Cubic Surfaces with 27 Lines over GF(4)

Orbiter

April 16, 2019

1 The field of order 4

The field $\mathbb{F}_4$:
polynomial: $X^2 + X + 1 = 7$
$Z_i = \log_\alpha(1 + \alpha^i)$

i	γ_i	$-\gamma_i$	γ_i^{-1}	$\log_\alpha(\gamma_i)$	α^i	Z_i	$\phi(\gamma_i)$	$T(\gamma_i)$	$N(\gamma_i)$
0	$0 = 0$	0	DNE	DNE	1	DNE	0	0	0
1	$1 = 1$	1	1	3	2	2	1	0	1
2	$\alpha = \alpha$	2	3	1	3	1	3	1	1
3	$\alpha + 1 = \alpha^2$	3	2	2	1	DNE	2	1	1

2 The groups

2.1 The semilinear group

Group action $\mathrm{P\Gamma L}(4, 4)$ of degree 85
Group order 1974067200
tl=$(85, 84, 80, 64, 27, 2)$
Base: $(0, 1, 2, 3, 4, 6)$
Strong generators for a group of order 1974067200:

$$\begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{bmatrix}_1, \begin{bmatrix} 1&0&0&0\\ 0&\alpha^2&0&0\\ 0&0&\alpha^2&0\\ 0&0&0&\alpha^2 \end{bmatrix}_0, \begin{bmatrix} 1&0&0&0\\ 0&\alpha&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{bmatrix}_0,$$

$$\begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&\alpha&0\\ 0&0&0&1 \end{bmatrix}_0, \begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 1&0&0&1 \end{bmatrix}_0, \begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ \alpha&0&0&1 \end{bmatrix}_0,$$

$$\begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&1&0&1 \end{bmatrix}_0, \begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&\alpha&0&1 \end{bmatrix}_0, \begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&0&1&1 \end{bmatrix}_0,$$

$$\begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&0&\alpha&1 \end{bmatrix}_0, \begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&0&1\\ 0&0&1&0 \end{bmatrix}_0, \begin{bmatrix} 1&0&0&0\\ 0&0&1&0\\ 0&1&0&0\\ 0&0&0&1 \end{bmatrix}_0,$$

$$\begin{bmatrix} 0&1&0&0\\ 1&0&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{bmatrix}_0$$

i	P_i
0	$(1,0,0,0)$
1	$(0,1,0,0)$
2	$(0,0,1,0)$
3	$(0,0,0,1)$
4	$(1,1,1,1)$
5	$(1,1,0,0)$
6	$(2,1,0,0)$
7	$(3,1,0,0)$
8	$(1,0,1,0)$
9	$(2,0,1,0)$

,

i	P_i
10	$(3,0,1,0)$
11	$(0,1,1,0)$
12	$(1,1,1,0)$
13	$(2,1,1,0)$
14	$(3,1,1,0)$
15	$(0,2,1,0)$
16	$(1,2,1,0)$
17	$(2,2,1,0)$
18	$(3,2,1,0)$
19	$(0,3,1,0)$

,

i	P_i
20	$(1,3,1,0)$
21	$(2,3,1,0)$
22	$(3,3,1,0)$
23	$(1,0,0,1)$
24	$(2,0,0,1)$
25	$(3,0,0,1)$
26	$(0,1,0,1)$
27	$(1,1,0,1)$
28	$(2,1,0,1)$
29	$(3,1,0,1)$

,

i	P_i
30	$(0,2,0,1)$
31	$(1,2,0,1)$
32	$(2,2,0,1)$
33	$(3,2,0,1)$
34	$(0,3,0,1)$
35	$(1,3,0,1)$
36	$(2,3,0,1)$
37	$(3,3,0,1)$
38	$(0,0,1,1)$
39	$(1,0,1,1)$

,

i	P_i
40	$(2,0,1,1)$
41	$(3,0,1,1)$
42	$(0,1,1,1)$
43	$(2,1,1,1)$
44	$(3,1,1,1)$
45	$(0,2,1,1)$
46	$(1,2,1,1)$
47	$(2,2,1,1)$
48	$(3,2,1,1)$
49	$(0,3,1,1)$

i	P_i
50	$(1,3,1,1)$
51	$(2,3,1,1)$
52	$(3,3,1,1)$
53	$(0,0,2,1)$
54	$(1,0,2,1)$
55	$(2,0,2,1)$
56	$(3,0,2,1)$
57	$(0,1,2,1)$
58	$(1,1,2,1)$
59	$(2,1,2,1)$

,

i	P_i
60	$(3,1,2,1)$
61	$(0,2,2,1)$
62	$(1,2,2,1)$
63	$(2,2,2,1)$
64	$(3,2,2,1)$
65	$(0,3,2,1)$
66	$(1,3,2,1)$
67	$(2,3,2,1)$
68	$(3,3,2,1)$
69	$(0,0,3,1)$

,

i	P_i
70	$(1,0,3,1)$
71	$(2,0,3,1)$
72	$(3,0,3,1)$
73	$(0,1,3,1)$
74	$(1,1,3,1)$
75	$(2,1,3,1)$
76	$(3,1,3,1)$
77	$(0,2,3,1)$
78	$(1,2,3,1)$
79	$(2,2,3,1)$

,

i	P_i
80	$(3,2,3,1)$
81	$(0,3,3,1)$
82	$(1,3,3,1)$
83	$(2,3,3,1)$
84	$(3,3,3,1)$

2.2 The orthogonal group

Group action $\mathrm{P\Gamma L}(4,4)Wedge$ of degree 1365
Does not have strong generators.

2.3 The group stabilizing the fixed line

Group action $\mathrm{P\Gamma L}(4,4)Wedgeres100$ of degree 100
Does not have strong generators.

i	P_i	i	P_i	i	P_i	i	P_i	i	P_i
0	$(0,1,0,0,0,0)$	10	$(0,1,1,0,0,0)$	20	$(2,3,1,0,0,0)$	30	$(1,2,0,1,0,0)$	40	$(0,0,1,0,1,0)$
1	$(0,0,1,0,0,0)$	11	$(1,1,1,0,0,0)$	21	$(3,3,1,0,0,0)$	31	$(2,2,0,1,0,0)$	41	$(1,0,1,0,1,0)$
2	$(0,0,0,1,0,0)$	12	$(2,1,1,0,0,0)$	22	$(1,0,0,1,0,0)$	32	$(3,2,0,1,0,0)$	42	$(2,0,1,0,1,0)$
3	$(0,0,0,0,1,0)$	13	$(3,1,1,0,0,0)$	23	$(2,0,0,1,0,0)$	33	$(0,3,0,1,0,0)$	43	$(3,0,1,0,1,0)$
4	$(1,1,0,0,0,0)$	14	$(0,2,1,0,0,0)$	24	$(3,0,0,1,0,0)$	34	$(1,3,0,1,0,0)$	44	$(0,0,2,0,1,0)$
5	$(2,1,0,0,0,0)$	15	$(1,2,1,0,0,0)$	25	$(0,1,0,1,0,0)$	35	$(2,3,0,1,0,0)$	45	$(1,0,2,0,1,0)$
6	$(3,1,0,0,0,0)$	16	$(2,2,1,0,0,0)$	26	$(1,1,0,1,0,0)$	36	$(3,3,0,1,0,0)$	46	$(2,0,2,0,1,0)$
7	$(1,0,1,0,0,0)$	17	$(3,2,1,0,0,0)$	27	$(2,1,0,1,0,0)$	37	$(1,0,0,0,1,0)$	47	$(3,0,2,0,1,0)$
8	$(2,0,1,0,0,0)$	18	$(0,3,1,0,0,0)$	28	$(3,1,0,1,0,0)$	38	$(2,0,0,0,1,0)$	48	$(0,0,3,0,1,0)$
9	$(3,0,1,0,0,0)$	19	$(1,3,1,0,0,0)$	29	$(0,2,0,1,0,0)$	39	$(3,0,0,0,1,0)$	49	$(1,0,3,0,1,0)$

i	P_i	i	P_i	i	P_i	i	P_i	i	P_i
50	$(2,0,3,0,1,0)$	60	$(0,2,2,1,1,0)$	70	$(2,0,0,2,1,0)$	80	$(0,1,3,2,1,0)$	90	$(2,3,1,3,1,0)$
51	$(3,0,3,0,1,0)$	61	$(1,2,2,1,1,0)$	71	$(3,0,0,2,1,0)$	81	$(1,1,3,2,1,0)$	91	$(3,3,1,3,1,0)$
52	$(0,0,0,1,1,0)$	62	$(2,2,2,1,1,0)$	72	$(0,2,1,2,1,0)$	82	$(2,1,3,2,1,0)$	92	$(0,1,2,3,1,0)$
53	$(1,0,0,1,1,0)$	63	$(3,2,2,1,1,0)$	73	$(1,2,1,2,1,0)$	83	$(3,1,3,2,1,0)$	93	$(1,1,2,3,1,0)$
54	$(2,0,0,1,1,0)$	64	$(0,3,3,1,1,0)$	74	$(2,2,1,2,1,0)$	84	$(0,0,0,3,1,0)$	94	$(2,1,2,3,1,0)$
55	$(3,0,0,1,1,0)$	65	$(1,3,3,1,1,0)$	75	$(3,2,1,2,1,0)$	85	$(1,0,0,3,1,0)$	95	$(3,1,2,3,1,0)$
56	$(0,1,1,1,1,0)$	66	$(2,3,3,1,1,0)$	76	$(0,3,2,2,1,0)$	86	$(2,0,0,3,1,0)$	96	$(0,2,3,3,1,0)$
57	$(1,1,1,1,1,0)$	67	$(3,3,3,1,1,0)$	77	$(1,3,2,2,1,0)$	87	$(3,0,0,3,1,0)$	97	$(1,2,3,3,1,0)$
58	$(2,1,1,1,1,0)$	68	$(0,0,0,2,1,0)$	78	$(2,3,2,2,1,0)$	88	$(0,3,1,3,1,0)$	98	$(2,2,3,3,1,0)$
59	$(3,1,1,1,1,0)$	69	$(1,0,0,2,1,0)$	79	$(3,3,2,2,1,0)$	89	$(1,3,1,3,1,0)$	99	$(3,2,3,3,1,0)$

i	P_i

Strong generators for a group of order 5529600:

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_1, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&\alpha&0\\0&0&0&1\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&\alpha^2&0&0\\0&0&\alpha^2&0\\0&0&0&\alpha^2\end{bmatrix}_0,$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&\alpha\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&\alpha&1\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&\alpha&0&1\end{bmatrix}_0,$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\\alpha&0&0&1\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&1&0&0\\\alpha&0&1&0\\0&0&0&1\end{bmatrix}_0,$$

$$\begin{bmatrix}1&0&0&0\\\alpha&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_0, \begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_0$$

Poset classification up to depth 5

3 The orbits

3.1 Number of orbits at depth

Depth	Nb of orbits
0	1
1	1
2	1
3	1
4	3
5	4

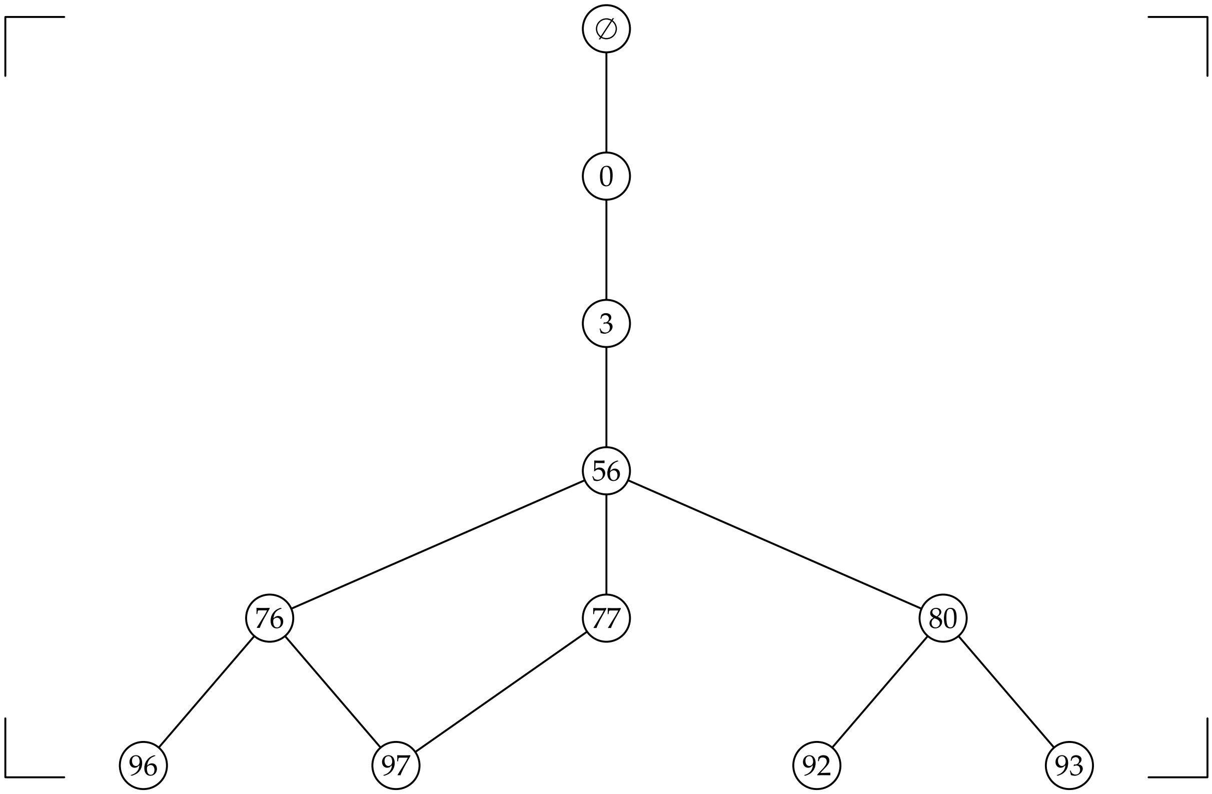
3.2 Orbit representatives: overview

N = node
D = depth or level
O = orbit with a level
Rep = orbit representative
SO = (order of stabilizer, orbit length)
L = number of live points
F = number of flags
Gen = number of generators for the stabilizer of the orbit rep.

Table 1: Orbit Representatives

N	D	O	Rep	SO	L	F	Gen
0	0	0	{ }	(5529600, 1)	100	1	11
1	1	0	{ 0 }	(55296, 100)	64	1	10
2	2	0	{ 0, 3 }	(1728, 3200)	36	1	7
3	3	0	{ 0, 3, 56 }	(144, 38400)	16	3	7
4	4	0	{ 0, 3, 56, 76 }	(288, 19200)	4	2	6
5	4	1	{ 0, 3, 56, 77 }	(96, 57600)	4	1	8
6	4	2	{ 0, 3, 56, 80 }	(72, 76800)	4	2	6
7	5	0	{ 0, 3, 56, 76, 96 }	(1440, 3840)			8
8	5	1	{ 0, 3, 56, 76, 97 }	(96, 57600)			5
9	5	2	{ 0, 3, 56, 80, 92 }	(360, 15360)			7
10	5	3	{ 0, 3, 56, 80, 93 }	(120, 46080)			6

4 The poset of orbits



5 Stabilizers and Schreier trees

5.1 Stabilizers and Schreier trees at level 0

Node 0 at Level 0 Orbit 0 / 1

$$\{\}_{5529600}$$

Strong generators for a group of order 5529600:

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_1, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&\alpha&0\\0&0&0&1\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&\alpha^2&0&0\\0&0&\alpha^2&0\\0&0&0&\alpha^2\end{bmatrix}_0,$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&\alpha\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&\alpha&1\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&\alpha&0&1\end{bmatrix}_0,$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\\alpha&0&0&1\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&1&0&0\\\alpha&0&1&0\\0&0&0&1\end{bmatrix}_0,$$

$$\begin{bmatrix}1&0&0&0\\\alpha&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_0, \begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_0$$

There are 1 extensions
Number of generators 11

Generators for the Schreier trees:
Generators for a group of order 5529600:

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_1 \begin{bmatrix}1&1&0&0\\\alpha&0&0&0\\\alpha^2&0&\alpha&1\\0&0&1&0\end{bmatrix}_0 \begin{bmatrix}1&1&0&0\\\alpha^2&0&0&0\\0&0&\alpha&1\\0&0&1&0\end{bmatrix}_0 \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&\alpha\end{bmatrix}_0$$

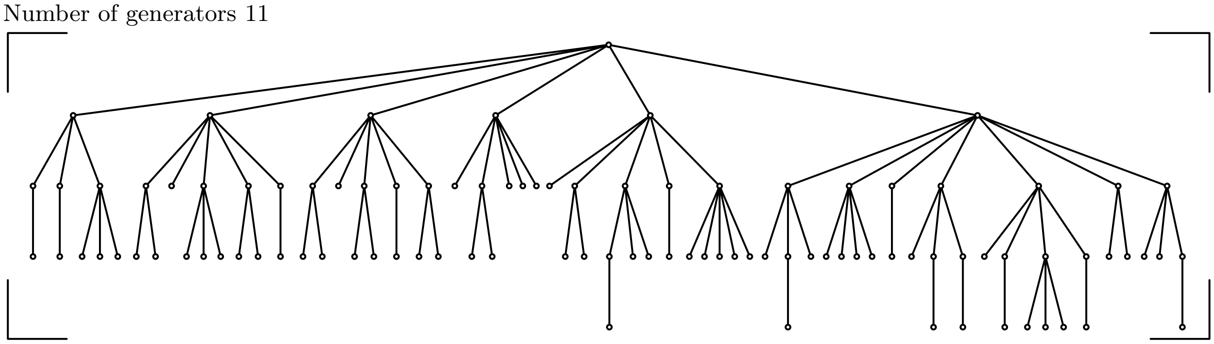
$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&\alpha&1&0\\0&0&0&1\end{bmatrix}_1 \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&\alpha&0&1\end{bmatrix}_0 \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\\alpha&0&0&1\end{bmatrix}_0 \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}_0$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\\alpha&0&1&0\\0&0&0&1\end{bmatrix}_0 \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&\alpha^2&\alpha&\alpha\\0&0&1&0\end{bmatrix}_0 \begin{bmatrix}1&\alpha^2&0&0\\\alpha^2&0&0&0\\1&0&1&1\\0&0&\alpha^2&0\end{bmatrix}_0$$

Orbit 0 / 1: Point 0 lies in an orbit of length 94 with average word length 3.75532 $H_{11} = 2.44651$

Node 0 at Level 0 Orbit 0 / 1 Tree 0 / 1

Number of generators 11



Extension number 0
Orbit representative 0
Flag orbit length 100
Flag orbit is defining new orbit 1 at level 1

5.2 Stabilizers and Schreier trees at level 1

Node 1 at Level 1 Orbit 0 / 1

$$\{0\}_{55296}$$

Strong generators for a group of order 55296:

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_1, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&\alpha&0\\0&0&0&1\end{bmatrix}_1, \begin{bmatrix}1&0&0&0\\0&\alpha^2&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_0,$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&\alpha\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&\alpha^2&0\\0&0&0&1\end{bmatrix}_1, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&\alpha&0\\0&\alpha&0&1\end{bmatrix}_0,$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\\alpha&0&0&1\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&\alpha&0&0\\0&0&\alpha&0\\0&0&1&\alpha^2\end{bmatrix}_1, \begin{bmatrix}1&0&0&0\\0&1&0&0\\\alpha^2&0&1&0\\0&0&0&1\end{bmatrix}_0,$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha & \alpha^2 & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & \alpha \end{bmatrix}_{0}$$

