension 1
Fusion element:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 5 & 4 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 4 & 3 & 1 & 5 \end{bmatrix}$$

Node 5 at Level 4 Orbit 1 / 8 Tree 4 / 6

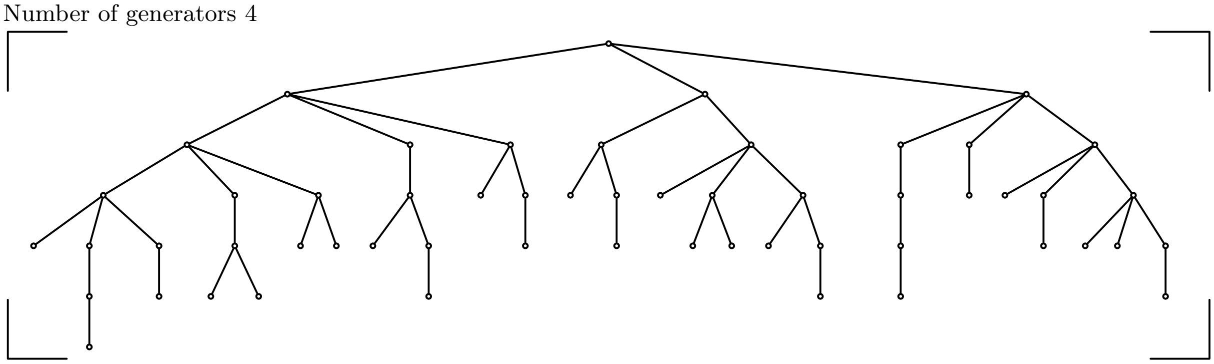
Number of generators 4



Extension number 4
Orbit representative 280
Flag orbit length 28
Flag orbit is defining new orbit 18 at level 5

Node 5 at Level 4 Orbit 1 / 8 Tree 5 / 6

Number of generators 4



Extension number 5
Orbit representative 287
Flag orbit length 56
Flag orbit is defining new orbit 19 at level 5

Node 6 at Level 4 Orbit 2 / 8

$$\{0, 3, 161, 224\}_{24}$$

Strong generators for a group of order 24:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 5 & 0 & 0 \\ 6 & 6 & 0 & 0 \\ 0 & 0 & 5 & 6 \\ 0 & 0 & 2 & 2 \end{bmatrix},$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 4 & 6 & 0 & 0 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 3 & 5 \end{bmatrix}$$

There are 8 extensions
Number of generators 4
Generators for the Schreier trees:

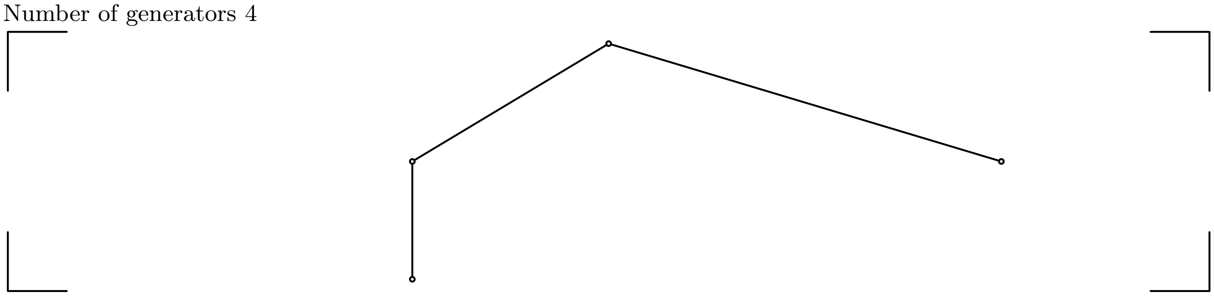
Generators for a group of order 24:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 5 & 0 & 0 \\ 6 & 6 & 0 & 0 \\ 0 & 0 & 5 & 6 \\ 0 & 0 & 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 4 & 6 & 0 & 0 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 3 & 5 \end{bmatrix}$$

Orbit 0 / 8: Point 266 lies in an orbit of length 4 with average word length 2 $H_4 = 1.5$
Orbit 1 / 8: Point 267 lies in an orbit of length 24 with average word length 3.5 $H_4 = 3.19616$
Orbit 2 / 8: Point 280 lies in an orbit of length 4 with average word length 2 $H_4 = 1.5$
Orbit 3 / 8: Point 281 lies in an orbit of length 24 with average word length 3.5 $H_4 = 3.19616$
Orbit 4 / 8: Point 364 lies in an orbit of length 4 with average word length 2 $H_4 = 1.5$
Orbit 5 / 8: Point 365 lies in an orbit of length 24 with average word length 3.5 $H_4 = 3.19616$
Orbit 6 / 8: Point 385 lies in an orbit of length 4 with average word length 2 $H_4 = 1.5$
Orbit 7 / 8: Point 386 lies in an orbit of length 24 with average word length 3.5 $H_4 = 3.19616$

Node 6 at Level 4 Orbit 2 / 8 Tree 0 / 8

Number of generators 4

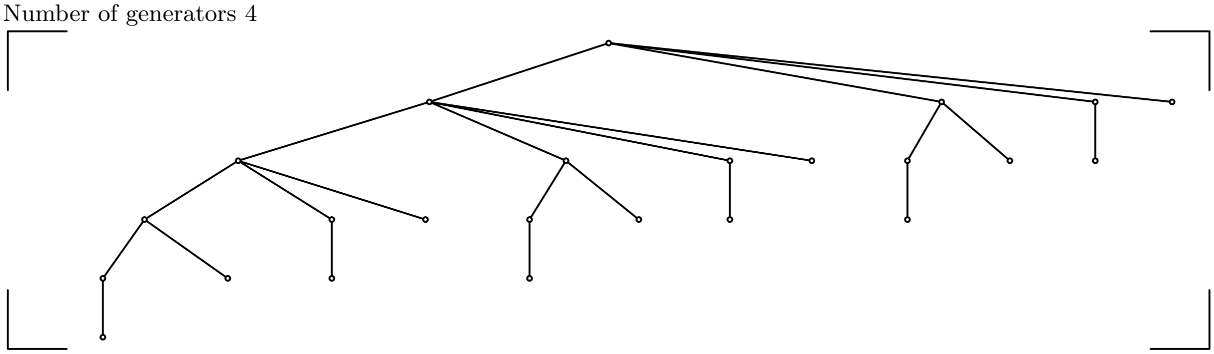


Extension number 0
Orbit representative 266
Flag orbit length 4
Flag orbit is fused to node 4 extension 3
Fusion element:

$$\begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

Node 6 at Level 4 Orbit 2 / 8 Tree 1 / 8

Number of generators 4



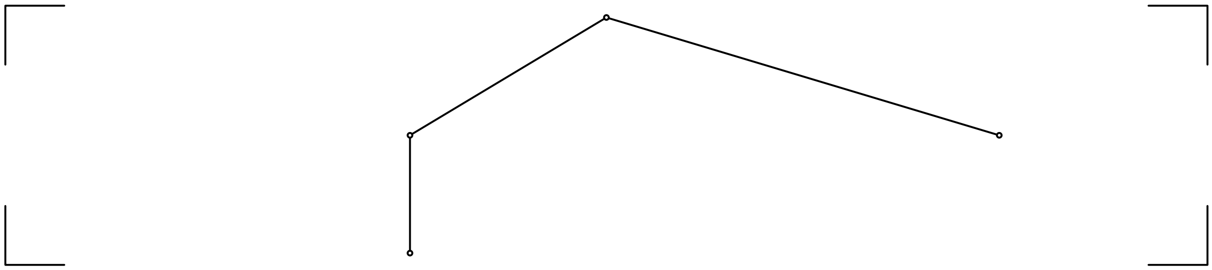
Extension number 1
Orbit representative 267
Flag orbit length 24
Flag orbit is fused to node 5 extension 5

Fusion element:

$$\begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 6 & 2 & 3 \\ 0 & 6 & 0 & 5 \end{bmatrix}$$

Node 6 at Level 4 Orbit 2 / 8 Tree 2 / 8

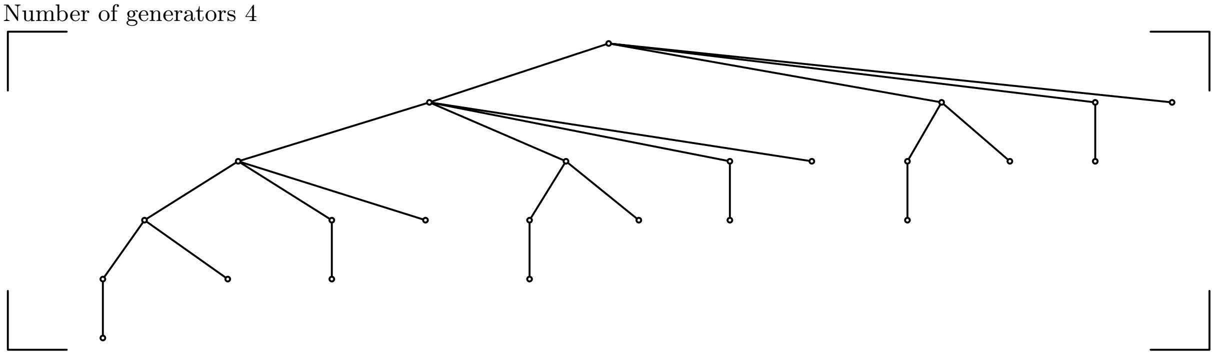
Number of generators 4



Extension number 2
Orbit representative 280
Flag orbit length 4
Flag orbit is defining new orbit 20 at level 5

Node 6 at Level 4 Orbit 2 / 8 Tree 3 / 8

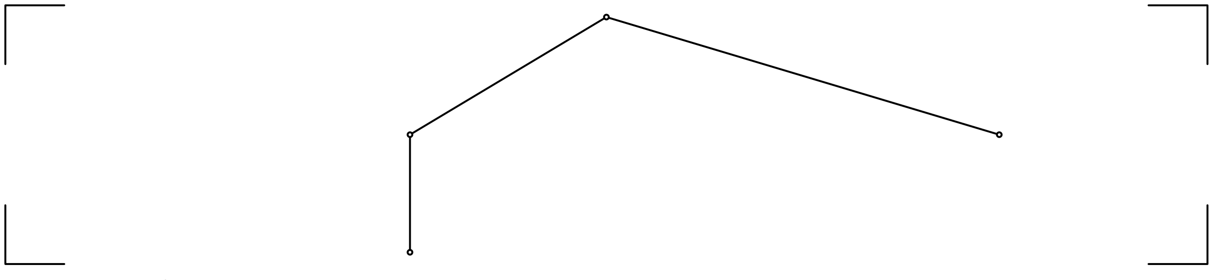
Number of generators 4



Extension number 3
Orbit representative 281
Flag orbit length 24
Flag orbit is defining new orbit 21 at level 5

Node 6 at Level 4 Orbit 2 / 8 Tree 4 / 8

Number of generators 4



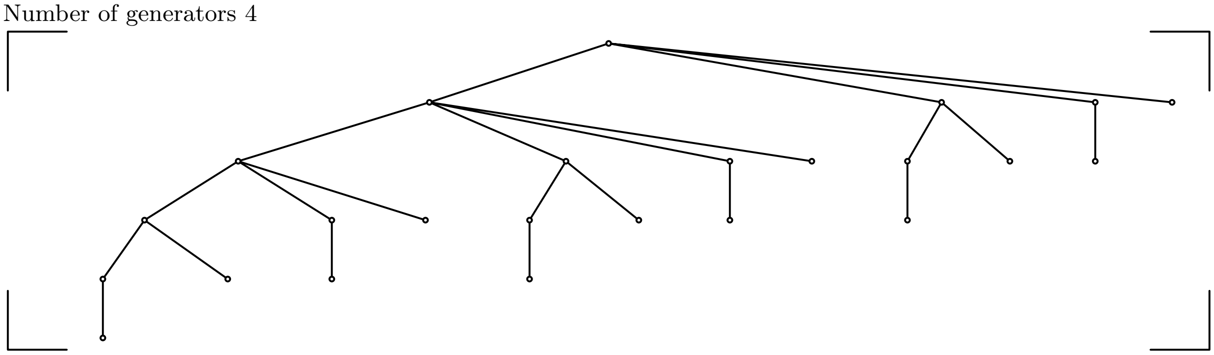
Extension number 4
Orbit representative 364

Flag orbit length 4
Flag orbit is fused to node 4 extension 2
Fusion element:

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 6 & 5 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 6 & 2 \end{bmatrix}$$

Node 6 at Level 4 Orbit 2 / 8 Tree 5 / 8

Number of generators 4

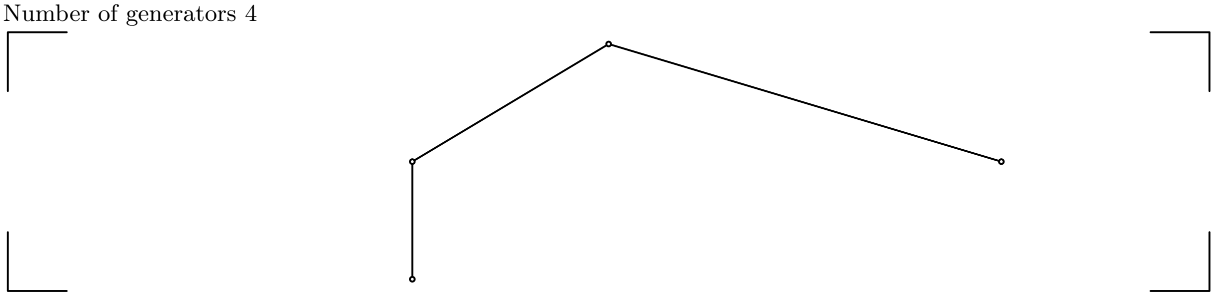


Extension number 5
Orbit representative 365
Flag orbit length 24
Flag orbit is fused to node 5 extension 4
Fusion element:

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 6 & 5 & 0 & 0 \\ 5 & 5 & 4 & 4 \\ 2 & 4 & 3 & 1 \end{bmatrix}$$

Node 6 at Level 4 Orbit 2 / 8 Tree 6 / 8

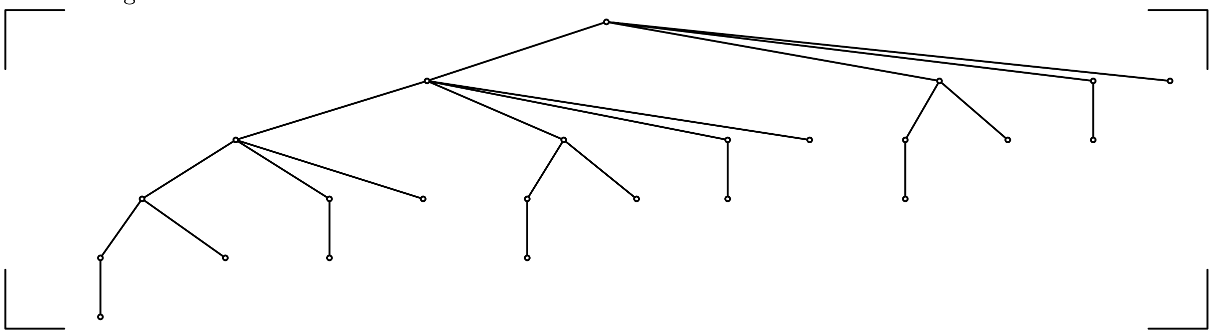
Number of generators 4



Extension number 6
Orbit representative 385
Flag orbit length 4
Flag orbit is defining new orbit 22 at level 5

Node 6 at Level 4 Orbit 2 / 8 Tree 7 / 8

Number of generators 4



Extension number 7
Orbit representative 386
Flag orbit length 24
Flag orbit is defining new orbit 23 at level 5

Node 7 at Level 4 Orbit 3 / 8

$$\{0, 3, 161, 231\}_{24}$$

Strong generators for a group of order 24:

$$\begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&3&0\\ 0&0&0&3 \end{bmatrix}, \begin{bmatrix} 0&1&0&0\\ 4&0&0&0\\ 0&0&0&2\\ 0&0&4&0 \end{bmatrix}, \begin{bmatrix} 1&2&0&0\\ 6&6&0&0\\ 0&0&3&5\\ 0&0&4&4 \end{bmatrix}$$

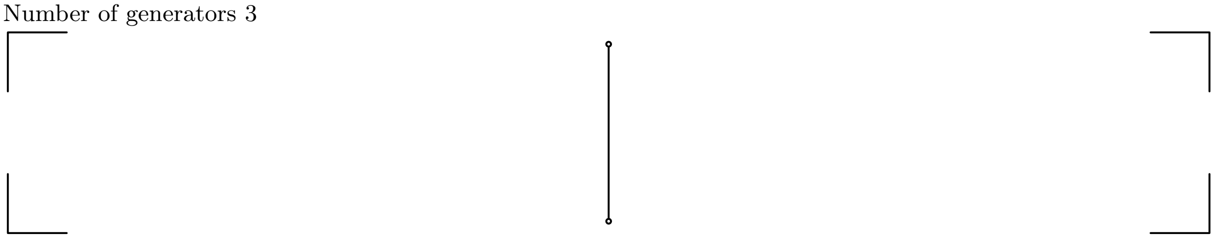
There are 10 extensions
Number of generators 3
Generators for the Schreier trees:
Generators for a group of order 24:

$$\begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&3&0\\ 0&0&0&3 \end{bmatrix} \begin{bmatrix} 0&1&0&0\\ 4&0&0&0\\ 0&0&0&2\\ 0&0&4&0 \end{bmatrix} \begin{bmatrix} 1&2&0&0\\ 6&6&0&0\\ 0&0&3&5\\ 0&0&4&4 \end{bmatrix}$$

Orbit 0 / 10: Point 266 lies in an orbit of length 2 with average word length 1.5 $H_3 = 1$
Orbit 1 / 10: Point 267 lies in an orbit of length 12 with average word length 2.83333 $H_3 = 3.20983$
Orbit 2 / 10: Point 273 lies in an orbit of length 4 with average word length 2 $H_3 = 1.89279$
Orbit 3 / 10: Point 274 lies in an orbit of length 24 with average word length 4 $H_3 = 4.15465$
Orbit 4 / 10: Point 287 lies in an orbit of length 2 with average word length 1.5 $H_3 = 1$
Orbit 5 / 10: Point 288 lies in an orbit of length 12 with average word length 3.5 $H_3 = 3.40217$
Orbit 6 / 10: Point 364 lies in an orbit of length 4 with average word length 2 $H_3 = 1.89279$
Orbit 7 / 10: Point 365 lies in an orbit of length 24 with average word length 4 $H_3 = 4.15465$
Orbit 8 / 10: Point 371 lies in an orbit of length 4 with average word length 2 $H_3 = 1.89279$
Orbit 9 / 10: Point 372 lies in an orbit of length 24 with average word length 4 $H_3 = 4.15465$

Node 7 at Level 4 Orbit 3 / 8 Tree 0 / 10

Number of generators 3

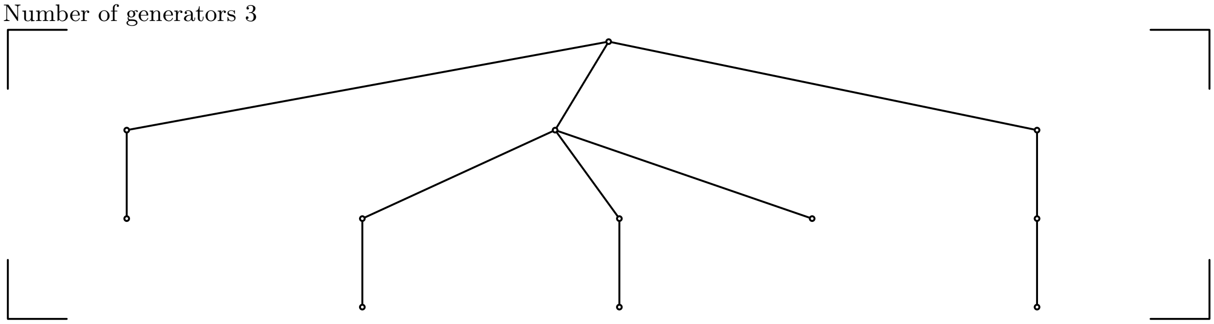


Extension number 0
Orbit representative 266
Flag orbit length 2
Flag orbit is fused to node 6 extension 2
Fusion element:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 6 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 3 & 1 \end{bmatrix}$$

Node 7 at Level 4 Orbit 3 / 8 Tree 1 / 10

Number of generators 3

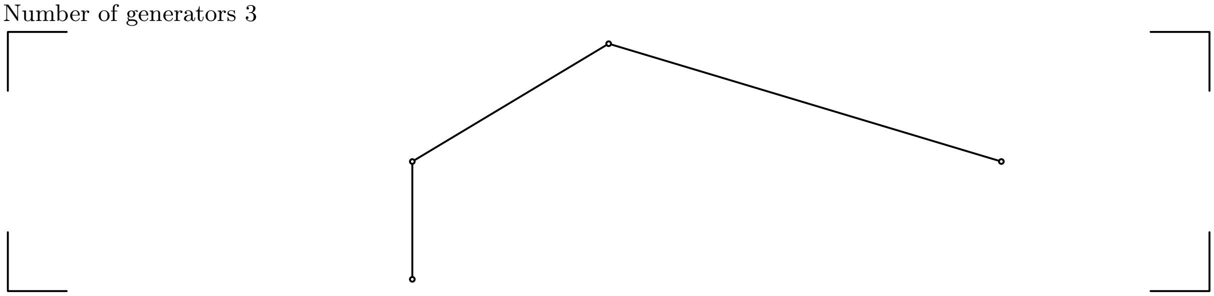


Extension number 1
Orbit representative 267
Flag orbit length 12
Flag orbit is fused to node 6 extension 3
Fusion element:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 6 & 5 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 4 & 2 & 5 & 4 \end{bmatrix}$$

Node 7 at Level 4 Orbit 3 / 8 Tree 2 / 10

Number of generators 3

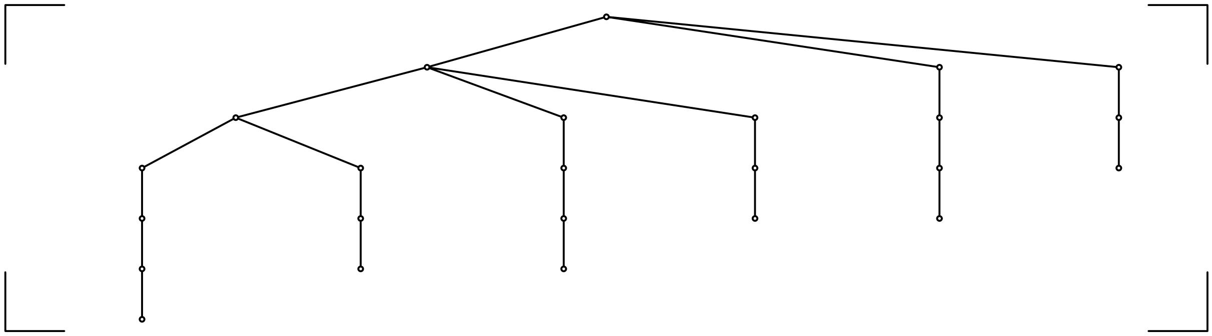


Extension number 2
Orbit representative 273
Flag orbit length 4
Flag orbit is fused to node 6 extension 6
Fusion element:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 2 & 6 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 6 & 4 \end{bmatrix}$$

Node 7 at Level 4 Orbit 3 / 8 Tree 3 / 10

Number of generators 3

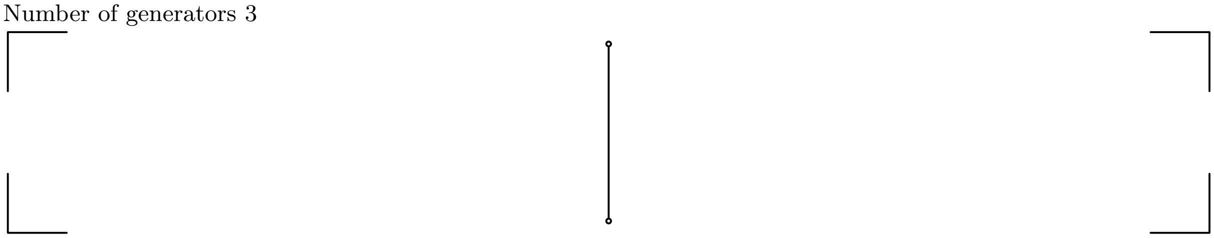


Extension number 3
Orbit representative 274
Flag orbit length 24
Flag orbit is fused to node 6 extension 7
Fusion element:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 2 & 6 & 0 & 0 \\ 0 & 6 & 0 & 6 \\ 4 & 1 & 5 & 1 \end{bmatrix}$$

Node 7 at Level 4 Orbit 3 / 8 Tree 4 / 10

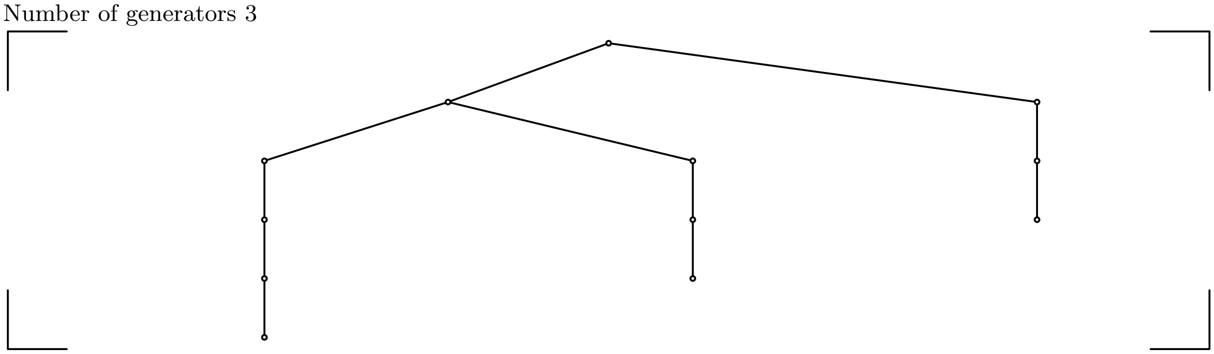
Number of generators 3



Extension number 4
Orbit representative 287
Flag orbit length 2
Flag orbit is defining new orbit 24 at level 5

Node 7 at Level 4 Orbit 3 / 8 Tree 5 / 10

Number of generators 3

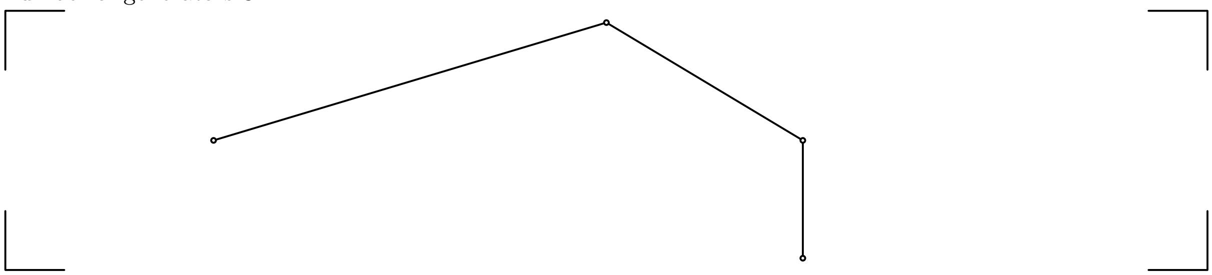


Extension number 5
Orbit representative 288
Flag orbit length 12

Flag orbit is defining new orbit 25 at level 5

Node 7 at Level 4 Orbit 3 / 8 Tree 6 / 10

Number of generators 3

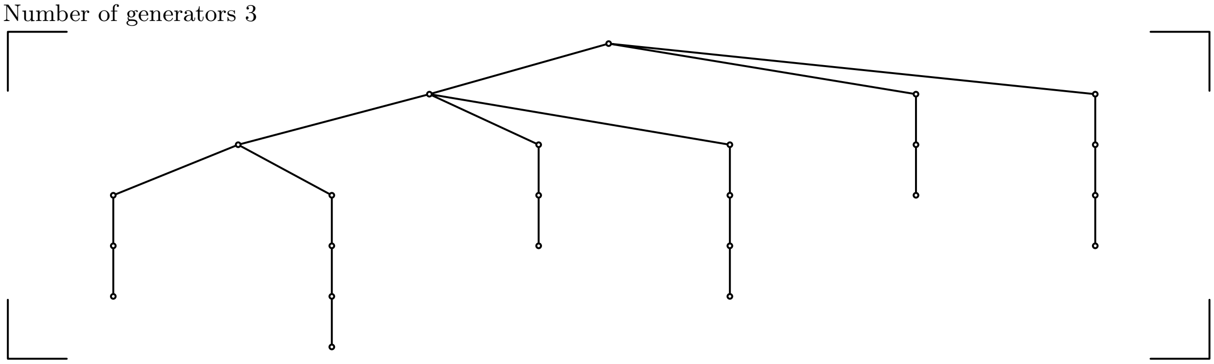


Extension number 6
Orbit representative 364
Flag orbit length 4
Flag orbit is fused to node 6 extension 6
Fusion element:

$$\begin{bmatrix} 1 & 5 & 0 & 0 \\ 5 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 2 & 6 \end{bmatrix}$$

Node 7 at Level 4 Orbit 3 / 8 Tree 7 / 10

Number of generators 3

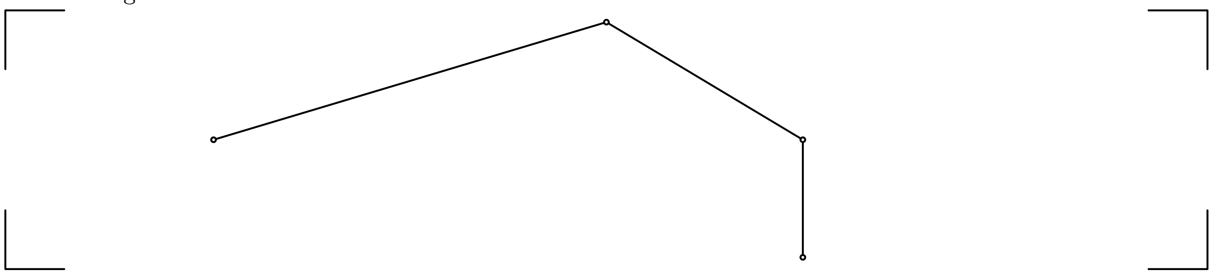


Extension number 7
Orbit representative 365
Flag orbit length 24
Flag orbit is fused to node 6 extension 7
Fusion element:

$$\begin{bmatrix} 1 & 5 & 0 & 0 \\ 5 & 1 & 0 & 0 \\ 6 & 2 & 5 & 6 \\ 6 & 3 & 3 & 2 \end{bmatrix}$$

Node 7 at Level 4 Orbit 3 / 8 Tree 8 / 10

Number of generators 3

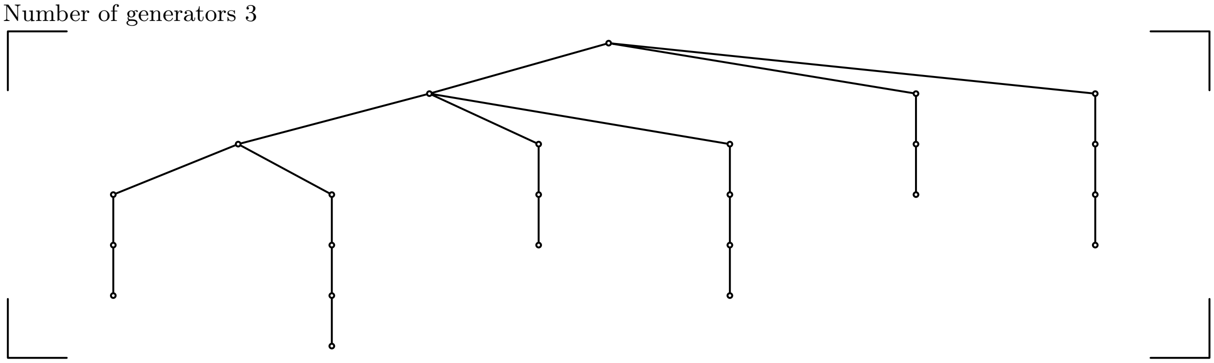


Extension number 8
Orbit representative 371
Flag orbit length 4
Flag orbit is fused to node 4 extension 3
Fusion element:

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

Node 7 at Level 4 Orbit 3 / 8 Tree 9 / 10

Number of generators 3



Extension number 9
Orbit representative 372
Flag orbit length 24
Flag orbit is fused to node 5 extension 5
Fusion element:

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 4 & 0 & 0 & 0 \\ 4 & 4 & 3 & 3 \\ 2 & 0 & 6 & 0 \end{bmatrix}$$

Node 8 at Level 4 Orbit 4 / 8

$$\{0, 3, 161, 266\}_{24}$$

Strong generators for a group of order 24:

$$\begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&2&0 \\ 0&0&0&2 \end{bmatrix}, \begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&3&0 \\ 0&0&0&3 \end{bmatrix}, \begin{bmatrix} 1&4&0&0 \\ 6&6&0&0 \\ 0&0&3&1 \\ 0&0&4&4 \end{bmatrix},$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 5 & 6 & 0 & 0 \\ 0 & 0 & 6 & 6 \\ 0 & 0 & 3 & 1 \end{bmatrix}$$

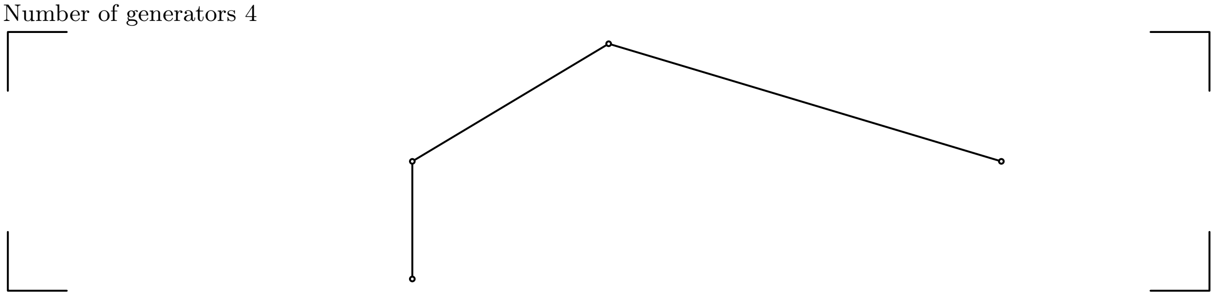
There are 8 extensions
Number of generators 4
Generators for the Schreier trees:
Generators for a group of order 24:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 4 & 0 & 0 \\ 6 & 6 & 0 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 4 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 5 & 6 & 0 & 0 \\ 0 & 0 & 6 & 6 \\ 0 & 0 & 3 & 1 \end{bmatrix}$$

Orbit 0 / 8: Point 224 lies in an orbit of length 4 with average word length 2 $H_4 = 1.5$
Orbit 1 / 8: Point 225 lies in an orbit of length 24 with average word length 3.33333 $H_4 = 3.16096$
Orbit 2 / 8: Point 231 lies in an orbit of length 4 with average word length 2 $H_4 = 1.5$
Orbit 3 / 8: Point 232 lies in an orbit of length 24 with average word length 3.33333 $H_4 = 3.16096$
Orbit 4 / 8: Point 238 lies in an orbit of length 4 with average word length 2 $H_4 = 1.5$
Orbit 5 / 8: Point 239 lies in an orbit of length 24 with average word length 3.33333 $H_4 = 3.16096$
Orbit 6 / 8: Point 245 lies in an orbit of length 4 with average word length 2 $H_4 = 1.5$
Orbit 7 / 8: Point 246 lies in an orbit of length 24 with average word length 3.33333 $H_4 = 3.16096$