There are 1 extensions
Number of generators 10
Generators for the Schreier trees:
Generators for a group of order 55296:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha^2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{0} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \alpha \end{bmatrix}_{0}$$

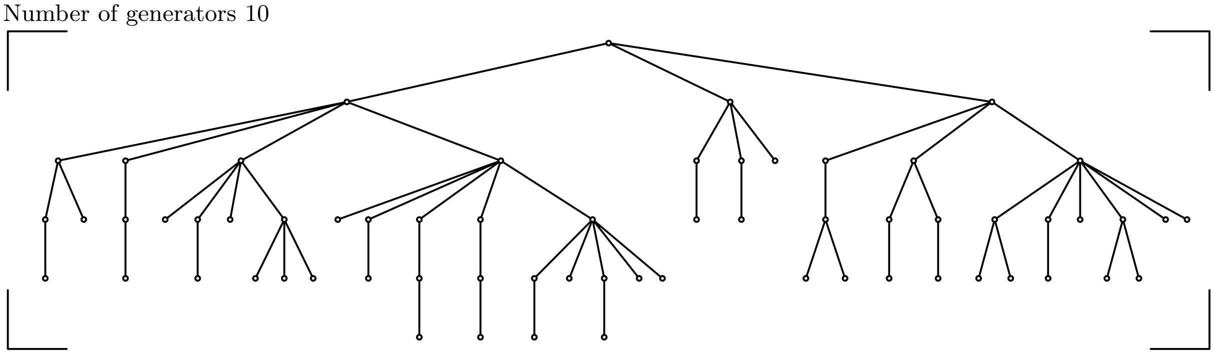
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & \alpha & 0 & 1 \end{bmatrix}_{0} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \alpha & 0 & 0 & 1 \end{bmatrix}_{0} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 1 & \alpha^2 \end{bmatrix}_{1}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \alpha^2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{0} \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha & \alpha^2 & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & \alpha \end{bmatrix}_{0}$$

Orbit 0 / 1: Point 3 lies in an orbit of length 64 with average word length 4.1875 $H_{10} = 2.42813$

Node 1 at Level 1 Orbit 0 / 1 Tree 0 / 1

Number of generators 10



Extension number 0
Orbit representative 3
Flag orbit length 64
Flag orbit is defining new orbit 2 at level 2

5.3 Stabilizers and Schreier trees at level 2

Node 2 at Level 2 Orbit 0 / 1

$$\{0,3\}_{1728}$$

Strong generators for a group of order 1728:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{1}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha^2 & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & \alpha \end{bmatrix}_{0}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha^2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{0},$$

$$\begin{bmatrix}1&0&0&0\\0&\alpha&0&0\\0&0&1&0\\0&0&0&\alpha^2\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&\alpha&0&0\\0&0&1&0\\0&\alpha^2&0&\alpha^2\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&\alpha&0&0\\\alpha&0&1&0\\0&\alpha^2&0&\alpha^2\end{bmatrix}_0,$$
$$\begin{bmatrix}0&1&0&0\\\alpha^2&0&0&0\\0&1&0&\alpha^2\\\alpha&0&\alpha&0\end{bmatrix}_0$$

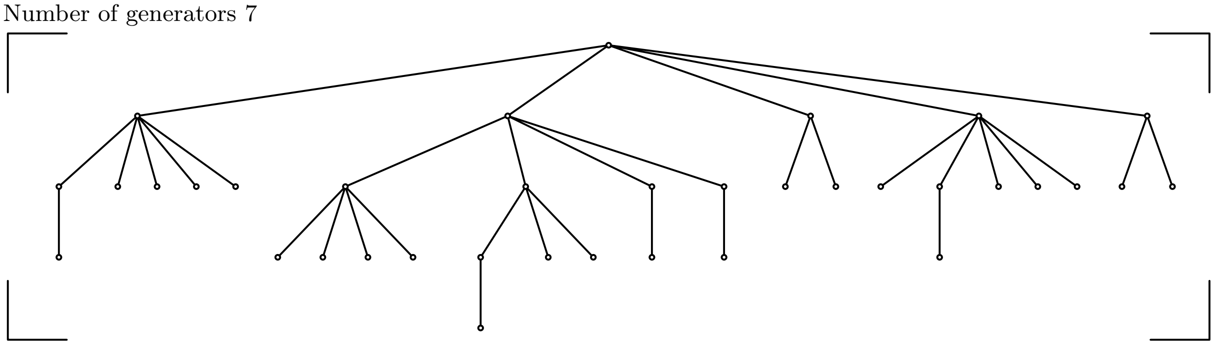
There are 1 extensions
Number of generators 7
Generators for the Schreier trees:
Generators for a group of order 1728:

$$\begin{bmatrix}0&1&0&0\\\alpha&0&0&0\\0&0&0&1\\1&0&1&0\end{bmatrix}_0 \begin{bmatrix}1&0&0&0\\0&\alpha^2&0&0\\0&0&\alpha&0\\0&0&0&\alpha\end{bmatrix}_0 \begin{bmatrix}1&0&0&0\\0&\alpha^2&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_0 \begin{bmatrix}1&0&0&0\\0&\alpha&0&0\\0&0&\alpha&0\\0&1&0&\alpha^2\end{bmatrix}_0$$
$$\begin{bmatrix}1&0&0&0\\0&\alpha&0&0\\0&0&1&0\\0&\alpha^2&0&\alpha^2\end{bmatrix}_0 \begin{bmatrix}0&1&0&0\\\alpha&0&0&0\\0&0&0&1\\1&0&1&0\end{bmatrix}_0 \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&\alpha&0&\alpha^2\end{bmatrix}_0$$

Orbit 0 / 1: Point 56 lies in an orbit of length 36 with average word length 3.16667 $H_7 = 2.43392$

Node 2 at Level 2 Orbit 0 / 1 Tree 0 / 1

Number of generators 7



Extension number 0
Orbit representative 56
Flag orbit length 36
Flag orbit is defining new orbit 3 at level 3

5.4 Stabilizers and Schreier trees at level 3

Node 3 at Level 3 Orbit 0 / 1

$$\{0, 3, 56\}_{144}$$

Strong generators for a group of order 144:

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_1, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&\alpha^2&0\\0&0&0&\alpha^2\end{bmatrix}_1, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&\alpha^2&0\\0&0&0&\alpha^2\end{bmatrix}_0,$$
$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\\alpha&0&\alpha^2&0\\0&\alpha&0&\alpha^2\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\1&1&0&0\\0&0&1&0\\0&0&1&1\end{bmatrix}_1, \begin{bmatrix}0&1&0&0\\1&0&0&0\\0&\alpha^2&0&\alpha^2\\\alpha^2&0&\alpha^2&0\end{bmatrix}_0,$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & \alpha^2 & \alpha^2 \\ 0 & 1 & 0 & \alpha^2 \end{bmatrix}_1$$

There are 3 extensions
Number of generators 7
Generators for the Schreier trees:
Generators for a group of order 144:

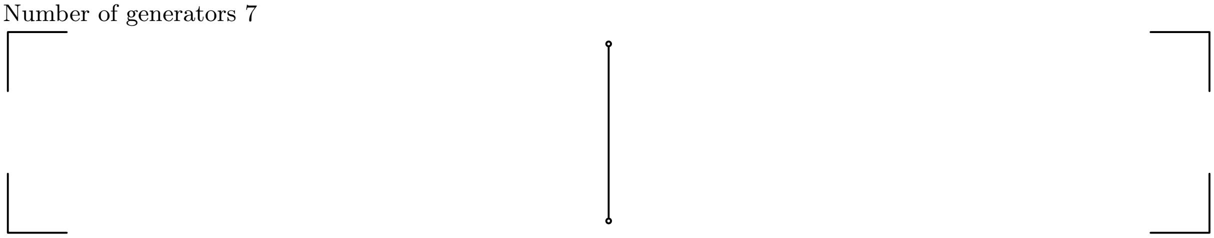
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha^2 & 0 \\ 0 & 0 & 0 & \alpha^2 \end{bmatrix}_0 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha^2 & 0 \\ 0 & 0 & 0 & \alpha^2 \end{bmatrix}_1 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha^2 & 0 \\ 0 & 0 & 0 & \alpha^2 \end{bmatrix}_0 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_1$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_1 \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & \alpha^2 & 0 & \alpha^2 \\ \alpha^2 & 0 & \alpha^2 & 0 \end{bmatrix}_0 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_0$$

Orbit 0 / 3: Point 76 lies in an orbit of length 2 with average word length 1.5 $H_7 = 0.564575$
Orbit 1 / 3: Point 77 lies in an orbit of length 6 with average word length 2.16667 $H_7 = 1.31812$
Orbit 2 / 3: Point 80 lies in an orbit of length 8 with average word length 2.75 $H_7 = 1.58848$

Node 3 at Level 3 Orbit 0 / 1 Tree 0 / 3

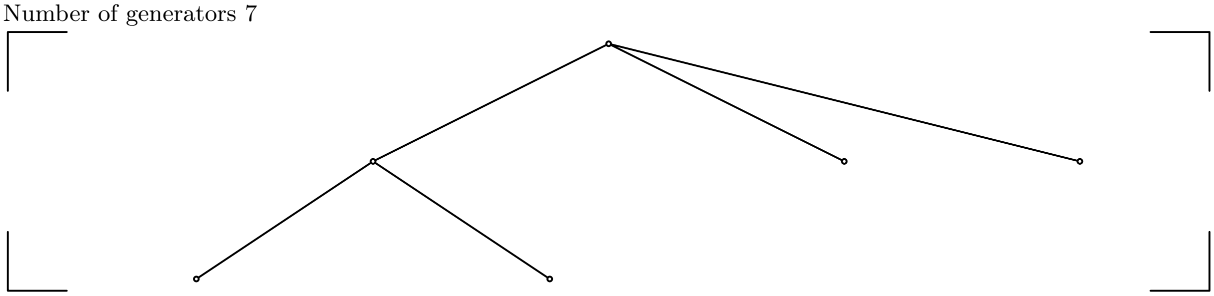
Number of generators 7



Extension number 0
Orbit representative 76
Flag orbit length 2
Flag orbit is defining new orbit 4 at level 4