Node 8 at Level 4 Orbit 4 / 8 Tree 0 / 8

Number of generators 4

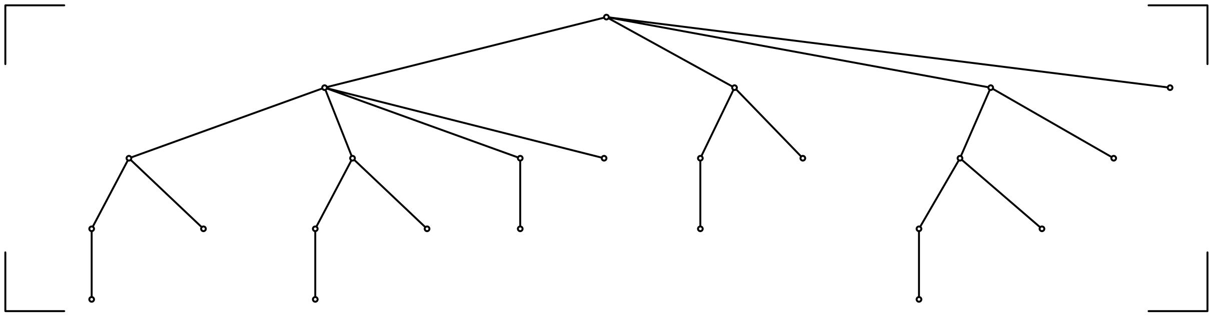


Extension number 0
Orbit representative 224
Flag orbit length 4
Flag orbit is fused to node 4 extension 3
Fusion element:

$$\begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 6 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Node 8 at Level 4 Orbit 4 / 8 Tree 1 / 8

Number of generators 4

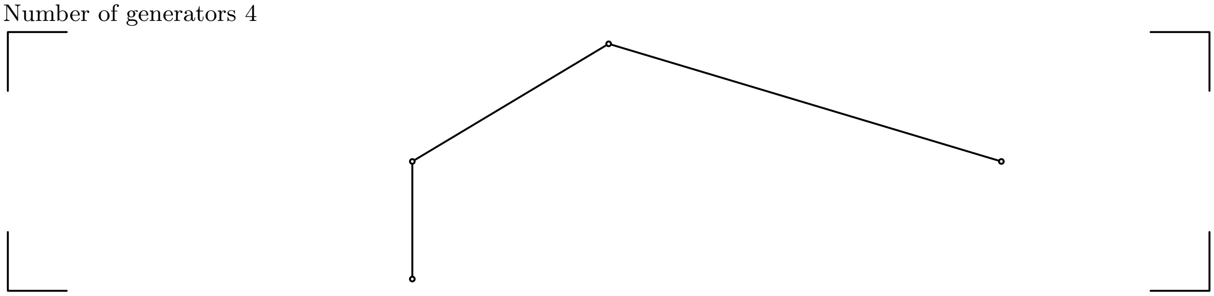


Extension number 1
Orbit representative 225
Flag orbit length 24
Flag orbit is fused to node 5 extension 5
Fusion element:

$$\begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 3 & 2 & 3 \\ 0 & 5 & 0 & 5 \end{bmatrix}$$

Node 8 at Level 4 Orbit 4 / 8 Tree 2 / 8

Number of generators 4

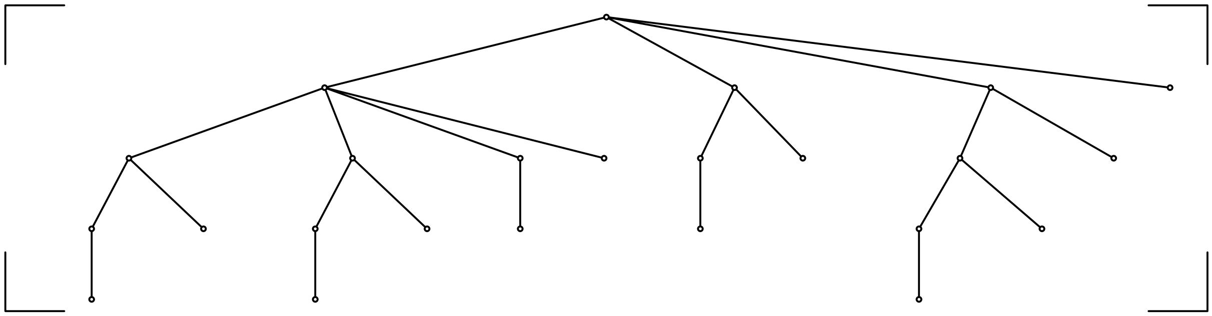


Extension number 2
Orbit representative 231
Flag orbit length 4
Flag orbit is fused to node 6 extension 2
Fusion element:

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

Node 8 at Level 4 Orbit 4 / 8 Tree 3 / 8

Number of generators 4

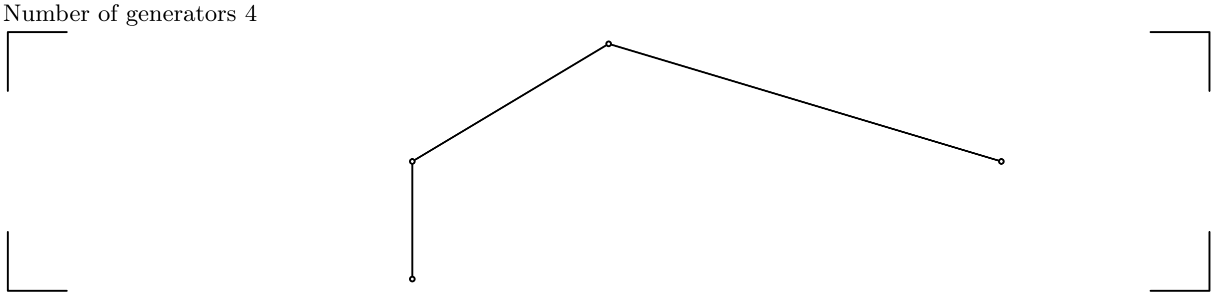


Extension number 3
Orbit representative 232
Flag orbit length 24
Flag orbit is fused to node 6 extension 3
Fusion element:

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 2 & 2 & 4 & 6 \\ 0 & 1 & 0 & 3 \end{bmatrix}$$

Node 8 at Level 4 Orbit 4 / 8 Tree 4 / 8

Number of generators 4

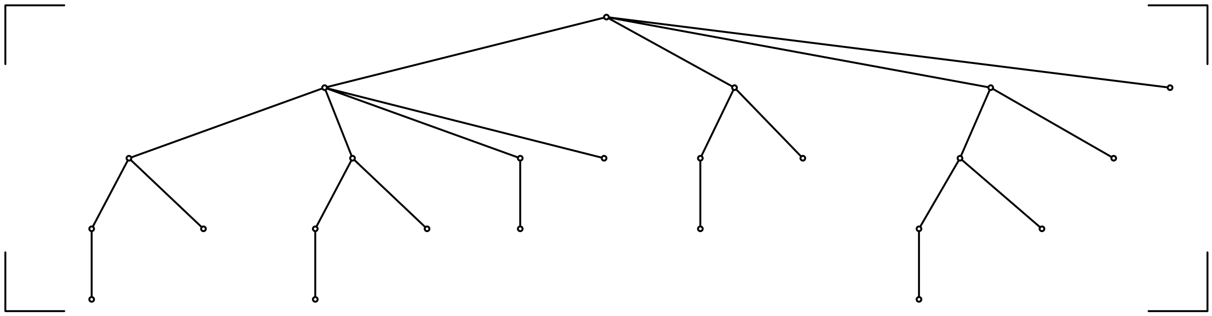


Extension number 4
Orbit representative 238
Flag orbit length 4
Flag orbit is fused to node 4 extension 2
Fusion element:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 5 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 4 & 6 \end{bmatrix}$$

Node 8 at Level 4 Orbit 4 / 8 Tree 5 / 8

Number of generators 4

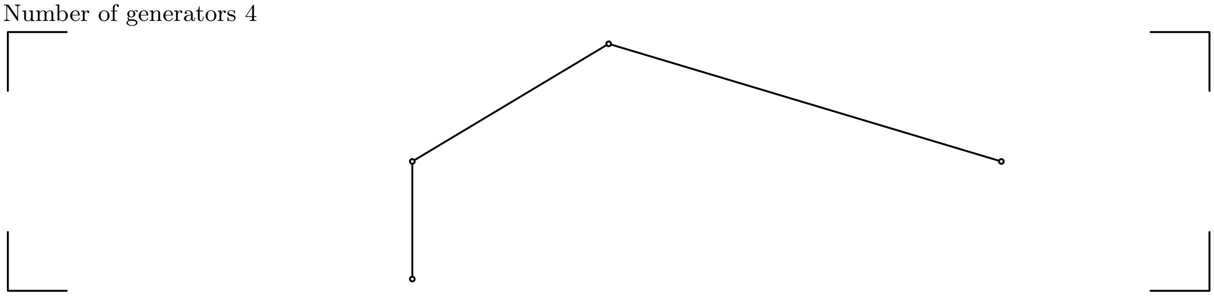


Extension number 5
Orbit representative 239
Flag orbit length 24
Flag orbit is fused to node 5 extension 4
Fusion element:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 5 & 3 & 0 & 0 \\ 3 & 0 & 2 & 0 \\ 5 & 3 & 1 & 5 \end{bmatrix}$$

Node 8 at Level 4 Orbit 4 / 8 Tree 6 / 8

Number of generators 4

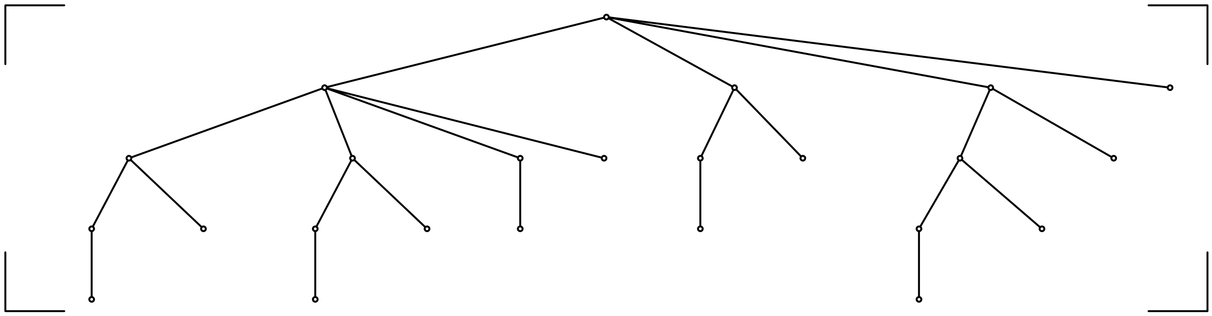


Extension number 6
Orbit representative 245
Flag orbit length 4
Flag orbit is fused to node 6 extension 6
Fusion element:

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 5 & 1 & 0 & 0 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 5 & 1 \end{bmatrix}$$

Node 8 at Level 4 Orbit 4 / 8 Tree 7 / 8

Number of generators 4



Extension number 7
Orbit representative 246
Flag orbit length 24
Flag orbit is fused to node 6 extension 7
Fusion element:

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 5 & 1 & 0 & 0 \\ 3 & 3 & 1 & 1 \\ 5 & 2 & 4 & 5 \end{bmatrix}$$

Node 9 at Level 4 Orbit 5 / 8

$$\{0, 3, 161, 273\}_{504}$$

Strong generators for a group of order 504:

$$\begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&3&0 \\ 0&0&0&3 \end{bmatrix}, \begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 1&0&5&0 \\ 0&1&0&5 \end{bmatrix}, \begin{bmatrix} 1&0&0&0 \\ 6&2&0&0 \\ 0&0&6&0 \\ 0&0&1&5 \end{bmatrix},$$

$$\begin{bmatrix} 1&1&0&0 \\ 6&0&0&0 \\ 6&6&6&6 \\ 1&0&1&0 \end{bmatrix}, \begin{bmatrix} 1&5&0&0 \\ 0&2&0&0 \\ 3&1&3&1 \\ 0&6&0&6 \end{bmatrix}$$

There are 3 extensions
Number of generators 5
Generators for the Schreier trees:
Generators for a group of order 504:

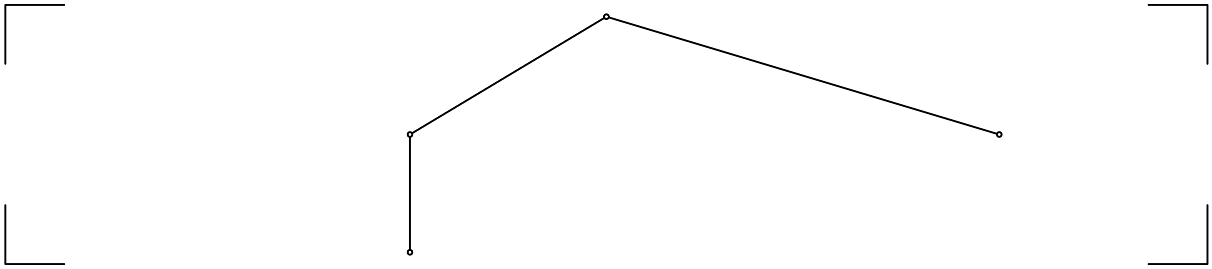
$$\begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&3&0 \\ 0&0&0&3 \end{bmatrix} \begin{bmatrix} 1&1&0&0 \\ 6&0&0&0 \\ 5&5&5&5 \\ 2&0&2&0 \end{bmatrix} \begin{bmatrix} 1&0&0&0 \\ 6&2&0&0 \\ 0&0&6&0 \\ 0&0&1&5 \end{bmatrix} \begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 2&0&1&0 \\ 0&2&0&1 \end{bmatrix}$$

$$\begin{bmatrix} 1&5&0&0 \\ 0&2&0&0 \\ 3&1&3&1 \\ 0&6&0&6 \end{bmatrix}$$

Orbit 0 / 3: Point 217 lies in an orbit of length 4 with average word length 2 $H_5 = 1.29203$
Orbit 1 / 3: Point 218 lies in an orbit of length 24 with average word length 3.25 $H_5 = 2.70698$
Orbit 2 / 3: Point 231 lies in an orbit of length 84 with average word length 3.79762 $H_5 = 3.58211$

Node 9 at Level 4 Orbit 5 / 8 Tree 0 / 3

Number of generators 5

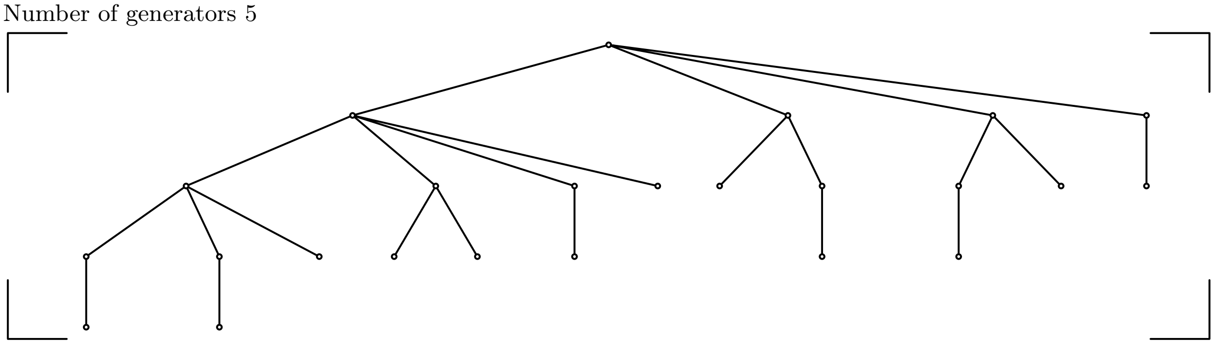


Extension number 0
Orbit representative 217
Flag orbit length 4
Flag orbit is fused to node 4 extension 0
Fusion element:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 4 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Node 9 at Level 4 Orbit 5 / 8 Tree 1 / 3

Number of generators 5

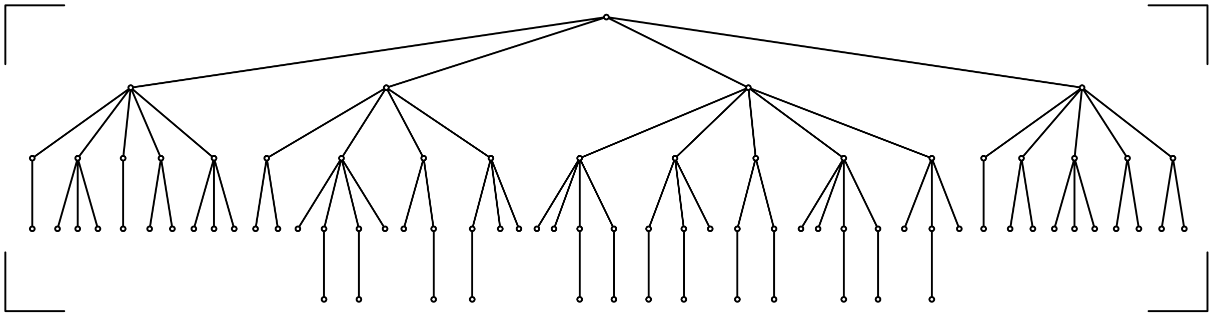


Extension number 1
Orbit representative 218
Flag orbit length 24
Flag orbit is fused to node 5 extension 0
Fusion element:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Node 9 at Level 4 Orbit 5 / 8 Tree 2 / 3