Node 3 at Level 3 Orbit 0 / 1 Tree 1 / 3

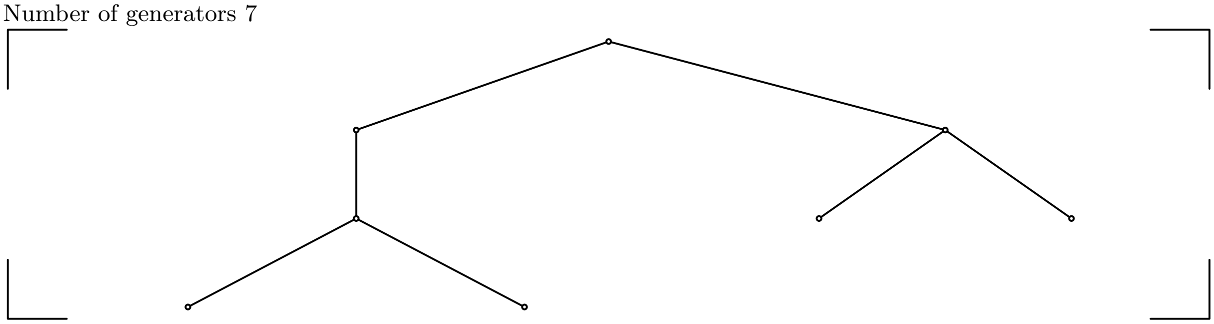
Number of generators 7



Extension number 1
Orbit representative 77
Flag orbit length 6
Flag orbit is defining new orbit 5 at level 4

Node 3 at Level 3 Orbit 0 / 1 Tree 2 / 3

Number of generators 7



Extension number 2
Orbit representative 80
Flag orbit length 8
Flag orbit is defining new orbit 6 at level 4

5.5 Stabilizers and Schreier trees at level 4

Node 4 at Level 4 Orbit 0 / 3

$$\{0, 3, 56, 76\}_{288}$$

Strong generators for a group of order 288:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha^2 & 0 \\ 0 & 0 & 0 & \alpha^2 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & \alpha^2 \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \alpha & 0 & \alpha^2 & 0 \\ 0 & \alpha & 0 & \alpha^2 \end{bmatrix}_0,$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_1, \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & \alpha^2 & 0 & \alpha^2 \\ \alpha^2 & 0 & \alpha^2 & 0 \end{bmatrix}_1, \begin{bmatrix} 1 & \alpha & 0 & 0 \\ \alpha & \alpha & 0 & 0 \\ 1 & \alpha & \alpha^2 & 1 \\ \alpha & \alpha & 1 & 1 \end{bmatrix}_1$$

There are 2 extensions
Number of generators 6
Generators for the Schreier trees:
Generators for a group of order 288:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_0 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & \alpha^2 \end{bmatrix}_1 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha^2 & 0 \\ 0 & 0 & 0 & \alpha^2 \end{bmatrix}_0 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_1$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha^2 & 0 \\ 0 & 0 & 0 & \alpha^2 \end{bmatrix}_0 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_0$$

Orbit 0 / 2: Point 96 lies in an orbit of length 1 with average word length 1 $H_6 = 0$
Orbit 1 / 2: Point 97 lies in an orbit of length 3 with average word length 1.66667 $H_6 = 0.898244$

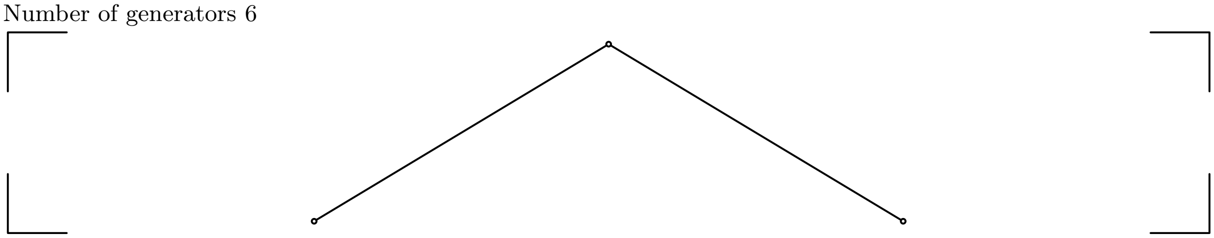
Node 4 at Level 4 Orbit 0 / 3 Tree 0 / 2

Number of generators 6

Extension number 0
Orbit representative 96
Flag orbit length 1
Flag orbit is defining new orbit 7 at level 5

Node 4 at Level 4 Orbit 0 / 3 Tree 1 / 2

Number of generators 6



Extension number 1
Orbit representative 97
Flag orbit length 3
Flag orbit is defining new orbit 8 at level 5

Node 5 at Level 4 Orbit 1 / 3

$$\{0,3,56,77\}_{96}$$

Strong generators for a group of order 96:

$$\begin{bmatrix}1&0&0&0\\0&\alpha&0&0\\0&0&\alpha&0\\0&1&0&\alpha^2\end{bmatrix}_1,\begin{bmatrix}1&0&0&0\\0&1&0&0\\\alpha^2&0&1&0\\0&\alpha^2&0&1\end{bmatrix}_0,\begin{bmatrix}1&0&0&0\\0&1&0&0\\\alpha&0&1&0\\0&\alpha&0&1\end{bmatrix}_0,$$

$$\begin{bmatrix}1&0&0&0\\1&1&0&0\\\alpha^2&0&1&0\\\alpha^2&\alpha^2&1&1\end{bmatrix}_1,\begin{bmatrix}1&0&0&0\\\alpha&\alpha^2&0&0\\1&0&\alpha^2&0\\\alpha^2&\alpha&1&\alpha\end{bmatrix}_1,\begin{bmatrix}1&1&0&0\\0&1&0&0\\1&1&\alpha^2&\alpha^2\\0&1&0&\alpha^2\end{bmatrix}_1,$$

$$\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&\alpha^2&0&\alpha\\\alpha^2&0&\alpha&0\end{bmatrix}_1,\begin{bmatrix}1&\alpha^2&0&0\\0&\alpha^2&0&0\\0&\alpha^2&\alpha&1\\0&\alpha^2&0&1\end{bmatrix}_0$$

There are 1 extensions
Number of generators 8
Generators for the Schreier trees:
Generators for a group of order 96:

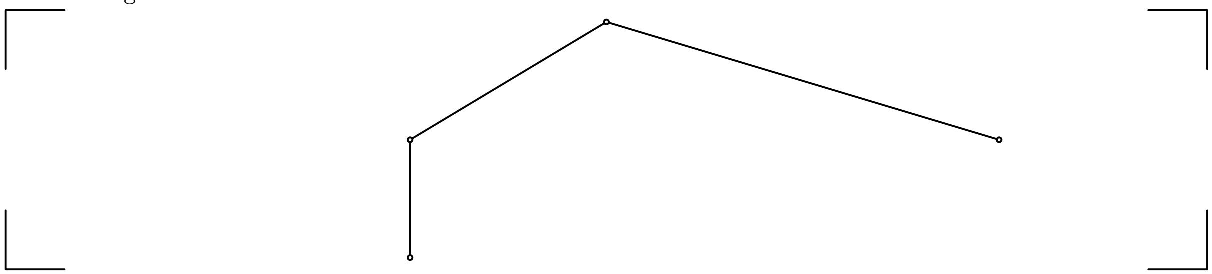
$$\begin{bmatrix}1&0&0&0\\0&\alpha&0&0\\0&0&\alpha&0\\0&1&0&\alpha^2\end{bmatrix}_1\begin{bmatrix}1&0&0&0\\0&1&0&0\\\alpha^2&0&1&0\\0&\alpha^2&0&1\end{bmatrix}_0\begin{bmatrix}1&0&0&0\\0&1&0&0\\\alpha&0&1&0\\0&\alpha&0&1\end{bmatrix}_0\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_0$$

$$\begin{bmatrix}1&0&0&0\\\alpha&\alpha^2&0&0\\1&0&\alpha^2&0\\\alpha^2&\alpha&1&\alpha\end{bmatrix}_1\begin{bmatrix}1&1&0&0\\0&1&0&0\\1&1&\alpha^2&\alpha^2\\0&1&0&\alpha^2\end{bmatrix}_1\begin{bmatrix}1&0&0&0\\\alpha&\alpha^2&0&0\\1&0&\alpha^2&0\\\alpha^2&\alpha&1&\alpha\end{bmatrix}_1\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_0$$

Orbit 0 / 1: Point 96 lies in an orbit of length 4 with average word length 2 $H_8 = 1$

Node 5 at Level 4 Orbit 1 / 3 Tree 0 / 1

Number of generators 8



Extension number 0
Orbit representative 96
Flag orbit length 4
Flag orbit is fused to node 4 extension 1
Fusion element:

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & \alpha^2 & 0 & 0 \\ 0 & 0 & \alpha^2 & \alpha^2 \\ 0 & 0 & 0 & \alpha \end{bmatrix}_1$$