Number of generators 5



Extension number 2
Orbit representative 231
Flag orbit length 84
Flag orbit is fused to node 6 extension 6
Fusion element:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 2 & 6 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

Node 10 at Level 4 Orbit 6 / 8

$$\{0, 3, 161, 274\}_{84}$$

Strong generators for a group of order 84:

$$\begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 6&0&1&0 \\ 0&6&0&1 \end{bmatrix}, \begin{bmatrix} 1&0&0&0 \\ 6&2&0&0 \\ 6&0&2&0 \\ 1&4&5&4 \end{bmatrix}, \begin{bmatrix} 1&1&0&0 \\ 6&0&0&0 \\ 0&0&4&4 \\ 0&0&3&0 \end{bmatrix},$$

$$\begin{bmatrix} 1&5&0&0 \\ 6&6&0&0 \\ 5&1&1&5 \\ 2&2&6&6 \end{bmatrix}$$

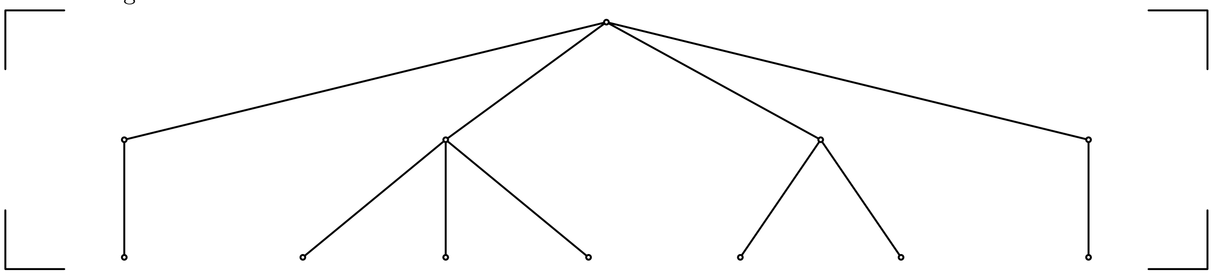
There are 4 extensions
Number of generators 4
Generators for the Schreier trees:
Generators for a group of order 84:

$$\begin{bmatrix} 1&5&0&0 \\ 6&6&0&0 \\ 5&1&1&5 \\ 2&2&6&6 \end{bmatrix} \begin{bmatrix} 1&0&0&0 \\ 6&2&0&0 \\ 6&0&2&0 \\ 1&4&5&4 \end{bmatrix} \begin{bmatrix} 0&1&0&0 \\ 5&4&0&0 \\ 0&4&0&4 \\ 0&3&6&2 \end{bmatrix} \begin{bmatrix} 0&1&0&0 \\ 3&0&0&0 \\ 0&1&0&1 \\ 2&0&3&0 \end{bmatrix}$$

Orbit 0 / 4: Point 217 lies in an orbit of length 12 with average word length 2.5 $H_4 = 2.45345$
Orbit 1 / 4: Point 218 lies in an orbit of length 12 with average word length 2.5 $H_4 = 2.45345$
Orbit 2 / 4: Point 220 lies in an orbit of length 4 with average word length 2 $H_4 = 1.5$
Orbit 3 / 4: Point 231 lies in an orbit of length 84 with average word length 4.04762 $H_4 = 4.2047$

Node 10 at Level 4 Orbit 6 / 8 Tree 0 / 4

Number of generators 4

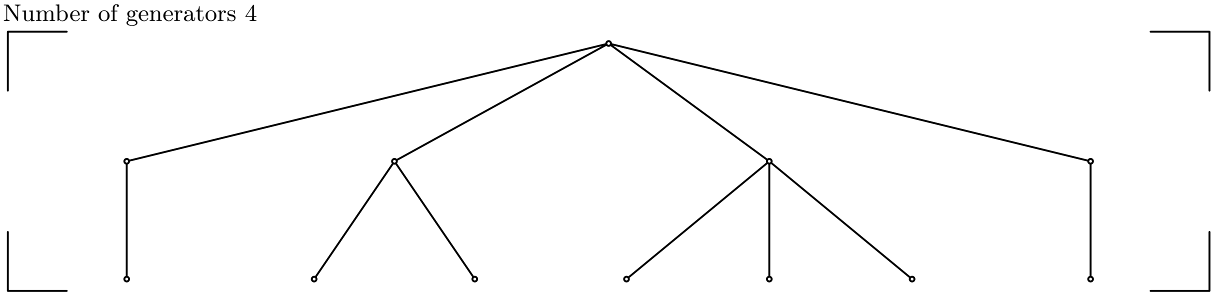


Extension number 0
Orbit representative 217
Flag orbit length 12
Flag orbit is fused to node 4 extension 1
Fusion element:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 6 \\ 6 & 0 & 5 & 0 \end{bmatrix}$$

Node 10 at Level 4 Orbit 6 / 8 Tree 1 / 4

Number of generators 4

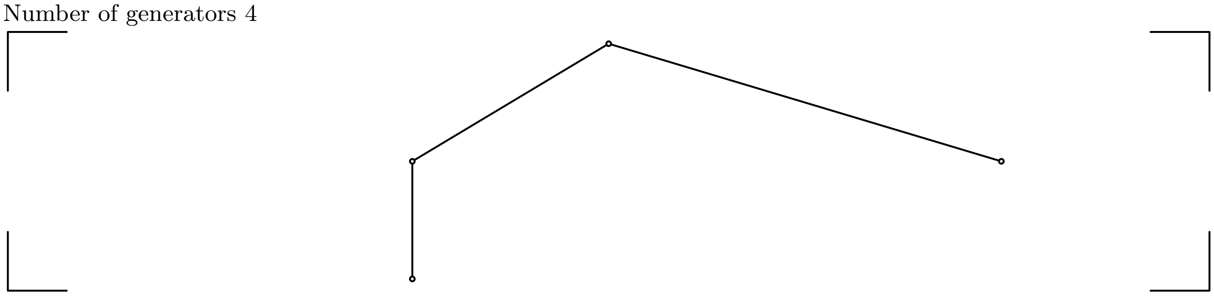


Extension number 1
Orbit representative 218
Flag orbit length 12
Flag orbit is fused to node 5 extension 1
Fusion element:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 4 & 6 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 6 & 4 & 4 & 6 \end{bmatrix}$$

Node 10 at Level 4 Orbit 6 / 8 Tree 2 / 4

Number of generators 4

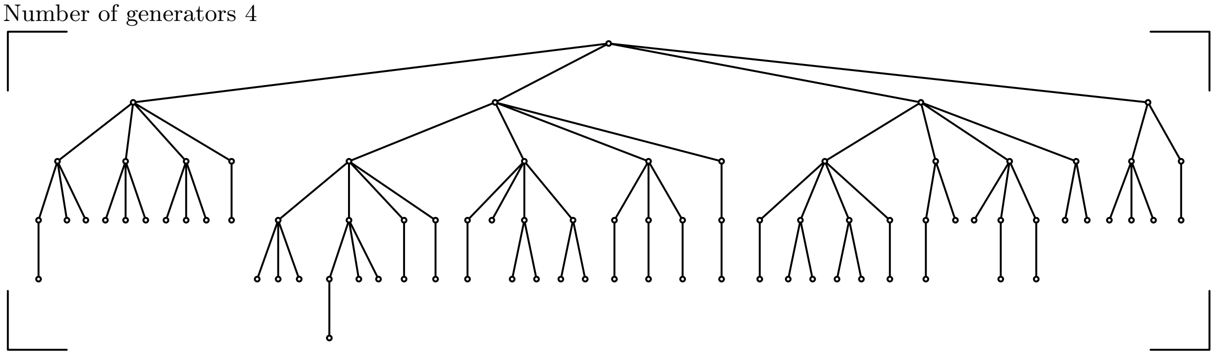


Extension number 2
Orbit representative 220
Flag orbit length 4
Flag orbit is fused to node 5 extension 0
Fusion element:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 4 & 6 & 0 & 0 \\ 4 & 0 & 5 & 0 \\ 3 & 1 & 6 & 2 \end{bmatrix}$$

Node 10 at Level 4 Orbit 6 / 8 Tree 3 / 4

Number of generators 4



Extension number 3
Orbit representative 231
Flag orbit length 84
Flag orbit is fused to node 6 extension 7
Fusion element:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 2 & 6 & 0 & 0 \\ 0 & 6 & 0 & 6 \\ 4 & 1 & 5 & 1 \end{bmatrix}$$

Node 11 at Level 4 Orbit 7 / 8

$$\{0, 3, 161, 287\}_{72}$$

Strong generators for a group of order 72:

$$\begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&2&0 \\ 0&0&0&2 \end{bmatrix}, \begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&5&0 \\ 0&0&0&5 \end{bmatrix}, \begin{bmatrix} 1&0&0&0 \\ 6&4&0&0 \\ 0&0&6&0 \\ 0&0&1&5 \end{bmatrix},$$

$$\begin{bmatrix} 1&1&0&0 \\ 6&0&0&0 \\ 0&0&4&4 \\ 0&0&3&0 \end{bmatrix}, \begin{bmatrix} 1&3&0&0 \\ 0&4&0&0 \\ 0&0&6&2 \\ 0&0&0&5 \end{bmatrix}$$

There are 4 extensions
Number of generators 5
Generators for the Schreier trees:
Generators for a group of order 72:

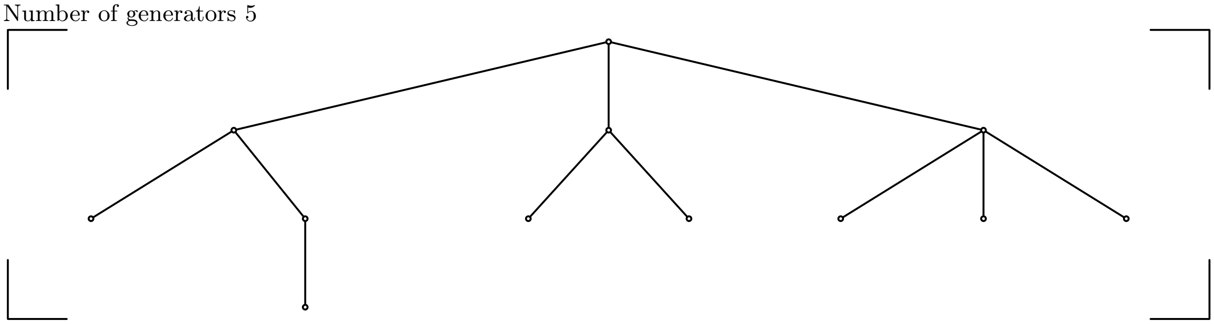
$$\begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&2&0 \\ 0&0&0&2 \end{bmatrix} \begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&5&0 \\ 0&0&0&5 \end{bmatrix} \begin{bmatrix} 0&1&0&0 \\ 3&2&0&0 \\ 0&0&0&5 \\ 0&0&4&6 \end{bmatrix} \begin{bmatrix} 1&1&0&0 \\ 6&0&0&0 \\ 0&0&4&4 \\ 0&0&3&0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 6 & 4 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

Orbit 0 / 4: Point 217 lies in an orbit of length 12 with average word length 2.66667 $H_5 = 2.15338$
Orbit 1 / 4: Point 218 lies in an orbit of length 72 with average word length 3.88889 $H_5 = 3.50109$
Orbit 2 / 4: Point 231 lies in an orbit of length 4 with average word length 2.25 $H_5 = 1.36521$
Orbit 3 / 4: Point 232 lies in an orbit of length 24 with average word length 3 $H_5 = 2.65724$

Node 11 at Level 4 Orbit 7 / 8 Tree 0 / 4

Number of generators 5

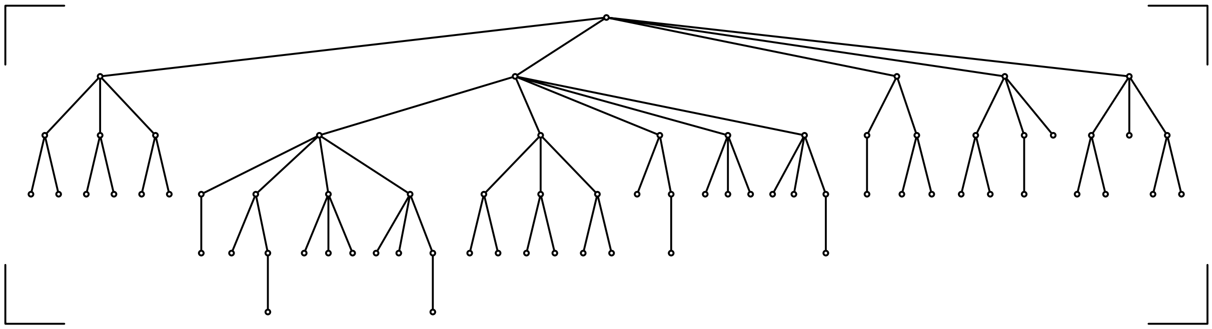


Extension number 0
Orbit representative 217
Flag orbit length 12
Flag orbit is fused to node 4 extension 3
Fusion element:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Node 11 at Level 4 Orbit 7 / 8 Tree 1 / 4

Number of generators 5

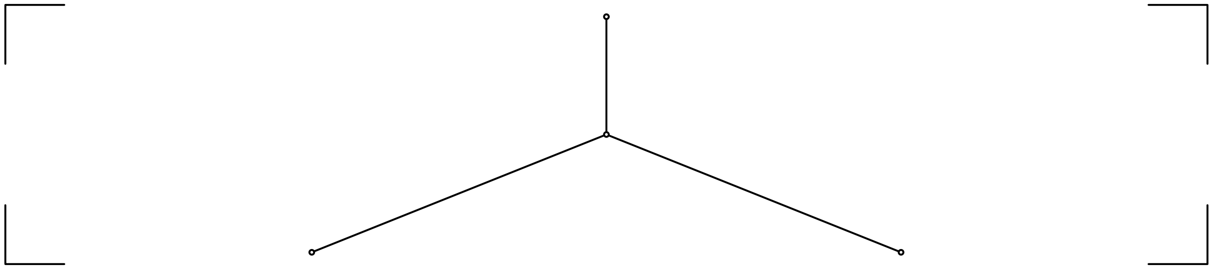


Extension number 1
Orbit representative 218
Flag orbit length 72
Flag orbit is fused to node 5 extension 5
Fusion element:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Node 11 at Level 4 Orbit 7 / 8 Tree 2 / 4

Number of generators 5

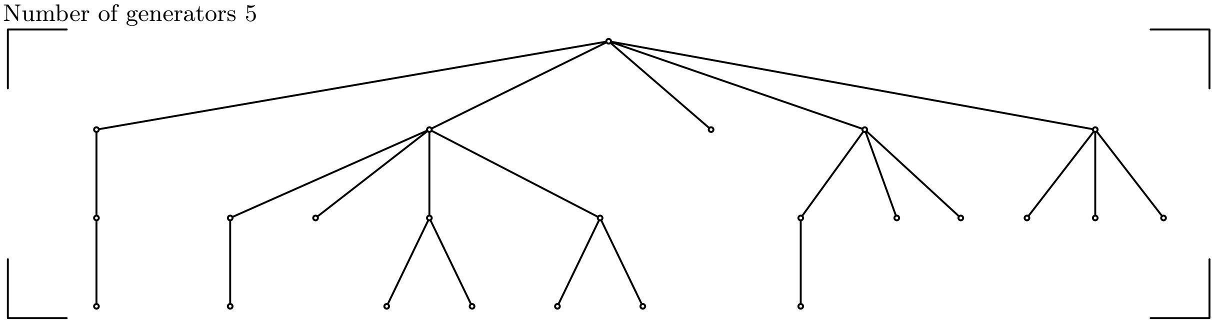


Extension number 2
Orbit representative 231
Flag orbit length 4
Flag orbit is fused to node 7 extension 4
Fusion element:

$$\begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 6 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Node 11 at Level 4 Orbit 7 / 8 Tree 3 / 4

Number of generators 5



Extension number 3
Orbit representative 232
Flag orbit length 24
Flag orbit is fused to node 7 extension 5
Fusion element:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 6 & 0 & 0 \\ 5 & 0 & 1 & 0 \\ 6 & 1 & 4 & 6 \end{bmatrix}$$

5.6 Stabilizers and Schreier trees at level 5

Node 12 at Level 5 Orbit 0 / 14

$$\{0, 3, 161, 217, 273\}_{252}$$

Strong generators for a group of order 252:

$$\begin{bmatrix} 1&0&0&0\\0&1&0&0\\0&0&3&0\\0&0&0&3 \end{bmatrix}, \begin{bmatrix} 1&0&0&0\\0&1&0&0\\2&0&1&0\\0&2&0&1 \end{bmatrix}, \begin{bmatrix} 1&0&0&0\\4&6&0&0\\5&0&2&0\\6&2&1&5 \end{bmatrix},$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 6 & 0 & 3 \\ 5 & 0 & 6 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 5 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

There are 0 extensions
Number of generators 5

Node 13 at Level 5 Orbit 1 / 14

$$\{0, 3, 161, 217, 274\}_{14}$$

Strong generators for a group of order 14:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 5 & 0 \end{bmatrix}$$

There are 0 extensions
Number of generators 2

Node 14 at Level 5 Orbit 2 / 14

$$\{0, 3, 161, 217, 280\}_{12}$$

Strong generators for a group of order 12:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 6 & 0 \end{bmatrix}$$

There are 0 extensions
Number of generators 2

Node 15 at Level 5 Orbit 3 / 14

$$\{0, 3, 161, 217, 287\}_{6}$$

Strong generators for a group of order 6:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

There are 0 extensions
Number of generators 1

Node 16 at Level 5 Orbit 4 / 14

$$\{0, 3, 161, 218, 273\}_{21}$$

Strong generators for a group of order 21:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 6 & 0 & 1 & 0 \\ 0 & 6 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 5 & 0 & 0 \\ 4 & 0 & 0 & 0 \\ 5 & 4 & 4 & 6 \\ 6 & 0 & 2 & 0 \end{bmatrix}$$

There are 0 extensions
Number of generators 2

Node 17 at Level 5 Orbit 5 / 14

$$\{0, 3, 161, 218, 274\}_{14}$$

Strong generators for a group of order 14:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 4 & 6 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 6 & 4 & 4 & 6 \end{bmatrix}$$

There are 0 extensions
Number of generators 2

Node 18 at Level 5 Orbit 6 / 14

$$\{0, 3, 161, 218, 280\}_{2}$$

Strong generators for a group of order 2:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 3 & 0 & 2 & 0 \end{bmatrix}$$

There are 0 extensions
Number of generators 1

Node 19 at Level 5 Orbit 7 / 14

$$\{0, 3, 161, 218, 287\}_{1}$$

Strong generators for a group of order 1:

There are 0 extensions
Number of generators 0

Node 20 at Level 5 Orbit 8 / 14

$$\{0, 3, 161, 224, 280\}_{12}$$

Strong generators for a group of order 12:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 6 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

There are 0 extensions
Number of generators 2

Node 21 at Level 5 Orbit 9 / 14

$$\{0, 3, 161, 224, 281\}_{2}$$

Strong generators for a group of order 2:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 6 & 0 & 0 \\ 2 & 0 & 6 & 0 \\ 2 & 6 & 3 & 1 \end{bmatrix}$$

There are 0 extensions
Number of generators 1

Node 22 at Level 5 Orbit 10 / 14

$$\{0, 3, 161, 224, 385\}_6$$

Strong generators for a group of order 6:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

There are 0 extensions
Number of generators 2

Node 23 at Level 5 Orbit 11 / 14

$$\{0, 3, 161, 224, 386\}_1$$

Strong generators for a group of order 1:

There are 0 extensions
Number of generators 0

Node 24 at Level 5 Orbit 12 / 14

$$\{0, 3, 161, 231, 287\}_{36}$$

Strong generators for a group of order 36:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 6 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 6 & 2 \end{bmatrix},$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 4 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

There are 0 extensions
Number of generators 5

Node 25 at Level 5 Orbit 13 / 14

$$\{0, 3, 161, 231, 288\}_6$$

Strong generators for a group of order 6:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 6 & 0 & 0 \\ 5 & 0 & 6 & 0 \\ 0 & 3 & 3 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 0 \\ 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 6 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 5 & 0 & 6 & 2 \\ 0 & 3 & 0 & 1 \end{bmatrix}$$

There are 0 extensions
Number of generators 3

Classification of $5+1$ Configurations in $\mathrm{PG}(3,7)$

The order of the group is 4635182361600
The group has 14 orbits on five plus one configurations in PG(3, 7).

Of these, 5 impose 19 conditions.
Of these, 2 are associated with double sixes. They are:
0/2 is orbit 9/14 $\{0, 3, 161, 224, 281\}_2$ orbit length 813189888
1/2 is orbit 13/14 $\{0, 3, 161, 231, 288\}_6$ orbit length 271063296
The overall number of five plus one configurations associated with double sixes in PG(3, 7) is: 1084253184

Flag orbits for double sixes

The number of primary orbits below is 14
The number of primary orbits above is 2
The number of flag orbits is 2
The flag orbits are:

(1) Flag orbit 0 / 2 down=(9,0) up=(0,-1) is (0, 3, 161, 224, 281, 49, 2800, 107, 338, 2655, 0, 49, 2800, 107, 338, 2655, 795, 2593, 216, 135, 64, 2849, 0) with a stabilizer of order 2
Strong generators for a group of order 2:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 6 & 0 & 0 \\ 2 & 0 & 6 & 0 \\ 2 & 6 & 3 & 1 \end{bmatrix}$$

(2) Flag orbit 1 / 2 down=(13,0) up=(1,-1) is (0, 3, 161, 231, 288, 49, 2800, 107, 167, 2658, 0, 49, 2800, 107, 167, 2658, 1875, 2657, 264, 213, 128, 2849, 0) with a stabilizer of order 6
Strong generators for a group of order 6:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 6 & 0 & 0 \\ 5 & 0 & 6 & 0 \\ 0 & 3 & 3 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 0 \\ 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 6 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 5 & 0 & 6 & 2 \\ 0 & 3 & 0 & 1 \end{bmatrix}$$

Double Sixes

The order of the group is 4635182361600
The group has 2 orbits:

The orbits are:

(1) 0/2 $\{49, 2800, 107, 338, 2655, 795, 2593, 216, 135, 64, 2849, 0\}_{24}$ orbit length 193132598400
Strong generators for a group of order 24:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 4 & 0 & 1 & 0 \\ 3 & 1 & 0 & 0 \\ 1 & 5 & 2 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 6 & 0 & 0 \\ 2 & 0 & 6 & 0 \\ 2 & 6 & 3 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 6 & 3 & 2 \\ 3 & 5 & 2 & 3 \\ 2 & 5 & 0 & 2 \\ 2 & 6 & 3 & 1 \end{bmatrix},$$

$$\begin{bmatrix} 1 & 5 & 2 & 1 \\ 4 & 0 & 1 & 0 \\ 3 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

(2) 1/2 $\{49, 2800, 107, 167, 2658, 1875, 2657, 264, 213, 128, 2849, 0\}_{72}$ orbit length 64377532800
Strong generators for a group of order 72:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 6 & 0 & 5 & 0 \\ 3 & 3 & 0 & 0 \\ 4 & 4 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 6 & 0 & 0 \\ 5 & 0 & 6 & 0 \\ 0 & 3 & 3 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 5 & 2 & 3 \\ 6 & 1 & 4 & 6 \\ 2 & 1 & 4 & 2 \\ 6 & 3 & 4 & 4 \end{bmatrix},$$

$$\begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 6 & 6 \\ 2 & 1 & 4 & 2 \\ 5 & 5 & 3 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 4 & 1 & 6 \\ 2 & 1 & 1 & 3 \\ 0 & 6 & 6 & 2 \\ 1 & 0 & 4 & 6 \end{bmatrix}$$

The overall number of objects is: 257510131200

Flag orbits for surfaces

The number of primary orbits below is 2
The number of primary orbits above is 1
The number of flag orbits is 2
The flag orbits are:

(1) Flag orbit 0 / 2 down=(0,0) up=(0,-1) is (49, 2800, 107, 338, 2655, 795, 2593, 216, 135, 64, 2849, 0, 213, 255, 293, 2443, 344, 2529, 2458, 2815, 2429, 2433, 1705, 2744, 1533, 2706, 396) with a stabilizer of order 24
Strong generators for a group of order 24:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 4 & 0 & 1 & 0 \\ 3 & 1 & 0 & 0 \\ 1 & 5 & 2 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 6 & 0 & 0 \\ 2 & 0 & 6 & 0 \\ 2 & 6 & 3 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 6 & 3 & 2 \\ 3 & 5 & 2 & 3 \\ 2 & 5 & 0 & 2 \\ 2 & 6 & 3 & 1 \end{bmatrix},$$

$$\begin{bmatrix} 1 & 5 & 2 & 1 \\ 4 & 0 & 1 & 0 \\ 3 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

(2) Flag orbit 1 / 2 down=(1,0) up=(0,-1) fuse to 0 is (49, 2800, 107, 167, 2658, 1875, 2657, 264, 213, 128, 2849, 0, 263, 328, 357, 2443, 58, 2593, 2515, 2808, 2415, 2473, 910, 2744, 2161, 2708, 282) with a stabilizer of order 72
Strong generators for a group of order 72:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 6 & 0 & 5 & 0 \\ 3 & 3 & 0 & 0 \\ 4 & 4 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 6 & 0 & 0 \\ 5 & 0 & 6 & 0 \\ 0 & 3 & 3 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 5 & 2 & 3 \\ 6 & 1 & 4 & 6 \\ 2 & 1 & 4 & 2 \\ 6 & 3 & 4 & 4 \end{bmatrix},$$

$$\begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 6 & 6 \\ 2 & 1 & 4 & 2 \\ 5 & 5 & 3 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 4 & 1 & 6 \\ 2 & 1 & 1 & 3 \\ 0 & 6 & 6 & 2 \\ 1 & 0 & 4 & 6 \end{bmatrix}$$

Fusion element:

$$\begin{bmatrix} 1 & 0 & 6 & 6 \\ 6 & 0 & 4 & 1 \\ 0 & 2 & 2 & 2 \\ 1 & 5 & 3 & 5 \end{bmatrix}$$

Surfaces

The order of the group is 4635182361600
The group has 1 orbits:

The orbits are:

(1) 0/1 $\{49, 2800, 107, 338, 2655, 795, 2593, 216, 135, 64, 2849, 0, 213, 255, 293, 2443, 344, 2529, 2458, 2815, 2429, 2433, 1705, 2744, 1533, 2706, 396\}_{648}$ orbit length 7153059200
Strong generators for a group of order 648:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 6 & 0 \\ 0 & 1 & 0 & 6 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 4 & 0 & 1 & 0 \\ 6 & 6 & 0 & 0 \\ 3 & 2 & 6 & 6 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 5 & 5 & 3 & 1 \\ 6 & 0 & 5 & 0 \end{bmatrix},$$

$$\begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 5 \\ 5 & 4 & 5 & 4 \\ 0 & 3 & 1 & 5 \end{bmatrix}$$

The overall number of objects is: 7153059200

The Group $\mathrm{P\Gamma L}(4,7)$

The order of the group is 4635182361600

Cubic Surfaces with 27 Lines in $\mathrm{PG}(3,7)$

The order of the group is 4635182361600
The group has 1 orbits:

The orbits are:

(1) 0/1 $\{49, 2800, 107, 338, 2655, 795, 2593, 216, 135, 64, 2849, 0, 213, 255, 293, 2443, 344, 2529, 2458, 2815, 2429, 2433, 1705, 2744, 1533, 2706, 396\}_{648}$ orbit length 7153059200
Strong generators for a group of order 648:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 6 & 0 \\ 0 & 1 & 0 & 6 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 4 & 0 & 1 & 0 \\ 6 & 6 & 0 & 0 \\ 3 & 2 & 6 & 6 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 5 & 5 & 3 & 1 \\ 6 & 0 & 5 & 0 \end{bmatrix},$$

$$\begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 5 \\ 5 & 4 & 5 & 4 \\ 0 & 3 & 1 & 5 \end{bmatrix}$$

The overall number of objects is: 7153059200

Surface 7#0

The equation

The equation of the surface is :

$$X_0^2X_3 + X_1^2X_2 + 3X_1X_2^2 + X_0X_3^2 + 3X_0X_1X_2 + 2X_0X_1X_3 + 3X_1X_2X_3 = 0$$

Number of points on the surface 99
The automorphism group of the surface has order 648
The automorphism group is the following group
Strong generators for a group of order 648:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 6 & 0 \\ 0 & 1 & 0 & 6 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 4 & 0 & 1 & 0 \\ 6 & 6 & 0 & 0 \\ 3 & 2 & 6 & 6 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 5 & 5 & 3 & 1 \\ 6 & 0 & 5 & 0 \end{bmatrix},$$

$$\begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 5 \\ 5 & 4 & 5 & 4 \\ 0 & 3 & 1 & 5 \end{bmatrix}$$

General information

Plane types by points:

$$22^{18},\ 21^{27},\ 16^{81},\ 13^{108},\ 12^{162},\ 9^{4}$$

Type of pts on lines:

$$8^{27}$$

Type of lines on point:

$$3^{18},\ 2^{81}$$

Type iso of tritangent planes:

$$3^{27},\ 2^{18}$$

The 27 lines

$$\ell_0 = a_1 = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 3 & 6 \end{bmatrix}_{216}$$

$$\ell_1 = a_2 = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 3 \end{bmatrix}_{135}$$

$$\ell_2 = a_3 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{64}$$

$$\ell_3 = a_4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2849}$$

$$\ell_4 = a_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{0}$$

$$\ell_5 = a_6 = \begin{bmatrix} 1 & 0 & 3 & 6 \\ 0 & 1 & 0 & 4 \end{bmatrix}_{2593}$$

$$\ell_6 = b_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{2800}$$

$$\ell_7 = b_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{107}$$

$$\ell_8 = b_3 = \begin{bmatrix} 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}_{338}$$

$$\ell_9 = b_4 = \begin{bmatrix} 1 & 0 & 4 & 6 \\ 0 & 1 & 5 & 4 \end{bmatrix}_{2655}$$

$$\ell_{10} = b_5 = \begin{bmatrix} 1 & 6 & 0 & 1 \\ 0 & 0 & 1 & 5 \end{bmatrix}_{795}$$

$$\ell_{11} = b_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{49}$$

$$\ell_{12} = c_{12} = \begin{bmatrix} 1 & 0 & 2 & 6 \\ 0 & 1 & 0 & 3 \end{bmatrix}_{2529}$$

$$\ell_{13} = c_{13} = \begin{bmatrix} 1 & 0 & 1 & 6 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{2458}$$

$$\ell_{14} = c_{14} = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 6 \end{bmatrix}_{2815}$$

$$\ell_{15} = c_{15} = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 5 \end{bmatrix}_{2429}$$

$$\ell_{16} = c_{16} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 6 \end{bmatrix}_{213}$$

$$\ell_{17} = c_{23} = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 4 & 5 \end{bmatrix}_{2433}$$

$$\ell_{18} = c_{24} = \begin{bmatrix} 1 & 1 & 0 & 4 \\ 0 & 0 & 1 & 3 \end{bmatrix}_{1705}$$

$$\ell_{19} = c_{25} = \begin{bmatrix} 1 & 0 & 6 & 6 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{2744}$$

$$\ell_{20} = c_{26} = \begin{bmatrix} 1 & 0 & 4 & 0 \\ 0 & 1 & 6 & 3 \end{bmatrix}_{255}$$

$$\ell_{21} = c_{34} = \begin{bmatrix} 1 & 5 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{1533}$$

$$\ell_{22} = c_{35} = \begin{bmatrix} 1 & 0 & 5 & 6 \\ 0 & 1 & 6 & 3 \end{bmatrix}_{2706}$$

$$\ell_{23} = c_{36} = \begin{bmatrix} 1 & 0 & 5 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{293}$$

$$\ell_{24} = c_{45} = \begin{bmatrix} 1 & 6 & 0 & 0 \\ 0 & 0 & 1 & 5 \end{bmatrix}_{396}$$

$$\ell_{25} = c_{46} = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{2443}$$

$$\ell_{26} = c_{56} = \begin{bmatrix} 1 & 0 & 6 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix}_{344}$$

Points on surface

All Points

The surface has 99 points:

	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	10	11	12	13	14	15	16	18	27	36
2	38	47	56	64	65	69	72	74	79	86
3	91	93	96	100	101	107	113	114	121	122
4	129	130	137	138	145	146	153	154	155	161
5	164	165	174	175	177	178	187	188	190	191
6	200	201	204	210	214	217	218	223	229	233
7	235	241	244	247	248	253	259	262	264	290
8	292	295	301	302	308	314	323	327	343	348
9	349	351	357	359	364	387	389	395	396	0

The points on the surface are:

$0: P_0 = P_0 = (1,0,0,0)$
$1: P_1 = P_1 = (0,1,0,0)$
$2: P_2 = P_2 = (0,0,1,0)$
$3: P_3 = P_3 = (0,0,0,1)$
$4: P_4 = P_4 = (1,1,1,1)$
$5: P_5 = P_5 = (1,1,0,0)$
$6: P_6 = P_6 = (2,1,0,0)$
$7: P_7 = P_7 = (3,1,0,0)$
$8: P_8 = P_8 = (4,1,0,0)$
$9: P_9 = P_9 = (5,1,0,0)$
$10: P_{10} = P_{10} = (6,1,0,0)$
$11: P_{11} = P_{11} = (1,0,1,0)$
$12: P_{12} = P_{12} = (2,0,1,0)$
$13: P_{13} = P_{13} = (3,0,1,0)$
$14: P_{14} = P_{14} = (4,0,1,0)$
$15: P_{15} = P_{15} = (5,0,1,0)$
$16: P_{16} = P_{16} = (6,0,1,0)$
$17: P_{17} = P_{18} = (1,1,1,0)$
$18: P_{18} = P_{27} = (3,2,1,0)$
$19: P_{19} = P_{36} = (5,3,1,0)$
$20: P_{20} = P_{38} = (0,4,1,0)$
$21: P_{21} = P_{47} = (2,5,1,0)$
$22: P_{22} = P_{56} = (4,6,1,0)$
$23: P_{23} = P_{64} = (6,0,0,1)$
$24: P_{24} = P_{65} = (0,1,0,1)$
$25: P_{25} = P_{69} = (4,1,0,1)$
$26: P_{26} = P_{72} = (0,2,0,1)$
$27: P_{27} = P_{74} = (2,2,0,1)$
$28: P_{28} = P_{79} = (0,3,0,1)$
$29: P_{29} = P_{86} = (0,4,0,1)$
$30: P_{30} = P_{91} = (5,4,0,1)$
$31: P_{31} = P_{93} = (0,5,0,1)$
$32: P_{32} = P_{96} = (3,5,0,1)$
$33: P_{33} = P_{100} = (0,6,0,1)$
$34: P_{34} = P_{101} = (1,6,0,1)$
$35: P_{35} = P_{107} = (0,0,1,1)$
$36: P_{36} = P_{113} = (6,0,1,1)$
$37: P_{37} = P_{114} = (0,1,1,1)$
$38: P_{38} = P_{121} = (1,2,1,1)$
$39: P_{39} = P_{122} = (2,2,1,1)$
$40: P_{40} = P_{129} = (2,3,1,1)$
$41: P_{41} = P_{130} = (3,3,1,1)$
$42: P_{42} = P_{137} = (3,4,1,1)$
$43: P_{43} = P_{138} = (4,4,1,1)$
$44: P_{44} = P_{145} = (4,5,1,1)$
$45: P_{45} = P_{146} = (5,5,1,1)$
$46: P_{46} = P_{153} = (5,6,1,1)$
$47: P_{47} = P_{154} = (6,6,1,1)$
$48: P_{48} = P_{155} = (0,0,2,1)$
$49: P_{49} = P_{161} = (6,0,2,1)$
$50: P_{50} = P_{164} = (2,1,2,1)$
$51: P_{51} = P_{165} = (3,1,2,1)$
$52: P_{52} = P_{174} = (5,2,2,1)$
$53: P_{53} = P_{175} = (6,2,2,1)$
$54: P_{54} = P_{177} = (1,3,2,1)$
$55: P_{55} = P_{178} = (2,3,2,1)$
$56: P_{56} = P_{187} = (4,4,2,1)$
$57: P_{57} = P_{188} = (5,4,2,1)$
$58: P_{58} = P_{190} = (0,5,2,1)$
$59: P_{59} = P_{191} = (1,5,2,1)$
$60: P_{60} = P_{200} = (3,6,2,1)$
$61: P_{61} = P_{201} = (4,6,2,1)$
$62: P_{62} = P_{204} = (0,0,3,1)$
$63: P_{63} = P_{210} = (6,0,3,1)$
$64: P_{64} = P_{214} = (3,1,3,1)$
$65: P_{65} = P_{217} = (6,1,3,1)$
$66: P_{66} = P_{218} = (0,2,3,1)$
$67: P_{67} = P_{223} = (5,2,3,1)$
$68: P_{68} = P_{229} = (4,3,3,1)$
$69: P_{69} = P_{233} = (1,4,3,1)$
$70: P_{70} = P_{235} = (3,4,3,1)$
$71: P_{71} = P_{241} = (2,5,3,1)$
$72: P_{72} = P_{244} = (5,5,3,1)$
$73: P_{73} = P_{247} = (1,6,3,1)$
$74: P_{74} = P_{248} = (2,6,3,1)$
$75: P_{75} = P_{253} = (0,0,4,1)$
$76: P_{76} = P_{259} = (6,0,4,1)$
$77: P_{77} = P_{262} = (2,1,4,1)$
$78: P_{78} = P_{264} = (4,1,4,1)$
$79: P_{79} = P_{290} = (2,5,4,1)$
$80: P_{80} = P_{292} = (4,5,4,1)$
$81: P_{81} = P_{295} = (0,6,4,1)$
$82: P_{82} = P_{301} = (6,6,4,1)$
$83: P_{83} = P_{302} = (0,0,5,1)$
$84: P_{84} = P_{308} = (6,0,5,1)$
$85: P_{85} = P_{314} = (5,1,5,1)$
$86: P_{86} = P_{323} = (0,3,5,1)$
$87: P_{87} = P_{327} = (4,3,5,1)$
$88: P_{88} = P_{343} = (6,5,5,1)$
$89: P_{89} = P_{348} = (4,6,5,1)$
$90: P_{90} = P_{349} = (5,6,5,1)$
$91: P_{91} = P_{351} = (0,0,6,1)$
$92: P_{92} = P_{357} = (6,0,6,1)$
$93: P_{93} = P_{359} = (1,1,6,1)$
$94: P_{94} = P_{364} = (6,1,6,1)$
$95: P_{95} = P_{387} = (1,5,6,1)$
$96: P_{96} = P_{389} = (3,5,6,1)$
$97: P_{97} = P_{395} = (2,6,6,1)$
$98: P_{98} = P_{396} = (3,6,6,1)$

Eckardt Points

The surface has 18 Eckardt points:

	0	1	2	3	4	5	6	7	8	9
0	2	5	7	15	18	56	64	65	93	101
1	114	146	165	190	204	214	244	247	0	0

	0	1	2	3	4	5	6	7	8	9
0	2	5	7	15	17	22	23	24	31	34
1	37	45	51	58	62	64	72	73	0	0

The Eckardt points on the surface are:

$$\begin{array}{c}
E_0 = P_2 = P_2 = (0,0,1,0) = \ell_3 \cap \ell_{11} \cap \ell_{25} = a_4 \cap b_6 \cap c_{46} \\
E_1 = P_5 = P_5 = (1,1,0,0) = \ell_4 \cap \ell_7 \cap \ell_{19} = a_5 \cap b_2 \cap c_{25} \\
E_2 = P_7 = P_7 = (3,1,0,0) = \ell_4 \cap \ell_8 \cap \ell_{22} = a_5 \cap b_3 \cap c_{35} \\
E_3 = P_{15} = P_{15} = (5,0,1,0) = \ell_0 \cap \ell_{11} \cap \ell_{16} = a_1 \cap b_6 \cap c_{16} \\
E_4 = P_{17} = P_{18} = (1,1,1,0) = \ell_{13} \cap \ell_{18} \cap \ell_{26} = c_{13} \cap c_{24} \cap c_{56} \\
E_5 = P_{22} = P_{56} = (4,6,1,0) = \ell_{12} \cap \ell_{21} \cap \ell_{26} = c_{12} \cap c_{34} \cap c_{56} \\
E_6 = P_{23} = P_{64} = (6,0,0,1) = \ell_{15} \cap \ell_{17} \cap \ell_{25} = c_{15} \cap c_{23} \cap c_{46} \\
E_7 = P_{24} = P_{65} = (0,1,0,1) = \ell_2 \cap \ell_6 \cap \ell_{13} = a_3 \cap b_1 \cap c_{13} \\
E_8 = P_{31} = P_{93} = (0,5,0,1) = \ell_1 \cap \ell_6 \cap \ell_{12} = a_2 \cap b_1 \cap c_{12} \\
E_9 = P_{34} = P_{101} = (1,6,0,1) = \ell_0 \cap \ell_{10} \cap \ell_{15} = a_1 \cap b_5 \cap c_{15} \\
E_{10} = P_{37} = P_{114} = (0,1,1,1) = \ell_{14} \cap \ell_{19} \cap \ell_{23} = c_{14} \cap c_{25} \cap c_{36} \\
E_{11} = P_{45} = P_{146} = (5,5,1,1) = \ell_5 \cap \ell_7 \cap \ell_{20} = a_6 \cap b_2 \cap c_{26} \\
E_{12} = P_{51} = P_{165} = (3,1,2,1) = \ell_5 \cap \ell_8 \cap \ell_{23} = a_6 \cap b_3 \cap c_{36} \\
E_{13} = P_{58} = P_{190} = (0,5,2,1) = \ell_{14} \cap \ell_{20} \cap \ell_{22} = c_{14} \cap c_{26} \cap c_{35} \\
E_{14} = P_{62} = P_{204} = (0,0,3,1) = \ell_3 \cap \ell_{10} \cap \ell_{24} = a_4 \cap b_5 \cap c_{45} \\
E_{15} = P_{64} = P_{214} = (3,1,3,1) = \ell_2 \cap \ell_9 \cap \ell_{21} = a_3 \cap b_4 \cap c_{34} \\
E_{16} = P_{72} = P_{244} = (5,5,3,1) = \ell_1 \cap \ell_9 \cap \ell_{18} = a_2 \cap b_4 \cap c_{24} \\
E_{17} = P_{73} = P_{247} = (1,6,3,1) = \ell_{16} \cap \ell_{17} \cap \ell_{24} = c_{16} \cap c_{23} \cap c_{45}
\end{array}$$

The Eckardt points on the surface are:

$$\begin{aligned}
E_0 &= P_2 = \mathbf{P}(0,0,1,0) = a_4 \cap b_6 \cap c_{46}, \\
E_1 &= P_5 = \mathbf{P}(1,1,0,0) = a_5 \cap b_2 \cap c_{25}, \\
E_2 &= P_7 = \mathbf{P}(3,1,0,0) = a_5 \cap b_3 \cap c_{35}, \\
E_3 &= P_{15} = \mathbf{P}(5,0,1,0) = a_1 \cap b_6 \cap c_{16}, \\
E_4 &= P_{18} = \mathbf{P}(1,1,1,0) = c_{13} \cap c_{24} \cap c_{56}, \\
E_5 &= P_{56} = \mathbf{P}(4,6,1,0) = c_{12} \cap c_{34} \cap c_{56}, \\
E_6 &= P_{64} = \mathbf{P}(6,0,0,1) = c_{15} \cap c_{23} \cap c_{46}, \\
E_7 &= P_{65} = \mathbf{P}(0,1,0,1) = a_3 \cap b_1 \cap c_{13}, \\
E_8 &= P_{93} = \mathbf{P}(0,5,0,1) = a_2 \cap b_1 \cap c_{12}, \\
E_9 &= P_{101} = \mathbf{P}(1,6,0,1) = a_1 \cap b_5 \cap c_{15}, \\
E_{10} &= P_{114} = \mathbf{P}(0,1,1,1) = c_{14} \cap c_{25} \cap c_{36}, \\
E_{11} &= P_{146} = \mathbf{P}(5,5,1,1) = a_6 \cap b_2 \cap c_{26}, \\
E_{12} &= P_{165} = \mathbf{P}(3,1,2,1) = a_6 \cap b_3 \cap c_{36}, \\
E_{13} &= P_{190} = \mathbf{P}(0,5,2,1) = c_{14} \cap c_{26} \cap c_{35}, \\
E_{14} &= P_{204} = \mathbf{P}(0,0,3,1) = a_4 \cap b_5 \cap c_{45}, \\
E_{15} &= P_{214} = \mathbf{P}(3,1,3,1) = a_3 \cap b_4 \cap c_{34}, \\
E_{16} &= P_{244} = \mathbf{P}(5,5,3,1) = a_2 \cap b_4 \cap c_{24}, \\
E_{17} &= P_{247} = \mathbf{P}(1,6,3,1) = c_{16} \cap c_{23} \cap c_{45}.
\end{aligned}$$

Double Points

The surface has 81 Double points:

	0	1	2	3	4	5	6	7	8	9
0	0	1	3	4	6	8	9	10	11	12
1	13	14	16	27	36	38	47	69	72	74
2	79	86	91	96	100	107	113	121	122	129
3	130	137	138	145	153	154	155	161	164	174
4	175	177	178	187	188	191	200	201	210	217
5	218	223	229	233	235	241	248	253	259	262
6	264	290	292	295	301	302	308	314	323	327
7	343	348	349	351	357	359	364	387	389	395
8	396	0	0	0	0	0	0	0	0	0

	0	1	2	3	4	5	6	7	8	9
0	0	1	3	4	6	8	9	10	11	12
1	13	14	16	18	19	20	21	25	26	27
2	28	29	30	32	33	35	36	38	39	40
3	41	42	43	44	46	47	48	49	50	52
4	53	54	55	56	57	59	60	61	63	65
5	66	67	68	69	70	71	74	75	76	77
6	78	79	80	81	82	83	84	85	86	87
7	88	89	90	91	92	93	94	95	96	97
8	98	0	0	0	0	0	0	0	0	0

The Double points on the surface are:

0 : $P_0 = P_0 = (1, 0, 0, 0)$
1 : $P_1 = P_1 = (0, 1, 0, 0)$
2 : $P_3 = P_3 = (0, 0, 0, 1)$
3 : $P_4 = P_4 = (1, 1, 1, 1)$
4 : $P_6 = P_6 = (2, 1, 0, 0)$
5 : $P_8 = P_8 = (4, 1, 0, 0)$
6 : $P_9 = P_9 = (5, 1, 0, 0)$
7 : $P_{10} = P_{10} = (6, 1, 0, 0)$
8 : $P_{11} = P_{11} = (1, 0, 1, 0)$
9 : $P_{12} = P_{12} = (2, 0, 1, 0)$
10 : $P_{13} = P_{13} = (3, 0, 1, 0)$
11 : $P_{14} = P_{14} = (4, 0, 1, 0)$
12 : $P_{16} = P_{16} = (6, 0, 1, 0)$
13 : $P_{18} = P_{27} = (3, 2, 1, 0)$
14 : $P_{19} = P_{36} = (5, 3, 1, 0)$
15 : $P_{20} = P_{38} = (0, 4, 1, 0)$
16 : $P_{21} = P_{47} = (2, 5, 1, 0)$
17 : $P_{25} = P_{69} = (4, 1, 0, 1)$
18 : $P_{26} = P_{72} = (0, 2, 0, 1)$
19 : $P_{27} = P_{74} = (2, 2, 0, 1)$
20 : $P_{28} = P_{79} = (0, 3, 0, 1)$
21 : $P_{29} = P_{86} = (0, 4, 0, 1)$
22 : $P_{30} = P_{91} = (5, 4, 0, 1)$
23 : $P_{32} = P_{96} = (3, 5, 0, 1)$
24 : $P_{33} = P_{100} = (0, 6, 0, 1)$
25 : $P_{35} = P_{107} = (0, 0, 1, 1)$
26 : $P_{36} = P_{113} = (6, 0, 1, 1)$
27 : $P_{38} = P_{121} = (1, 2, 1, 1)$
28 : $P_{39} = P_{122} = (2, 2, 1, 1)$
29 : $P_{40} = P_{129} = (2, 3, 1, 1)$
30 : $P_{41} = P_{130} = (3, 3, 1, 1)$
31 : $P_{42} = P_{137} = (3, 4, 1, 1)$
32 : $P_{43} = P_{138} = (4, 4, 1, 1)$
33 : $P_{44} = P_{145} = (4, 5, 1, 1)$
34 : $P_{46} = P_{153} = (5, 6, 1, 1)$
35 : $P_{47} = P_{154} = (6, 6, 1, 1)$
36 : $P_{48} = P_{155} = (0, 0, 2, 1)$
37 : $P_{49} = P_{161} = (6, 0, 2, 1)$
38 : $P_{50} = P_{164} = (2, 1, 2, 1)$
39 : $P_{52} = P_{174} = (5, 2, 2, 1)$
40 : $P_{53} = P_{175} = (6, 2, 2, 1)$
41 : $P_{54} = P_{177} = (1, 3, 2, 1)$
42 : $P_{55} = P_{178} = (2, 3, 2, 1)$
43 : $P_{56} = P_{187} = (4, 4, 2, 1)$
44 : $P_{57} = P_{188} = (5, 4, 2, 1)$
45 : $P_{59} = P_{191} = (1, 5, 2, 1)$
46 : $P_{60} = P_{200} = (3, 6, 2, 1)$
47 : $P_{61} = P_{201} = (4, 6, 2, 1)$
48 : $P_{63} = P_{210} = (6, 0, 3, 1)$
49 : $P_{65} = P_{217} = (6, 1, 3, 1)$
50 : $P_{66} = P_{218} = (0, 2, 3, 1)$
51 : $P_{67} = P_{223} = (5, 2, 3, 1)$
52 : $P_{68} = P_{229} = (4, 3, 3, 1)$
53 : $P_{69} = P_{233} = (1, 4, 3, 1)$
54 : $P_{70} = P_{235} = (3, 4, 3, 1)$
55 : $P_{71} = P_{241} = (2, 5, 3, 1)$
56 : $P_{74} = P_{248} = (2, 6, 3, 1)$
57 : $P_{75} = P_{253} = (0, 0, 4, 1)$
58 : $P_{76} = P_{259} = (6, 0, 4, 1)$
59 : $P_{77} = P_{262} = (2, 1, 4, 1)$
60 : $P_{78} = P_{264} = (4, 1, 4, 1)$
61 : $P_{79} = P_{290} = (2, 5, 4, 1)$

$62: P_{80} = P_{292} = (4,5,4,1)$
$63: P_{81} = P_{295} = (0,6,4,1)$
$64: P_{82} = P_{301} = (6,6,4,1)$
$65: P_{83} = P_{302} = (0,0,5,1)$
$66: P_{84} = P_{308} = (6,0,5,1)$
$67: P_{85} = P_{314} = (5,1,5,1)$
$68: P_{86} = P_{323} = (0,3,5,1)$
$69: P_{87} = P_{327} = (4,3,5,1)$
$70: P_{88} = P_{343} = (6,5,5,1)$
$71: P_{89} = P_{348} = (4,6,5,1)$
$72: P_{90} = P_{349} = (5,6,5,1)$
$73: P_{91} = P_{351} = (0,0,6,1)$
$74: P_{92} = P_{357} = (6,0,6,1)$
$75: P_{93} = P_{359} = (1,1,6,1)$
$76: P_{94} = P_{364} = (6,1,6,1)$
$77: P_{95} = P_{387} = (1,5,6,1)$
$78: P_{96} = P_{389} = (3,5,6,1)$
$79: P_{97} = P_{395} = (2,6,6,1)$
$80: P_{98} = P_{396} = (3,6,6,1)$