Node 6 at Level 4 Orbit 2 / 3

$$\{0, 3, 56, 80\}_{72}$$

Strong generators for a group of order 72:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & \alpha \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha^2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \alpha \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_1,$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & \alpha \\ 0 & 0 & 0 & \alpha \end{bmatrix}_1, \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_1, \begin{bmatrix} 1 & \alpha & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & \alpha^2 & \alpha \\ 0 & 0 & \alpha^2 & \alpha^2 \end{bmatrix}_0$$

There are 2 extensions
Number of generators 6
Generators for the Schreier trees:
Generators for a group of order 72:

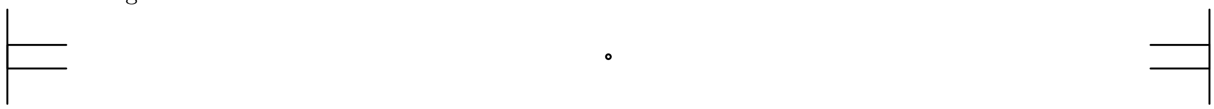
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & \alpha \end{bmatrix}_0 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_0 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_0 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & \alpha \end{bmatrix}_0$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & \alpha \end{bmatrix}_0 \begin{bmatrix} 1 & \alpha & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & \alpha^2 & \alpha \\ 0 & 0 & \alpha^2 & \alpha^2 \end{bmatrix}_0$$

Orbit 0 / 2: Point 92 lies in an orbit of length 1 with average word length 1 $H_6 = 0$
Orbit 1 / 2: Point 93 lies in an orbit of length 3 with average word length 2 $H_6 = 1$

Node 6 at Level 4 Orbit 2 / 3 Tree 0 / 2

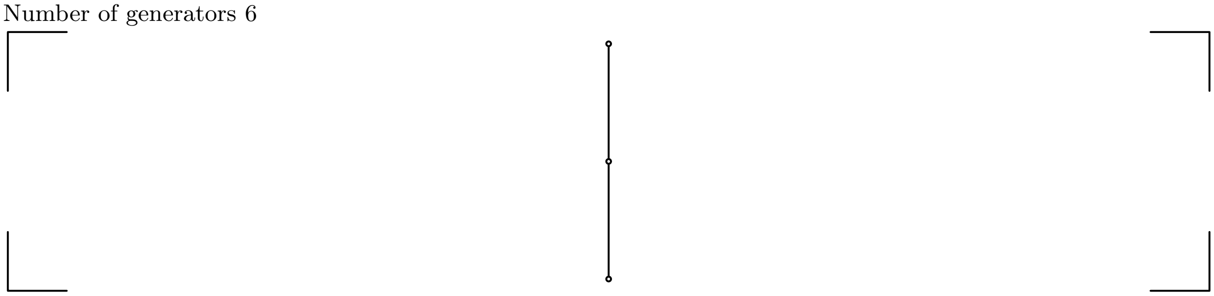
Number of generators 6



Extension number 0
Orbit representative 92
Flag orbit length 1
Flag orbit is defining new orbit 9 at level 5

Node 6 at Level 4 Orbit 2 / 3 Tree 1 / 2

Number of generators 6



Extension number 1
Orbit representative 93
Flag orbit length 3
Flag orbit is defining new orbit 10 at level 5

5.6 Stabilizers and Schreier trees at level 5

Node 7 at Level 5 Orbit 0 / 4

$$\{0, 3, 56, 76, 96\}_{1440}$$

Strong generators for a group of order 1440:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha^2 & 0 \\ 0 & 0 & 0 & \alpha^2 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & \alpha^2 \end{bmatrix}_1,$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \alpha & 0 & \alpha^2 & 0 \\ 0 & \alpha & 0 & \alpha^2 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha & \alpha^2 & 0 & 0 \\ \alpha^2 & 0 & \alpha^2 & 0 \\ 1 & \alpha & 1 & \alpha \end{bmatrix}_0,$$

$$\begin{bmatrix} 1 & \alpha & 0 & 0 \\ \alpha & \alpha & 0 & 0 \\ 1 & \alpha & \alpha^2 & 1 \\ \alpha & \alpha & 1 & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & \alpha^2 & 0 & 0 \\ 1 & \alpha & 0 & 0 \\ \alpha^2 & \alpha & 1 & \alpha^2 \\ \alpha^2 & 1 & 1 & \alpha \end{bmatrix}_1$$

There are 0 extensions
Number of generators 8

Node 8 at Level 5 Orbit 1 / 4

$$\{0, 3, 56, 76, 97\}_{96}$$

Strong generators for a group of order 96:

$$\begin{bmatrix}1&0&0&0\\0&\alpha&0&0\\0&0&\alpha^2&0\\0&0&0&1\end{bmatrix}_1, \begin{bmatrix}1&0&0&0\\0&1&0&0\\\alpha&0&1&0\\0&\alpha&0&1\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\0&1&0&0\\1&0&1&0\\0&1&0&1\end{bmatrix}_0,$$

$$\begin{bmatrix}1&0&0&0\\1&1&0&0\\0&0&1&0\\0&0&1&1\end{bmatrix}_1, \begin{bmatrix}0&1&0&0\\\alpha&0&0&0\\0&\alpha&0&1\\\alpha^2&0&\alpha&0\end{bmatrix}_0$$

There are 0 extensions
Number of generators 5

Node 9 at Level 5 Orbit 2 / 4

$$\{0,3,56,80,92\}_{360}$$

Strong generators for a group of order 360:

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}_1, \begin{bmatrix}1&0&0&0\\0&\alpha^2&0&0\\0&0&1&0\\0&0&0&\alpha\end{bmatrix}_1, \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&\alpha&0\\0&0&0&\alpha\end{bmatrix}_0,$$

$$\begin{bmatrix}1&0&0&0\\0&\alpha^2&0&0\\0&0&1&0\\0&0&0&\alpha\end{bmatrix}_0, \begin{bmatrix}1&0&0&0\\1&\alpha&0&0\\0&0&\alpha&0\\0&0&\alpha&1\end{bmatrix}_0, \begin{bmatrix}1&1&0&0\\0&1&0&0\\0&0&\alpha&\alpha\\0&0&0&\alpha\end{bmatrix}_1,$$

$$\begin{bmatrix}1&\alpha^2&0&0\\1&0&0&0\\0&0&\alpha&\alpha^2\\0&0&\alpha&0\end{bmatrix}_0$$

There are 0 extensions
Number of generators 7

Node 10 at Level 5 Orbit 3 / 4

$$\{0,3,56,80,93\}_{120}$$

Strong generators for a group of order 120:

$$\begin{bmatrix}1&0&0&0\\0&\alpha^2&0&0\\0&0&\alpha^2&0\\0&0&0&1\end{bmatrix}_1, \begin{bmatrix}1&0&0&0\\0&\alpha&0&0\\\alpha^2&0&\alpha&0\\0&\alpha^2&0&1\end{bmatrix}_1, \begin{bmatrix}1&0&0&0\\1&1&0&0\\0&0&1&0\\0&0&1&1\end{bmatrix}_1,$$

$$\begin{bmatrix}0&1&0&0\\\alpha^2&0&0&0\\0&0&0&\alpha^2\\0&0&1&0\end{bmatrix}_0, \begin{bmatrix}1&1&0&0\\0&1&0&0\\0&0&1&1\\0&0&0&1\end{bmatrix}_1, \begin{bmatrix}1&\alpha&0&0\\1&\alpha^2&0&0\\\alpha^2&0&1&\alpha^2\\\alpha^2&\alpha&1&\alpha\end{bmatrix}_1$$

There are 0 extensions
Number of generators 6

Classification of $5+1$ Configurations in $\mathrm{PG}(3,4)$

The order of the group is 1974067200
The group has 4 orbits on five plus one configurations in $\mathrm{PG}(3,4)$.

Of these, 1 impose 19 conditions.
Of these, 1 are associated with double sixes. They are:
0/1 is orbit 3/4 $\{0,3,56,80,93\}_{120}$ orbit length 46080

The overall number of five plus one configurations associated with double sixes in $\mathrm{PG}(3,4)$ is: 46080

Flag orbits for double sixes

The number of primary orbits below is 4
The number of primary orbits above is 1
The number of flag orbits is 1
The flag orbits are:

(1) Flag orbit 0 / 1 down=(3,0) up=(0,-1) is (0, 3, 56, 80, 93, 16, 340, 38, 61, 156, 0, 16, 340, 38, 61, 156, 165, 155, 72, 54, 25, 356, 0) with a stabilizer of order 120
Strong generators for a group of order 120:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha^2 & 0 & 0 \\ 0 & 0 & \alpha^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ \alpha^2 & 0 & \alpha & 0 \\ 0 & \alpha^2 & 0 & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_1,$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ \alpha^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha^2 \\ 0 & 0 & 1 & 0 \end{bmatrix}_0, \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & \alpha & 0 & 0 \\ 1 & \alpha^2 & 0 & 0 \\ \alpha^2 & 0 & 1 & \alpha^2 \\ \alpha^2 & \alpha & 1 & \alpha \end{bmatrix}_1$$

Double Sixes

The order of the group is 1974067200
The group has 1 orbits:

The orbits are:

(1) 0/1 {16, 340, 38, 61, 156, 165, 155, 72, 54, 25, 356, 0}$_{1440}$ orbit length 1370880
Strong generators for a group of order 1440:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha^2 & 0 & 0 \\ 0 & 0 & \alpha^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 1 & 0 & \alpha & 0 \\ 0 & \alpha & 0 & 1 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_1,$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^2 & 0 & \alpha^2 & 0 \\ \alpha^2 & \alpha^2 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}_1, \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & \alpha \\ \alpha & 0 & \alpha & 0 \\ \alpha & 1 & \alpha & 1 \end{bmatrix}_0$$

The overall number of objects is: 1370880

Flag orbits for surfaces

The number of primary orbits below is 1
The number of primary orbits above is 1
The number of flag orbits is 1
The flag orbits are:

(1) Flag orbit 0 / 1 down=(0,0) up=(0,-1) is (16, 340, 38, 61, 156, 165, 155, 72, 54, 25, 356, 0, 71, 55, 26, 100, 1, 138, 109, 345, 84, 85, 122, 110, 145, 139, 81) with a stabilizer of order 1440
Strong generators for a group of order 1440:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha^2 & 0 & 0 \\ 0 & 0 & \alpha^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 1 & 0 & \alpha & 0 \\ 0 & \alpha & 0 & 1 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_1,$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^2 & 0 & \alpha^2 & 0 \\ \alpha^2 & \alpha^2 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}_1, \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & \alpha \\ \alpha & 0 & \alpha & 0 \\ \alpha & 1 & \alpha & 1 \end{bmatrix}_0$$

Surfaces

The order of the group is 1974067200
The group has 1 orbits:

The orbits are:

(1) 0/1 {16, 340, 38, 61, 156, 165, 155, 72, 54, 25, 356, 0, 71, 55, 26, 100, 1, 138, 109, 345, 84, 85, 122, 110, 145, 139, 81}$_{51840}$ orbit length 38080
Strong generators for a group of order 51840:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_0,$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha^2 & 0 & 0 \\ 0 & 0 & \alpha^2 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 1 & 0 & \alpha & 0 \\ 0 & \alpha & 0 & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ \alpha & \alpha & \alpha & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_0,$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ \alpha & 1 & \alpha & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & \alpha^2 & \alpha^2 & 0 \\ 0 & 0 & \alpha^2 & 0 \\ 1 & 0 & \alpha^2 & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^2 & 0 & \alpha^2 & 0 \\ \alpha & \alpha^2 & 0 & 0 \\ \alpha^2 & 1 & \alpha & 1 \end{bmatrix}_1,$$

$$\begin{bmatrix} 1 & \alpha & \alpha & 0 \\ \alpha & 0 & 0 & 0 \\ \alpha & 0 & \alpha^2 & \alpha \\ 0 & 0 & \alpha & 0 \end{bmatrix}_0$$

The overall number of objects is: 38080

The Group $P\Gamma L(4,4)$

The order of the group is 1974067200

Cubic Surfaces with 27 Lines in $PG(3,4)$

The order of the group is 1974067200
The group has 1 orbits:

The orbits are:

(1) 0/1 {16, 340, 38, 61, 156, 165, 155, 72, 54, 25, 356, 0, 71, 55, 26, 100, 1, 138, 109, 345, 84, 85, 122, 110, 145, 139, 81}$_{51840}$ orbit length 38080
Strong generators for a group of order 51840:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_0,$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha^2 & 0 & 0 \\ 0 & 0 & \alpha^2 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}_0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 1 & 0 & \alpha & 0 \\ 0 & \alpha & 0 & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ \alpha & \alpha & \alpha & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}_0,$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ \alpha & 1 & \alpha & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & \alpha^2 & \alpha^2 & 0 \\ 0 & 0 & \alpha^2 & 0 \\ 1 & 0 & \alpha^2 & 1 \end{bmatrix}_1, \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha^2 & 0 & \alpha^2 & 0 \\ \alpha & \alpha^2 & 0 & 0 \\ \alpha^2 & 1 & \alpha & 1 \end{bmatrix}_1,$$

$$\begin{bmatrix} 1 & \alpha & \alpha & 0 \\ \alpha & 0 & 0 & 0 \\ \alpha & 0 & \alpha^2 & \alpha \\ 0 & 0 & \alpha & 0 \end{bmatrix}_0$$

The overall number of objects is: 38080

Surface 4#0

The equation

The equation of the surface is :

$$X_0^2X_3 + X_1^2X_2 + X_1X_2^2 + X_0X_3^2 = 0$$

Number of points on the surface 45
The automorphism group of the surface has order 51840
The automorphism group is the following group
Strong generators for a group of order 51840:

$$\left[\begin{array}{cccc}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{array}\right]_1, \left[\begin{array}{cccc}1&0&0&0\\0&\alpha&0&0\\0&0&\alpha&0\\0&0&0&1\end{array}\right]_1, \left[\begin{array}{cccc}1&0&0&0\\0&\alpha&0&0\\0&0&\alpha&0\\0&0&0&1\end{array}\right]_0,$$

$$\left[\begin{array}{cccc}1&0&0&0\\0&\alpha^2&0&0\\0&0&\alpha^2&0\\1&0&0&1\end{array}\right]_0, \left[\begin{array}{cccc}1&0&0&0\\0&\alpha&0&0\\1&0&\alpha&0\\0&\alpha&0&1\end{array}\right]_1, \left[\begin{array}{cccc}1&0&0&0\\0&\alpha&0&0\\\alpha&\alpha&\alpha&0\\0&1&0&1\end{array}\right]_0,$$

$$\left[\begin{array}{cccc}1&0&0&0\\\alpha^2&1&0&0\\1&0&1&0\\\alpha&1&\alpha&1\end{array}\right]_1, \left[\begin{array}{cccc}1&0&0&0\\1&\alpha^2&\alpha^2&0\\0&0&\alpha^2&0\\1&0&\alpha^2&1\end{array}\right]_1, \left[\begin{array}{cccc}1&0&0&0\\\alpha^2&0&\alpha^2&0\\\alpha&\alpha^2&0&0\\\alpha^2&1&\alpha&1\end{array}\right]_1,$$

$$\left[\begin{array}{cccc}1&\alpha&\alpha&0\\\alpha&0&0&0\\\alpha&0&\alpha^2&\alpha\\0&0&\alpha&0\end{array}\right]_0$$

General information

Plane types by points:

$$13^{45},\ 9^{40}$$

Type of pts on lines:

$$5^{27}$$

Type of lines on point:

$$3^{45}$$

Type iso of tritangent planes:

$$2^{45}$$

The 27 lines

$$\ell_0 = a_1 = \left[\begin{array}{cccc}1&0&\alpha^2&0\\0&1&1&\alpha\end{array}\right]_{72}$$

$$\ell_1 = a_2 = \left[\begin{array}{cccc}1&0&\alpha&0\\0&1&0&\alpha^2\end{array}\right]_{54}$$

$$\ell_2 = a_3 = \left[\begin{array}{cccc}1&0&1&0\\0&1&0&1\end{array}\right]_{25}$$

$$\ell_3 = a_4 = \left[\begin{array}{cccc}0&0&1&0\\0&0&0&1\end{array}\right]_{356}$$

$$\ell_4 = a_5 = \left[\begin{array}{cccc}1&0&0&0\\0&1&0&0\end{array}\right]_0$$

$$\ell_5 = a_6 = \left[\begin{array}{cccc}1&0&\alpha^2&1\\0&1&0&\alpha\end{array}\right]_{155}$$

$$\ell_6 = b_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{340}$$

$$\ell_7 = b_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{38}$$

$$\ell_8 = b_3 = \begin{bmatrix} 1 & \alpha & 0 & 0 \\ 0 & 0 & 1 & \alpha^2 \end{bmatrix}_{61}$$

$$\ell_9 = b_4 = \begin{bmatrix} 1 & 0 & \alpha^2 & 1 \\ 0 & 1 & 1 & \alpha \end{bmatrix}_{156}$$

$$\ell_{10} = b_5 = \begin{bmatrix} 1 & \alpha^2 & 0 & 1 \\ 0 & 0 & 1 & \alpha \end{bmatrix}_{165}$$

$$\ell_{11} = b_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{16}$$

$$\ell_{12} = c_{12} = \begin{bmatrix} 1 & 0 & \alpha & 1 \\ 0 & 1 & 0 & \alpha^2 \end{bmatrix}_{138}$$

$$\ell_{13} = c_{13} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}_{109}$$

$$\ell_{14} = c_{14} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{345}$$

$$\ell_{15} = c_{15} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{84}$$

$$\ell_{16} = c_{16} = \begin{bmatrix} 1 & 0 & \alpha^2 & 0 \\ 0 & 1 & 0 & \alpha \end{bmatrix}_{71}$$

$$\ell_{17} = c_{23} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{85}$$

$$\ell_{18} = c_{24} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{122}$$

$$\ell_{19} = c_{25} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{110}$$

$$\ell_{20} = c_{26} = \begin{bmatrix} 1 & 0 & \alpha & 0 \\ 0 & 1 & 1 & \alpha^2 \end{bmatrix}_{55}$$

$$\ell_{21} = c_{34} = \begin{bmatrix} 1 & \alpha & 0 & 1 \\ 0 & 0 & 1 & \alpha^2 \end{bmatrix}_{145}$$

$$\ell_{22} = c_{35} = \begin{bmatrix} 1 & 0 & \alpha & 1 \\ 0 & 1 & 1 & \alpha^2 \end{bmatrix}_{139}$$

$$\ell_{23} = c_{36} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}_{26}$$

$$\ell_{24} = c_{45} = \begin{bmatrix} 1 & \alpha^2 & 0 & 0 \\ 0 & 0 & 1 & \alpha \end{bmatrix}_{81}$$

$$\ell_{25} = c_{46} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{100}$$

$$\ell_{26} = c_{56} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}_{1}$$

Points on surface

All Points

The surface has 45 points:

	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	10	11	12	13	14	23	26	27	30	31
2	34	35	38	39	42	47	48	51	52	53
3	54	59	60	61	62	67	68	69	70	75
4	76	79	80	81	82	0	0	0	0	0

The points on the surface are:

$0 : P_0 = P_0 = (1,0,0,0)$
$1 : P_1 = P_1 = (0,1,0,0)$
$2 : P_2 = P_2 = (0,0,1,0)$
$3 : P_3 = P_3 = (0,0,0,1)$
$4 : P_4 = P_4 = (1,1,1,1)$
$5 : P_5 = P_5 = (1,1,0,0)$
$6 : P_6 = P_6 = (2,1,0,0)$
$7 : P_7 = P_7 = (3,1,0,0)$
$8 : P_8 = P_8 = (1,0,1,0)$
$9 : P_9 = P_9 = (2,0,1,0)$
$10 : P_{10} = P_{10} = (3,0,1,0)$
$11 : P_{11} = P_{11} = (0,1,1,0)$
$12 : P_{12} = P_{12} = (1,1,1,0)$
$13 : P_{13} = P_{13} = (2,1,1,0)$
$14 : P_{14} = P_{14} = (3,1,1,0)$
$15 : P_{15} = P_{23} = (1,0,0,1)$
$16 : P_{16} = P_{26} = (0,1,0,1)$
$17 : P_{17} = P_{27} = (1,1,0,1)$
$18 : P_{18} = P_{30} = (0,2,0,1)$
$19 : P_{19} = P_{31} = (1,2,0,1)$
$20 : P_{20} = P_{34} = (0,3,0,1)$
$21 : P_{21} = P_{35} = (1,3,0,1)$
$22 : P_{22} = P_{38} = (0,0,1,1)$
$23 : P_{23} = P_{39} = (1,0,1,1)$
$24 : P_{24} = P_{42} = (0,1,1,1)$
$25 : P_{25} = P_{47} = (2,2,1,1)$
$26 : P_{26} = P_{48} = (3,2,1,1)$
$27 : P_{27} = P_{51} = (2,3,1,1)$
$28 : P_{28} = P_{52} = (3,3,1,1)$
$29 : P_{29} = P_{53} = (0,0,2,1)$
$30 : P_{30} = P_{54} = (1,0,2,1)$
$31 : P_{31} = P_{59} = (2,1,2,1)$
$32 : P_{32} = P_{60} = (3,1,2,1)$
$33 : P_{33} = P_{61} = (0,2,2,1)$
$34 : P_{34} = P_{62} = (1,2,2,1)$
$35 : P_{35} = P_{67} = (2,3,2,1)$
$36 : P_{36} = P_{68} = (3,3,2,1)$
$37 : P_{37} = P_{69} = (0,0,3,1)$
$38 : P_{38} = P_{70} = (1,0,3,1)$
$39 : P_{39} = P_{75} = (2,1,3,1)$
$40 : P_{40} = P_{76} = (3,1,3,1)$
$41 : P_{41} = P_{79} = (2,2,3,1)$
$42 : P_{42} = P_{80} = (3,2,3,1)$
$43 : P_{43} = P_{81} = (0,3,3,1)$
$44 : P_{44} = P_{82} = (1,3,3,1)$

Eckardt Points

The surface has 45 Eckardt points:

	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	10	11	12	13	14	23	26	27	30	31
2	34	35	38	39	42	47	48	51	52	53
3	54	59	60	61	62	67	68	69	70	75
4	76	79	80	81	82	0	0	0	0	0

	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	10	11	12	13	14	15	16	17	18	19
2	20	21	22	23	24	25	26	27	28	29
3	30	31	32	33	34	35	36	37	38	39
4	40	41	42	43	44	0	0	0	0	0

The Eckardt points on the surface are:

$$E_0 = P_0 = P_0 = (1,0,0,0) = \ell_4 \cap \ell_{11} \cap \ell_{26} = a_5 \cap b_6 \cap c_{56}$$

$$E_1 = P_1 = P_1 = (0,1,0,0) = \ell_4 \cap \ell_6 \cap \ell_{15} = a_5 \cap b_1 \cap c_{15}$$
$$E_2 = P_2 = P_2 = (0,0,1,0) = \ell_3 \cap \ell_{11} \cap \ell_{25} = a_4 \cap b_6 \cap c_{46}$$
$$E_3 = P_3 = P_3 = (0,0,0,1) = \ell_3 \cap \ell_6 \cap \ell_{14} = a_4 \cap b_1 \cap c_{14}$$
$$E_4 = P_4 = P_4 = (1,1,1,1) = \ell_2 \cap \ell_7 \cap \ell_{17} = a_3 \cap b_2 \cap c_{23}$$
$$E_5 = P_5 = P_5 = (1,1,0,0) = \ell_4 \cap \ell_7 \cap \ell_{19} = a_5 \cap b_2 \cap c_{25}$$
$$E_6 = P_6 = P_6 = (2,1,0,0) = \ell_4 \cap \ell_9 \cap \ell_{24} = a_5 \cap b_4 \cap c_{45}$$
$$E_7 = P_7 = P_7 = (3,1,0,0) = \ell_4 \cap \ell_8 \cap \ell_{22} = a_5 \cap b_3 \cap c_{35}$$
$$E_8 = P_8 = P_8 = (1,0,1,0) = \ell_2 \cap \ell_{11} \cap \ell_{23} = a_3 \cap b_6 \cap c_{36}$$
$$E_9 = P_9 = P_9 = (2,0,1,0) = \ell_0 \cap \ell_{11} \cap \ell_{16} = a_1 \cap b_6 \cap c_{16}$$
$$E_{10} = P_{10} = P_{10} = (3,0,1,0) = \ell_1 \cap \ell_{11} \cap \ell_{20} = a_2 \cap b_6 \cap c_{26}$$
$$E_{11} = P_{11} = P_{11} = (0,1,1,0) = \ell_{14} \cap \ell_{17} \cap \ell_{26} = c_{14} \cap c_{23} \cap c_{56}$$
$$E_{12} = P_{12} = P_{12} = (1,1,1,0) = \ell_{13} \cap \ell_{18} \cap \ell_{26} = c_{13} \cap c_{24} \cap c_{56}$$
$$E_{13} = P_{13} = P_{13} = (2,1,1,0) = \ell_5 \cap \ell_{10} \cap \ell_{26} = a_6 \cap b_5 \cap c_{56}$$
$$E_{14} = P_{14} = P_{14} = (3,1,1,0) = \ell_{12} \cap \ell_{21} \cap \ell_{26} = c_{12} \cap c_{34} \cap c_{56}$$
$$E_{15} = P_{15} = P_{23} = (1,0,0,1) = \ell_{15} \cap \ell_{17} \cap \ell_{25} = c_{15} \cap c_{23} \cap c_{46}$$
$$E_{16} = P_{16} = P_{26} = (0,1,0,1) = \ell_2 \cap \ell_6 \cap \ell_{13} = a_3 \cap b_1 \cap c_{13}$$
$$E_{17} = P_{17} = P_{27} = (1,1,0,1) = \ell_{15} \cap \ell_{18} \cap \ell_{23} = c_{15} \cap c_{24} \cap c_{36}$$
$$E_{18} = P_{18} = P_{30} = (0,2,0,1) = \ell_1 \cap \ell_6 \cap \ell_{12} = a_2 \cap b_1 \cap c_{12}$$
$$E_{19} = P_{19} = P_{31} = (1,2,0,1) = \ell_{15} \cap \ell_{20} \cap \ell_{21} = c_{15} \cap c_{26} \cap c_{34}$$
$$E_{20} = P_{20} = P_{34} = (0,3,0,1) = \ell_5 \cap \ell_6 \cap \ell_{16} = a_6 \cap b_1 \cap c_{16}$$
$$E_{21} = P_{21} = P_{35} = (1,3,0,1) = \ell_0 \cap \ell_{10} \cap \ell_{15} = a_1 \cap b_5 \cap c_{15}$$
$$E_{22} = P_{22} = P_{38} = (0,0,1,1) = \ell_3 \cap \ell_7 \cap \ell_{18} = a_4 \cap b_2 \cap c_{24}$$
$$E_{23} = P_{23} = P_{39} = (1,0,1,1) = \ell_{13} \cap \ell_{19} \cap \ell_{25} = c_{13} \cap c_{25} \cap c_{46}$$
$$E_{24} = P_{24} = P_{42} = (0,1,1,1) = \ell_{14} \cap \ell_{19} \cap \ell_{23} = c_{14} \cap c_{25} \cap c_{36}$$
$$E_{25} = P_{25} = P_{47} = (2,2,1,1) = \ell_5 \cap \ell_7 \cap \ell_{20} = a_6 \cap b_2 \cap c_{26}$$
$$E_{26} = P_{26} = P_{48} = (3,2,1,1) = \ell_1 \cap \ell_{10} \cap \ell_{19} = a_2 \cap b_5 \cap c_{25}$$
$$E_{27} = P_{27} = P_{51} = (2,3,1,1) = \ell_{16} \cap \ell_{19} \cap \ell_{21} = c_{16} \cap c_{25} \cap c_{34}$$
$$E_{28} = P_{28} = P_{52} = (3,3,1,1) = \ell_0 \cap \ell_7 \cap \ell_{12} = a_1 \cap b_2 \cap c_{12}$$
$$E_{29} = P_{29} = P_{53} = (0,0,2,1) = \ell_3 \cap \ell_8 \cap \ell_{21} = a_4 \cap b_3 \cap c_{34}$$
$$E_{30} = P_{30} = P_{54} = (1,0,2,1) = \ell_{12} \cap \ell_{22} \cap \ell_{25} = c_{12} \cap c_{35} \cap c_{46}$$
$$E_{31} = P_{31} = P_{59} = (2,1,2,1) = \ell_2 \cap \ell_{10} \cap \ell_{22} = a_3 \cap b_5 \cap c_{35}$$
$$E_{32} = P_{32} = P_{60} = (3,1,2,1) = \ell_5 \cap \ell_8 \cap \ell_{23} = a_6 \cap b_3 \cap c_{36}$$
$$E_{33} = P_{33} = P_{61} = (0,2,2,1) = \ell_{14} \cap \ell_{20} \cap \ell_{22} = c_{14} \cap c_{26} \cap c_{35}$$
$$E_{34} = P_{34} = P_{62} = (1,2,2,1) = \ell_1 \cap \ell_8 \cap \ell_{17} = a_2 \cap b_3 \cap c_{23}$$
$$E_{35} = P_{35} = P_{67} = (2,3,2,1) = \ell_0 \cap \ell_8 \cap \ell_{13} = a_1 \cap b_3 \cap c_{13}$$
$$E_{36} = P_{36} = P_{68} = (3,3,2,1) = \ell_{16} \cap \ell_{18} \cap \ell_{22} = c_{16} \cap c_{24} \cap c_{35}$$
$$E_{37} = P_{37} = P_{69} = (0,0,3,1) = \ell_3 \cap \ell_{10} \cap \ell_{24} = a_4 \cap b_5 \cap c_{45}$$
$$E_{38} = P_{38} = P_{70} = (1,0,3,1) = \ell_5 \cap \ell_9 \cap \ell_{25} = a_6 \cap b_4 \cap c_{46}$$
$$E_{39} = P_{39} = P_{75} = (2,1,3,1) = \ell_{12} \cap \ell_{23} \cap \ell_{24} = c_{12} \cap c_{36} \cap c_{45}$$
$$E_{40} = P_{40} = P_{76} = (3,1,3,1) = \ell_2 \cap \ell_9 \cap \ell_{21} = a_3 \cap b_4 \cap c_{34}$$
$$E_{41} = P_{41} = P_{79} = (2,2,3,1) = \ell_1 \cap \ell_9 \cap \ell_{18} = a_2 \cap b_4 \cap c_{24}$$
$$E_{42} = P_{42} = P_{80} = (3,2,3,1) = \ell_{13} \cap \ell_{20} \cap \ell_{24} = c_{13} \cap c_{26} \cap c_{45}$$
$$E_{43} = P_{43} = P_{81} = (0,3,3,1) = \ell_0 \cap \ell_9 \cap \ell_{14} = a_1 \cap b_4 \cap c_{14}$$
$$E_{44} = P_{44} = P_{82} = (1,3,3,1) = \ell_{16} \cap \ell_{17} \cap \ell_{24} = c_{16} \cap c_{23} \cap c_{45}$$