The double points on the surface are:

$$
\begin{aligned}
P_{47} = P_{154} = (6,6,1,1) &= \ell_0 \cap \ell_7 = a_1 \cap b_2 \\
P_{61} = P_{201} = (4,6,2,1) &= \ell_0 \cap \ell_8 = a_1 \cap b_3 \\
P_{74} = P_{248} = (2,6,3,1) &= \ell_0 \cap \ell_9 = a_1 \cap b_4 \\
P_{98} = P_{396} = (3,6,6,1) &= \ell_0 \cap \ell_{12} = a_1 \cap c_{12} \\
P_{90} = P_{349} = (5,6,5,1) &= \ell_0 \cap \ell_{13} = a_1 \cap c_{13} \\
P_{81} = P_{295} = (0,6,4,1) &= \ell_0 \cap \ell_{14} = a_1 \cap c_{14} \\
P_{59} = P_{191} = (1,5,2,1) &= \ell_1 \cap \ell_8 = a_2 \cap b_3 \\
P_{79} = P_{290} = (2,5,4,1) &= \ell_1 \cap \ell_{10} = a_2 \cap b_5 \\
P_{14} = P_{14} = (4,0,1,0) &= \ell_1 \cap \ell_{11} = a_2 \cap b_6 \\
P_{96} = P_{389} = (3,5,6,1) &= \ell_1 \cap \ell_{17} = a_2 \cap c_{23} \\
P_{44} = P_{145} = (4,5,1,1) &= \ell_1 \cap \ell_{19} = a_2 \cap c_{25} \\
P_{88} = P_{343} = (6,5,5,1) &= \ell_1 \cap \ell_{20} = a_2 \cap c_{26} \\
P_4 = P_4 = (1,1,1,1) &= \ell_2 \cap \ell_7 = a_3 \cap b_2 \\
P_{94} = P_{364} = (6,1,6,1) &= \ell_2 \cap \ell_{10} = a_3 \cap b_5 \\
P_{11} = P_{11} = (1,0,1,0) &= \ell_2 \cap \ell_{11} = a_3 \cap b_6 \\
P_{78} = P_{264} = (4,1,4,1) &= \ell_2 \cap \ell_{17} = a_3 \cap c_{23} \\
P_{50} = P_{164} = (2,1,2,1) &= \ell_2 \cap \ell_{22} = a_3 \cap c_{35} \\
P_{85} = P_{314} = (5,1,5,1) &= \ell_2 \cap \ell_{23} = a_3 \cap c_{36} \\
P_3 = P_3 = (0,0,0,1) &= \ell_3 \cap \ell_6 = a_4 \cap b_1 \\
P_{35} = P_{107} = (0,0,1,1) &= \ell_3 \cap \ell_7 = a_4 \cap b_2 \\
P_{48} = P_{155} = (0,0,2,1) &= \ell_3 \cap \ell_8 = a_4 \cap b_3 \\
P_{91} = P_{351} = (0,0,6,1) &= \ell_3 \cap \ell_{14} = a_4 \cap c_{14} \\
P_{83} = P_{302} = (0,0,5,1) &= \ell_3 \cap \ell_{18} = a_4 \cap c_{24} \\
P_{75} = P_{253} = (0,0,4,1) &= \ell_3 \cap \ell_{21} = a_4 \cap c_{34} \\
P_1 = P_1 = (0,1,0,0) &= \ell_4 \cap \ell_6 = a_5 \cap b_1 \\
P_8 = P_8 = (4,1,0,0) &= \ell_4 \cap \ell_9 = a_5 \cap b_4 \\
P_0 = P_0 = (1,0,0,0) &= \ell_4 \cap \ell_{11} = a_5 \cap b_6 \\
P_9 = P_9 = (5,1,0,0) &= \ell_4 \cap \ell_{15} = a_5 \cap c_{15} \\
P_{10} = P_{10} = (6,1,0,0) &= \ell_4 \cap \ell_{24} = a_5 \cap c_{45} \\
P_6 = P_6 = (2,1,0,0) &= \ell_4 \cap \ell_{26} = a_5 \cap c_{56} \\
P_{26} = P_{72} = (0,2,0,1) &= \ell_5 \cap \ell_6 = a_6 \cap b_1 \\
P_{69} = P_{233} = (1,4,3,1) &= \ell_5 \cap \ell_9 = a_6 \cap b_4 \\
P_{87} = P_{327} = (4,3,5,1) &= \ell_5 \cap \ell_{10} = a_6 \cap b_5 \\
P_{97} = P_{395} = (2,6,6,1) &= \ell_5 \cap \ell_{16} = a_6 \cap c_{16} \\
P_{76} = P_{259} = (6,0,4,1) &= \ell_5 \cap \ell_{25} = a_6 \cap c_{46} \\
P_{19} = P_{36} = (5,3,1,0) &= \ell_5 \cap \ell_{26} = a_6 \cap c_{56}
\end{aligned}
$$

$$P_{29} = P_{86} = (0,4,0,1) = \ell_6 \cap \ell_{14} = b_1 \cap c_{14}$$
$$P_{28} = P_{79} = (0,3,0,1) = \ell_6 \cap \ell_{15} = b_1 \cap c_{15}$$
$$P_{33} = P_{100} = (0,6,0,1) = \ell_6 \cap \ell_{16} = b_1 \cap c_{16}$$
$$P_{43} = P_{138} = (4,4,1,1) = \ell_7 \cap \ell_{12} = b_2 \cap c_{12}$$
$$P_{39} = P_{122} = (2,2,1,1) = \ell_7 \cap \ell_{17} = b_2 \cap c_{23}$$
$$P_{41} = P_{130} = (3,3,1,1) = \ell_7 \cap \ell_{18} = b_2 \cap c_{24}$$
$$P_{55} = P_{178} = (2,3,2,1) = \ell_8 \cap \ell_{13} = b_3 \cap c_{13}$$
$$P_{57} = P_{188} = (5,4,2,1) = \ell_8 \cap \ell_{17} = b_3 \cap c_{23}$$
$$P_{53} = P_{175} = (6,2,2,1) = \ell_8 \cap \ell_{21} = b_3 \cap c_{34}$$
$$P_{66} = P_{218} = (0,2,3,1) = \ell_9 \cap \ell_{14} = b_4 \cap c_{14}$$
$$P_{68} = P_{229} = (4,3,3,1) = \ell_9 \cap \ell_{24} = b_4 \cap c_{45}$$
$$P_{63} = P_{210} = (6,0,3,1) = \ell_9 \cap \ell_{25} = b_4 \cap c_{46}$$
$$P_{42} = P_{137} = (3,4,1,1) = \ell_{10} \cap \ell_{19} = b_5 \cap c_{25}$$
$$P_{52} = P_{174} = (5,2,2,1) = \ell_{10} \cap \ell_{22} = b_5 \cap c_{35}$$
$$P_{21} = P_{47} = (2,5,1,0) = \ell_{10} \cap \ell_{26} = b_5 \cap c_{56}$$
$$P_{12} = P_{12} = (2,0,1,0) = \ell_{11} \cap \ell_{20} = b_6 \cap c_{26}$$
$$P_{13} = P_{13} = (3,0,1,0) = \ell_{11} \cap \ell_{23} = b_6 \cap c_{36}$$
$$P_{16} = P_{16} = (6,0,1,0) = \ell_{11} \cap \ell_{26} = b_6 \cap c_{56}$$
$$P_{54} = P_{177} = (1,3,2,1) = \ell_{12} \cap \ell_{22} = c_{12} \cap c_{35}$$
$$P_{77} = P_{262} = (2,1,4,1) = \ell_{12} \cap \ell_{23} = c_{12} \cap c_{36}$$
$$P_{67} = P_{223} = (5,2,3,1) = \ell_{12} \cap \ell_{24} = c_{12} \cap c_{45}$$
$$P_{84} = P_{308} = (6,0,5,1) = \ell_{12} \cap \ell_{25} = c_{12} \cap c_{46}$$
$$P_{38} = P_{121} = (1,2,1,1) = \ell_{13} \cap \ell_{19} = c_{13} \cap c_{25}$$
$$P_{80} = P_{292} = (4,5,4,1) = \ell_{13} \cap \ell_{20} = c_{13} \cap c_{26}$$
$$P_{70} = P_{235} = (3,4,3,1) = \ell_{13} \cap \ell_{24} = c_{13} \cap c_{45}$$
$$P_{92} = P_{357} = (6,0,6,1) = \ell_{13} \cap \ell_{25} = c_{13} \cap c_{46}$$
$$P_{86} = P_{323} = (0,3,5,1) = \ell_{14} \cap \ell_{17} = c_{14} \cap c_{23}$$
$$P_{20} = P_{38} = (0,4,1,0) = \ell_{14} \cap \ell_{26} = c_{14} \cap c_{56}$$
$$P_{27} = P_{74} = (2,2,0,1) = \ell_{15} \cap \ell_{18} = c_{15} \cap c_{24}$$
$$P_{32} = P_{96} = (3,5,0,1) = \ell_{15} \cap \ell_{20} = c_{15} \cap c_{26}$$
$$P_{30} = P_{91} = (5,4,0,1) = \ell_{15} \cap \ell_{21} = c_{15} \cap c_{34}$$
$$P_{25} = P_{69} = (4,1,0,1) = \ell_{15} \cap \ell_{23} = c_{15} \cap c_{36}$$
$$P_{82} = P_{301} = (6,6,4,1) = \ell_{16} \cap \ell_{18} = c_{16} \cap c_{24}$$
$$P_{46} = P_{153} = (5,6,1,1) = \ell_{16} \cap \ell_{19} = c_{16} \cap c_{25}$$
$$P_{89} = P_{348} = (4,6,5,1) = \ell_{16} \cap \ell_{21} = c_{16} \cap c_{34}$$
$$P_{60} = P_{200} = (3,6,2,1) = \ell_{16} \cap \ell_{22} = c_{16} \cap c_{35}$$
$$P_{18} = P_{27} = (3,2,1,0) = \ell_{17} \cap \ell_{26} = c_{23} \cap c_{56}$$
$$P_{56} = P_{187} = (4,4,2,1) = \ell_{18} \cap \ell_{22} = c_{24} \cap c_{35}$$
$$P_{93} = P_{359} = (1,1,6,1) = \ell_{18} \cap \ell_{23} = c_{24} \cap c_{36}$$
$$P_{40} = P_{129} = (2,3,1,1) = \ell_{19} \cap \ell_{21} = c_{25} \cap c_{34}$$
$$P_{36} = P_{113} = (6,0,1,1) = \ell_{19} \cap \ell_{25} = c_{25} \cap c_{46}$$
$$P_{95} = P_{387} = (1,5,6,1) = \ell_{20} \cap \ell_{21} = c_{26} \cap c_{34}$$
$$P_{71} = P_{241} = (2,5,3,1) = \ell_{20} \cap \ell_{24} = c_{26} \cap c_{45}$$
$$P_{49} = P_{161} = (6,0,2,1) = \ell_{22} \cap \ell_{25} = c_{35} \cap c_{46}$$
$$P_{65} = P_{217} = (6,1,3,1) = \ell_{23} \cap \ell_{24} = c_{36} \cap c_{45}$$

Points on lines

Line 0 = a_1 has 8 points: $\{P_i \mid i \in \{15, 34, 47, 61, 74, 81, 90, 98\}\}$
Line 1 = a_2 has 8 points: $\{P_i \mid i \in \{14, 31, 44, 59, 72, 79, 88, 96\}\}$
Line 2 = a_3 has 8 points: $\{P_i \mid i \in \{4, 11, 24, 50, 64, 78, 85, 94\}\}$
Line 3 = a_4 has 8 points: $\{P_i \mid i \in \{2, 3, 35, 48, 62, 75, 83, 91\}\}$
Line 4 = a_5 has 8 points: $\{P_i \mid i \in \{0, 1, 5, 6, 7, 8, 9, 10\}\}$
Line 5 = a_6 has 8 points: $\{P_i \mid i \in \{19, 26, 45, 51, 69, 76, 87, 97\}\}$
Line 6 = b_1 has 8 points: $\{P_i \mid i \in \{1, 3, 24, 26, 28, 29, 31, 33\}\}$
Line 7 = b_2 has 8 points: $\{P_i \mid i \in \{4, 5, 35, 39, 41, 43, 45, 47\}\}$
Line 8 = b_3 has 8 points: $\{P_i \mid i \in \{7, 48, 51, 53, 55, 57, 59, 61\}\}$
Line 9 = b_4 has 8 points: $\{P_i \mid i \in \{8, 63, 64, 66, 68, 69, 72, 74\}\}$
Line 10 = b_5 has 8 points: $\{P_i \mid i \in \{21, 34, 42, 52, 62, 79, 87, 94\}\}$
Line 11 = b_6 has 8 points: $\{P_i \mid i \in \{0, 2, 11, 12, 13, 14, 15, 16\}\}$
Line 12 = c_{12} has 8 points: $\{P_i \mid i \in \{22, 31, 43, 54, 67, 77, 84, 98\}\}$
Line 13 = c_{13} has 8 points: $\{P_i \mid i \in \{17, 24, 38, 55, 70, 80, 90, 92\}\}$
Line 14 = c_{14} has 8 points: $\{P_i \mid i \in \{20, 29, 37, 58, 66, 81, 86, 91\}\}$
Line 15 = c_{15} has 8 points: $\{P_i \mid i \in \{9, 23, 25, 27, 28, 30, 32, 34\}\}$
Line 16 = c_{16} has 8 points: $\{P_i \mid i \in \{15, 33, 46, 60, 73, 82, 89, 97\}\}$
Line 17 = c_{23} has 8 points: $\{P_i \mid i \in \{18, 23, 39, 57, 73, 78, 86, 96\}\}$
Line 18 = c_{24} has 8 points: $\{P_i \mid i \in \{17, 27, 41, 56, 72, 82, 83, 93\}\}$
Line 19 = c_{25} has 8 points: $\{P_i \mid i \in \{5, 36, 37, 38, 40, 42, 44, 46\}\}$
Line 20 = c_{26} has 8 points: $\{P_i \mid i \in \{12, 32, 45, 58, 71, 80, 88, 95\}\}$
Line 21 = c_{34} has 8 points: $\{P_i \mid i \in \{22, 30, 40, 53, 64, 75, 89, 95\}\}$
Line 22 = c_{35} has 8 points: $\{P_i \mid i \in \{7, 49, 50, 52, 54, 56, 58, 60\}\}$
Line 23 = c_{36} has 8 points: $\{P_i \mid i \in \{13, 25, 37, 51, 65, 77, 85, 93\}\}$
Line 24 = c_{45} has 8 points: $\{P_i \mid i \in \{10, 62, 65, 67, 68, 70, 71, 73\}\}$
Line 25 = c_{46} has 8 points: $\{P_i \mid i \in \{2, 23, 36, 49, 63, 76, 84, 92\}\}$
Line 26 = c_{56} has 8 points: $\{P_i \mid i \in \{6, 16, 17, 18, 19, 20, 21, 22\}\}$

Points on surface but on no line

The surface has 0 points not on any line:

	0	1	2	3	4	5	6	7	8	9

The points on the surface but not on lines are:

Tritangent planes

The 45 tritangent planes are:

$$\pi_{12} = \pi_0 = 253 = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix} = V\big(3X_0 + 4X_1 + 6X_2 + X_3\big)$$

dual pt rank = 382 = (3, 4, 6, 1).

$$\pi_{21} = \pi_1 = 121 = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(5X_0 + X_2\big)$$

dual pt rank = 15 = (5, 0, 1, 0).

$$\pi_{13} = \pi_2 = 126 = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 4 \end{bmatrix} = V\big(5X_0 + 6X_1 + 3X_2 + X_3\big)$$

dual pt rank $= 251 = (5,6,3,1)$.

$$\pi_{31} = \pi_3 = 64 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(6X_0 + X_2\big)$$

dual pt rank $= 16 = (6,0,1,0)$.

$$\pi_{14} = \pi_4 = 193 = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 6 \end{bmatrix} = V\big(4X_0 + 5X_1 + X_2 + X_3\big)$$

dual pt rank $= 145 = (4,5,1,1)$.

$$\pi_{41} = \pi_5 = 399 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(X_0\big)$$

dual pt rank $= 0 = (1,0,0,0)$.

$$\pi_{15} = \pi_6 = 387 = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 5 \end{bmatrix} = V\big(X_0 + 2X_1 + 2X_2 + X_3\big)$$

dual pt rank $= 170 = (1,2,2,1)$.

$$\pi_{51} = \pi_7 = 7 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(X_2\big)$$

dual pt rank $= 2 = (0,0,1,0)$.

$$\pi_{16} = \pi_8 = 48 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 0 \end{bmatrix} = V\big(X_1 + X_3\big)$$

dual pt rank $= 65 = (0,1,0,1)$.

$$\pi_{61} = \pi_9 = 178 = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(4X_0 + X_2\big)$$

dual pt rank $= 14 = (4,0,1,0)$.

$$\pi_{23} = \pi_{10} = 370 = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{bmatrix} = V\big(X_0 + 4X_1 + 3X_2 + X_3\big)$$

dual pt rank $= 233 = (1,4,3,1)$.

$$\pi_{32} = \pi_{11} = 351 = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = V\big(X_0 + 6X_1 + 6X_2 + X_3\big)$$

dual pt rank $= 394 = (1,6,6,1)$.

$$\pi_{24} = \pi_{12} = 84 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \end{bmatrix} = V\big(6X_0 + 4X_1 + 4X_2 + X_3\big)$$

dual pt rank $= 287 = (6,4,4,1)$.

$$\pi_{42} = \pi_{13} = 113 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(6X_0 + X_1\big)$$

dual pt rank $= 10 = (6,1,0,0)$.

$$\pi_{25} = \pi_{14} = 257 = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{bmatrix} = V\big(3X_0 + 4X_1 + 2X_2 + X_3\big)$$

dual pt rank $= 186 = (3,4,2,1)$.

$$\pi_{52} = \pi_{15} = 1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} = V\big(6X_2 + X_3\big)$$

dual pt rank $= 351 = (0,0,6,1)$.

$$\pi_{26} = \pi_{16} = 24 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix} = V\big(4X_1 + X_3\big)$$

dual pt rank $= 86 = (0,4,0,1)$.

$$\pi_{62} = \pi_{17} = 204 = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 1 \end{bmatrix} = V\big(4X_0 + 3X_1 + 6X_2 + X_3\big)$$

dual pt rank $= 376 = (4,3,6,1)$.

$$\pi_{34} = \pi_{18} = 295 = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} = V\big(2X_0 + 6X_1 + 5X_2 + X_3\big)$$

dual pt rank $= 346 = (2,6,5,1)$.

$$\pi_{43} = \pi_{19} = 341 = \begin{bmatrix} 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(2X_0 + X_1\big)$$

dual pt rank $= 6 = (2,1,0,0)$.

$$\pi_{35} = \pi_{20} = 127 = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 5 \end{bmatrix} = V\big(5X_0 + 6X_1 + 2X_2 + X_3\big)$$

dual pt rank $= 202 = (5,6,2,1)$.

$$\pi_{53} = \pi_{21} = 4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix} = V\big(3X_2 + X_3\big)$$

dual pt rank $= 204 = (0,0,3,1)$.

$$\pi_{36} = \pi_{22} = 8 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = V\big(6X_1 + X_3\big)$$

dual pt rank $= 100 = (0,6,0,1)$.

$$\pi_{63} = \pi_{23} = 93 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 4 \end{bmatrix} = V\big(6X_0 + 3X_1 + 3X_2 + X_3\big)$$

dual pt rank $= 231 = (6,3,3,1)$.

$$\pi_{45} = \pi_{24} = 398 = \begin{bmatrix} 1 & 6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(X_0 + X_1\big)$$

dual pt rank = 5 = (1, 1, 0, 0).

$$\pi_{54} = \pi_{25} = 5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 5 \end{bmatrix} = V\big(2X_2 + X_3\big)$$

dual pt rank = 155 = (0, 0, 2, 1).

$$\pi_{46} = \pi_{26} = 56 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(X_1\big)$$

dual pt rank = 1 = (0, 1, 0, 0).

$$\pi_{64} = \pi_{27} = 374 = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 0 \end{bmatrix} = V\big(X_0 + 3X_1 + X_3\big)$$

dual pt rank = 80 = (1, 3, 0, 1).

$$\pi_{56} = \pi_{28} = 0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = V\big(X_3\big)$$

dual pt rank = 3 = (0, 0, 0, 1).

$$\pi_{65} = \pi_{29} = 322 = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix} = V\big(2X_0 + 3X_1 + 2X_2 + X_3\big)$$

dual pt rank = 178 = (2, 3, 2, 1).

$$\pi_{12,34,56} = \pi_{30} = 140 = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix} = V\big(5X_0 + 4X_1 + 5X_2 + X_3\big)$$

dual pt rank = 335 = (5, 4, 5, 1).

$$\pi_{12,35,46} = \pi_{31} = 366 = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix} = V\big(X_0 + 4X_1 + X_3\big)$$

dual pt rank = 87 = (1, 4, 0, 1).

$$\pi_{12,36,45} = \pi_{32} = 200 = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{bmatrix} = V\big(4X_0 + 4X_1 + 2X_2 + X_3\big)$$

dual pt rank = 187 = (4, 4, 2, 1).

$$\pi_{13,24,56} = \pi_{33} = 182 = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{bmatrix} = V\big(4X_0 + 6X_1 + 4X_2 + X_3\big)$$

dual pt rank = 299 = (4, 6, 4, 1).

$$\pi_{13,25,46} = \pi_{34} = 350 = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = V\big(X_0 + 6X_1 + X_3\big)$$

dual pt rank = 101 = (1, 6, 0, 1).

$$\pi_{13,26,45} = \pi_{35} = 70 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 5 \end{bmatrix} = V\big(6X_0 + 6X_1 + 2X_2 + X_3\big)$$

dual pt rank = 203 = (6, 6, 2, 1).

$$\pi_{14,23,56} = \pi_{36} = 364 = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 6 \end{bmatrix} = V\big(X_0 + 5X_1 + X_2 + X_3\big)$$

dual pt rank = 142 = (1, 5, 1, 1).

$$\pi_{14,25,36} = \pi_{37} = 307 = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 6 \end{bmatrix} = V\big(2X_0 + 5X_1 + X_2 + X_3\big)$$

dual pt rank = 143 = (2, 5, 1, 1).

$$\pi_{14,26,35} = \pi_{38} = 250 = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 6 \end{bmatrix} = V\big(3X_0 + 5X_1 + X_2 + X_3\big)$$

dual pt rank = 144 = (3, 5, 1, 1).

$$\pi_{15,23,46} = \pi_{39} = 382 = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \end{bmatrix} = V\big(X_0 + 2X_1 + X_3\big)$$

dual pt rank = 73 = (1, 2, 0, 1).

$$\pi_{15,24,36} = \pi_{40} = 385 = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{bmatrix} = V\big(X_0 + 2X_1 + 4X_2 + X_3\big)$$

dual pt rank = 268 = (1, 2, 4, 1).

$$\pi_{15,26,34} = \pi_{41} = 384 = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 2 \end{bmatrix} = V\big(X_0 + 2X_1 + 5X_2 + X_3\big)$$

dual pt rank = 317 = (1, 2, 5, 1).

$$\pi_{16,23,45} = \pi_{42} = 395 = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 5 \end{bmatrix} = V\big(X_0 + X_1 + 2X_2 + X_3\big)$$

dual pt rank = 163 = (1, 1, 2, 1).

$$\pi_{16,24,35} = \pi_{43} = 336 = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 3 \end{bmatrix} = V\big(2X_0 + X_1 + 4X_2 + X_3\big)$$

dual pt rank = 262 = (2, 1, 4, 1).

$$\pi_{16,25,34} = \pi_{44} = 107 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 2 \end{bmatrix} = V\big(6X_0 + X_1 + 5X_2 + X_3\big)$$

dual pt rank = 315 = (6, 1, 5, 1).

The Generalized Quadrangle

The lines in the tritangent planes are:
$\pi_0 = \pi_{12} = \{\ell_i \mid i = 0, 7, 12\} = \{a_1, b_2, c_{12}\}$
$\pi_1 = \pi_{21} = \{\ell_i \mid i = 1, 6, 12\} = \{a_2, b_1, c_{12}\}$
$\pi_2 = \pi_{13} = \{\ell_i \mid i = 0, 8, 13\} = \{a_1, b_3, c_{13}\}$
$\pi_3 = \pi_{31} = \{\ell_i \mid i = 2, 6, 13\} = \{a_3, b_1, c_{13}\}$
$\pi_4 = \pi_{14} = \{\ell_i \mid i = 0, 9, 14\} = \{a_1, b_4, c_{14}\}$
$\pi_5 = \pi_{41} = \{\ell_i \mid i = 3, 6, 14\} = \{a_4, b_1, c_{14}\}$
$\pi_6 = \pi_{15} = \{\ell_i \mid i = 0, 10, 15\} = \{a_1, b_5, c_{15}\}$

$\pi_7 = \pi_{51} = \{\ell_i \mid i = 4, 6, 15\} = \{a_5, b_1, c_{15}\}$
$\pi_8 = \pi_{16} = \{\ell_i \mid i = 0, 11, 16\} = \{a_1, b_6, c_{16}\}$
$\pi_9 = \pi_{61} = \{\ell_i \mid i = 5, 6, 16\} = \{a_6, b_1, c_{16}\}$
$\pi_{10} = \pi_{23} = \{\ell_i \mid i = 1, 8, 17\} = \{a_2, b_3, c_{23}\}$
$\pi_{11} = \pi_{32} = \{\ell_i \mid i = 2, 7, 17\} = \{a_3, b_2, c_{23}\}$
$\pi_{12} = \pi_{24} = \{\ell_i \mid i = 1, 9, 18\} = \{a_2, b_4, c_{24}\}$
$\pi_{13} = \pi_{42} = \{\ell_i \mid i = 3, 7, 18\} = \{a_4, b_2, c_{24}\}$
$\pi_{14} = \pi_{25} = \{\ell_i \mid i = 1, 10, 19\} = \{a_2, b_5, c_{25}\}$
$\pi_{15} = \pi_{52} = \{\ell_i \mid i = 4, 7, 19\} = \{a_5, b_2, c_{25}\}$
$\pi_{16} = \pi_{26} = \{\ell_i \mid i = 1, 11, 20\} = \{a_2, b_6, c_{26}\}$
$\pi_{17} = \pi_{62} = \{\ell_i \mid i = 5, 7, 20\} = \{a_6, b_2, c_{26}\}$
$\pi_{18} = \pi_{34} = \{\ell_i \mid i = 2, 9, 21\} = \{a_3, b_4, c_{34}\}$
$\pi_{19} = \pi_{43} = \{\ell_i \mid i = 3, 8, 21\} = \{a_4, b_3, c_{34}\}$
$\pi_{20} = \pi_{35} = \{\ell_i \mid i = 2, 10, 22\} = \{a_3, b_5, c_{35}\}$
$\pi_{21} = \pi_{53} = \{\ell_i \mid i = 4, 8, 22\} = \{a_5, b_3, c_{35}\}$
$\pi_{22} = \pi_{36} = \{\ell_i \mid i = 2, 11, 23\} = \{a_3, b_6, c_{36}\}$
$\pi_{23} = \pi_{63} = \{\ell_i \mid i = 5, 8, 23\} = \{a_6, b_3, c_{36}\}$
$\pi_{24} = \pi_{45} = \{\ell_i \mid i = 3, 10, 24\} = \{a_4, b_5, c_{45}\}$
$\pi_{25} = \pi_{54} = \{\ell_i \mid i = 4, 9, 24\} = \{a_5, b_4, c_{45}\}$
$\pi_{26} = \pi_{46} = \{\ell_i \mid i = 3, 11, 25\} = \{a_4, b_6, c_{46}\}$
$\pi_{27} = \pi_{64} = \{\ell_i \mid i = 5, 9, 25\} = \{a_6, b_4, c_{46}\}$
$\pi_{28} = \pi_{56} = \{\ell_i \mid i = 4, 11, 26\} = \{a_5, b_6, c_{56}\}$
$\pi_{29} = \pi_{65} = \{\ell_i \mid i = 5, 10, 26\} = \{a_6, b_5, c_{56}\}$
$\pi_{30} = \pi_{12,34,56} = \{\ell_i \mid i = 12, 21, 26\} = \{c_{12}, c_{34}, c_{56}\}$
$\pi_{31} = \pi_{12,35,46} = \{\ell_i \mid i = 12, 22, 25\} = \{c_{12}, c_{35}, c_{46}\}$
$\pi_{32} = \pi_{12,36,45} = \{\ell_i \mid i = 12, 23, 24\} = \{c_{12}, c_{36}, c_{45}\}$
$\pi_{33} = \pi_{13,24,56} = \{\ell_i \mid i = 13, 18, 26\} = \{c_{13}, c_{24}, c_{56}\}$
$\pi_{34} = \pi_{13,25,46} = \{\ell_i \mid i = 13, 19, 25\} = \{c_{13}, c_{25}, c_{46}\}$
$\pi_{35} = \pi_{13,26,45} = \{\ell_i \mid i = 13, 20, 24\} = \{c_{13}, c_{26}, c_{45}\}$
$\pi_{36} = \pi_{14,23,56} = \{\ell_i \mid i = 14, 17, 26\} = \{c_{14}, c_{23}, c_{56}\}$
$\pi_{37} = \pi_{14,25,36} = \{\ell_i \mid i = 14, 19, 23\} = \{c_{14}, c_{25}, c_{36}\}$
$\pi_{38} = \pi_{14,26,35} = \{\ell_i \mid i = 14, 20, 22\} = \{c_{14}, c_{26}, c_{35}\}$
$\pi_{39} = \pi_{15,23,46} = \{\ell_i \mid i = 15, 17, 25\} = \{c_{15}, c_{23}, c_{46}\}$
$\pi_{40} = \pi_{15,24,36} = \{\ell_i \mid i = 15, 18, 23\} = \{c_{15}, c_{24}, c_{36}\}$
$\pi_{41} = \pi_{15,26,34} = \{\ell_i \mid i = 15, 20, 21\} = \{c_{15}, c_{26}, c_{34}\}$
$\pi_{42} = \pi_{16,23,45} = \{\ell_i \mid i = 16, 17, 24\} = \{c_{16}, c_{23}, c_{45}\}$
$\pi_{43} = \pi_{16,24,35} = \{\ell_i \mid i = 16, 18, 22\} = \{c_{16}, c_{24}, c_{35}\}$
$\pi_{44} = \pi_{16,25,34} = \{\ell_i \mid i = 16, 19, 21\} = \{c_{16}, c_{25}, c_{34}\}$
The tritangent planes through the 27 lines are:
$a_1 = \ell_0 \in \{\pi_i \mid i = 0, 2, 8, 6, 4\} = \{\pi_{12}, \pi_{13}, \pi_{16}, \pi_{15}, \pi_{14}\}$
$a_2 = \ell_1 \in \{\pi_i \mid i = 1, 16, 10, 12, 14\} = \{\pi_{21}, \pi_{26}, \pi_{23}, \pi_{24}, \pi_{25}\}$
$a_3 = \ell_2 \in \{\pi_i \mid i = 11, 18, 22, 3, 20\} = \{\pi_{32}, \pi_{34}, \pi_{36}, \pi_{31}, \pi_{35}\}$
$a_4 = \ell_3 \in \{\pi_i \mid i = 13, 26, 5, 24, 19\} = \{\pi_{42}, \pi_{46}, \pi_{41}, \pi_{45}, \pi_{43}\}$
$a_5 = \ell_4 \in \{\pi_i \mid i = 25, 21, 7, 15, 28\} = \{\pi_{54}, \pi_{53}, \pi_{51}, \pi_{52}, \pi_{56}\}$
$a_6 = \ell_5 \in \{\pi_i \mid i = 17, 23, 27, 9, 29\} = \{\pi_{62}, \pi_{63}, \pi_{64}, \pi_{61}, \pi_{65}\}$
$b_1 = \ell_6 \in \{\pi_i \mid i = 1, 5, 9, 3, 7\} = \{\pi_{21}, \pi_{41}, \pi_{61}, \pi_{31}, \pi_{51}\}$
$b_2 = \ell_7 \in \{\pi_i \mid i = 0, 13, 17, 11, 15\} = \{\pi_{12}, \pi_{42}, \pi_{62}, \pi_{32}, \pi_{52}\}$
$b_3 = \ell_8 \in \{\pi_i \mid i = 2, 23, 10, 19, 21\} = \{\pi_{13}, \pi_{63}, \pi_{23}, \pi_{43}, \pi_{53}\}$
$b_4 = \ell_9 \in \{\pi_i \mid i = 4, 25, 27, 12, 18\} = \{\pi_{14}, \pi_{54}, \pi_{64}, \pi_{24}, \pi_{34}\}$
$b_5 = \ell_{10} \in \{\pi_i \mid i = 24, 6, 29, 14, 20\} = \{\pi_{45}, \pi_{15}, \pi_{65}, \pi_{25}, \pi_{35}\}$
$b_6 = \ell_{11} \in \{\pi_i \mid i = 26, 16, 8, 22, 28\} = \{\pi_{46}, \pi_{26}, \pi_{16}, \pi_{36}, \pi_{56}\}$
$c_{12} = \ell_{12} \in \{\pi_i \mid i = 0, 1, 32, 31, 30\} = \{\pi_{12}, \pi_{21}, \pi_{12,36,45}, \pi_{12,35,46}, \pi_{12,34,56}\}$
$c_{13} = \ell_{13} \in \{\pi_i \mid i = 2, 33, 34, 35, 3\} = \{\pi_{13}, \pi_{13,24,56}, \pi_{13,25,46}, \pi_{13,26,45}, \pi_{31}\}$
$c_{14} = \ell_{14} \in \{\pi_i \mid i = 38, 5, 4, 36, 37\} = \{\pi_{14,26,35}, \pi_{41}, \pi_{14}, \pi_{14,23,56}, \pi_{14,25,36}\}$
$c_{15} = \ell_{15} \in \{\pi_i \mid i = 6, 40, 41, 39, 7\} = \{\pi_{15}, \pi_{15,24,36}, \pi_{15,26,34}, \pi_{15,23,46}, \pi_{51}\}$
$c_{16} = \ell_{16} \in \{\pi_i \mid i = 44, 42, 8, 9, 43\} = \{\pi_{16,25,34}, \pi_{16,23,45}, \pi_{16}, \pi_{61}, \pi_{16,24,35}\}$
$c_{23} = \ell_{17} \in \{\pi_i \mid i = 42, 39, 10, 36, 11\} = \{\pi_{16,23,45}, \pi_{15,23,46}, \pi_{23}, \pi_{14,23,56}, \pi_{32}\}$
$c_{24} = \ell_{18} \in \{\pi_i \mid i = 13, 40, 33, 12, 43\} = \{\pi_{42}, \pi_{15,24,36}, \pi_{13,24,56}, \pi_{24}, \pi_{16,24,35}\}$
$c_{25} = \ell_{19} \in \{\pi_i \mid i = 44, 34, 37, 14, 15\} = \{\pi_{16,25,34}, \pi_{13,25,46}, \pi_{14,25,36}, \pi_{25}, \pi_{52}\}$

$$c_{26} = \ell_{20} \in \{\pi_i \mid i = 38, 17, 16, 41, 35\} = \{\pi_{14,26,35}, \pi_{62}, \pi_{26}, \pi_{15,26,34}, \pi_{13,26,45}\}$$
$$c_{34} = \ell_{21} \in \{\pi_i \mid i = 44, 41, 19, 18, 30\} = \{\pi_{16,25,34}, \pi_{15,26,34}, \pi_{43}, \pi_{34}, \pi_{12,34,56}\}$$
$$c_{35} = \ell_{22} \in \{\pi_i \mid i = 38, 31, 43, 21, 20\} = \{\pi_{14,26,35}, \pi_{12,35,46}, \pi_{16,24,35}, \pi_{53}, \pi_{35}\}$$
$$c_{36} = \ell_{23} \in \{\pi_i \mid i = 32, 40, 23, 37, 22\} = \{\pi_{12,36,45}, \pi_{15,24,36}, \pi_{63}, \pi_{14,25,36}, \pi_{36}\}$$
$$c_{45} = \ell_{24} \in \{\pi_i \mid i = 32, 24, 42, 25, 35\} = \{\pi_{12,36,45}, \pi_{45}, \pi_{16,23,45}, \pi_{54}, \pi_{13,26,45}\}$$
$$c_{46} = \ell_{25} \in \{\pi_i \mid i = 26, 39, 27, 31, 34\} = \{\pi_{46}, \pi_{15,23,46}, \pi_{64}, \pi_{12,35,46}, \pi_{13,25,46}\}$$
$$c_{56} = \ell_{26} \in \{\pi_i \mid i = 33, 36, 29, 30, 28\} = \{\pi_{13,24,56}, \pi_{14,23,56}, \pi_{65}, \pi_{12,34,56}, \pi_{56}\}$$