The Eckardt points on the surface are:

$$
\begin{aligned}
E_0 &= P_0 = \mathbf{P}(1,0,0,0) = a_5 \cap b_6 \cap c_{56},\\
E_1 &= P_1 = \mathbf{P}(0,1,0,0) = a_5 \cap b_1 \cap c_{15},\\
E_2 &= P_2 = \mathbf{P}(0,0,1,0) = a_4 \cap b_6 \cap c_{46},\\
E_3 &= P_3 = \mathbf{P}(0,0,0,1) = a_4 \cap b_1 \cap c_{14},\\
E_4 &= P_4 = \mathbf{P}(1,1,1,1) = a_3 \cap b_2 \cap c_{23},\\
E_5 &= P_5 = \mathbf{P}(1,1,0,0) = a_5 \cap b_2 \cap c_{25},\\
E_6 &= P_6 = \mathbf{P}(\alpha,1,0,0) = a_5 \cap b_4 \cap c_{45},\\
E_7 &= P_7 = \mathbf{P}(\alpha^2,1,0,0) = a_5 \cap b_3 \cap c_{35},\\
E_8 &= P_8 = \mathbf{P}(1,0,1,0) = a_3 \cap b_6 \cap c_{36},\\
E_9 &= P_9 = \mathbf{P}(\alpha,0,1,0) = a_1 \cap b_6 \cap c_{16},\\
E_{10} &= P_{10} = \mathbf{P}(\alpha^2,0,1,0) = a_2 \cap b_6 \cap c_{26},\\
E_{11} &= P_{11} = \mathbf{P}(0,1,1,0) = c_{14} \cap c_{23} \cap c_{56},\\
E_{12} &= P_{12} = \mathbf{P}(1,1,1,0) = c_{13} \cap c_{24} \cap c_{56},\\
E_{13} &= P_{13} = \mathbf{P}(\alpha,1,1,0) = a_6 \cap b_5 \cap c_{56},\\
E_{14} &= P_{14} = \mathbf{P}(\alpha^2,1,1,0) = c_{12} \cap c_{34} \cap c_{56},\\
E_{15} &= P_{23} = \mathbf{P}(1,0,0,1) = c_{15} \cap c_{23} \cap c_{46},\\
E_{16} &= P_{26} = \mathbf{P}(0,1,0,1) = a_3 \cap b_1 \cap c_{13},\\
E_{17} &= P_{27} = \mathbf{P}(1,1,0,1) = c_{15} \cap c_{24} \cap c_{36},\\
E_{18} &= P_{30} = \mathbf{P}(0,\alpha,0,1) = a_2 \cap b_1 \cap c_{12},\\
E_{19} &= P_{31} = \mathbf{P}(1,\alpha,0,1) = c_{15} \cap c_{26} \cap c_{34},\\
E_{20} &= P_{34} = \mathbf{P}(0,\alpha^2,0,1) = a_6 \cap b_1 \cap c_{16},\\
E_{21} &= P_{35} = \mathbf{P}(1,\alpha^2,0,1) = a_1 \cap b_5 \cap c_{15},\\
E_{22} &= P_{38} = \mathbf{P}(0,0,1,1) = a_4 \cap b_2 \cap c_{24},\\
E_{23} &= P_{39} = \mathbf{P}(1,0,1,1) = c_{13} \cap c_{25} \cap c_{46},\\
E_{24} &= P_{42} = \mathbf{P}(0,1,1,1) = c_{14} \cap c_{25} \cap c_{36},\\
E_{25} &= P_{47} = \mathbf{P}(\alpha,\alpha,1,1) = a_6 \cap b_2 \cap c_{26},\\
E_{26} &= P_{48} = \mathbf{P}(\alpha^2,\alpha,1,1) = a_2 \cap b_5 \cap c_{25},\\
E_{27} &= P_{51} = \mathbf{P}(\alpha,\alpha^2,1,1) = c_{16} \cap c_{25} \cap c_{34},\\
E_{28} &= P_{52} = \mathbf{P}(\alpha^2,\alpha^2,1,1) = a_1 \cap b_2 \cap c_{12},\\
E_{29} &= P_{53} = \mathbf{P}(0,0,\alpha,1) = a_4 \cap b_3 \cap c_{34},\\
E_{30} &= P_{54} = \mathbf{P}(1,0,\alpha,1) = c_{12} \cap c_{35} \cap c_{46},\\
E_{31} &= P_{59} = \mathbf{P}(\alpha,1,\alpha,1) = a_3 \cap b_5 \cap c_{35},\\
E_{32} &= P_{60} = \mathbf{P}(\alpha^2,1,\alpha,1) = a_6 \cap b_3 \cap c_{36},\\
E_{33} &= P_{61} = \mathbf{P}(0,\alpha,\alpha,1) = c_{14} \cap c_{26} \cap c_{35},\\
E_{34} &= P_{62} = \mathbf{P}(1,\alpha,\alpha,1) = a_2 \cap b_3 \cap c_{23},\\
E_{35} &= P_{67} = \mathbf{P}(\alpha,\alpha^2,\alpha,1) = a_1 \cap b_3 \cap c_{13},\\
E_{36} &= P_{68} = \mathbf{P}(\alpha^2,\alpha^2,\alpha,1) = c_{16} \cap c_{24} \cap c_{35},\\
E_{37} &= P_{69} = \mathbf{P}(0,0,\alpha^2,1) = a_4 \cap b_5 \cap c_{45},\\
E_{38} &= P_{70} = \mathbf{P}(1,0,\alpha^2,1) = a_6 \cap b_4 \cap c_{46},\\
E_{39} &= P_{75} = \mathbf{P}(\alpha,1,\alpha^2,1) = c_{12} \cap c_{36} \cap c_{45},\\
E_{40} &= P_{76} = \mathbf{P}(\alpha^2,1,\alpha^2,1) = a_3 \cap b_4 \cap c_{34},\\
E_{41} &= P_{79} = \mathbf{P}(\alpha,\alpha,\alpha^2,1) = a_2 \cap b_4 \cap c_{24},\\
E_{42} &= P_{80} = \mathbf{P}(\alpha^2,\alpha,\alpha^2,1) = c_{13} \cap c_{26} \cap c_{45},
\end{aligned}
$$

$$E_{43} = P_{81} = \mathbf{P}(0, \alpha^2, \alpha^2, 1) = a_1 \cap b_4 \cap c_{14},$$
$$E_{44} = P_{82} = \mathbf{P}(1, \alpha^2, \alpha^2, 1) = c_{16} \cap c_{23} \cap c_{45}.$$

Double Points

The surface has 0 Double points:

	0	1	2	3	4	5	6	7	8	9
	0	1	2	3	4	5	6	7	8	9

The Double points on the surface are:

The double points on the surface are:

Points on lines

Line 0 = a_1 has 5 points: $\{P_i \mid i \in \{9, 21, 28, 35, 43\}\}$
Line 1 = a_2 has 5 points: $\{P_i \mid i \in \{10, 18, 26, 34, 41\}\}$
Line 2 = a_3 has 5 points: $\{P_i \mid i \in \{4, 8, 16, 31, 40\}\}$
Line 3 = a_4 has 5 points: $\{P_i \mid i \in \{2, 3, 22, 29, 37\}\}$
Line 4 = a_5 has 5 points: $\{P_i \mid i \in \{0, 1, 5, 6, 7\}\}$
Line 5 = a_6 has 5 points: $\{P_i \mid i \in \{13, 20, 25, 32, 38\}\}$
Line 6 = b_1 has 5 points: $\{P_i \mid i \in \{1, 3, 16, 18, 20\}\}$
Line 7 = b_2 has 5 points: $\{P_i \mid i \in \{4, 5, 22, 25, 28\}\}$
Line 8 = b_3 has 5 points: $\{P_i \mid i \in \{7, 29, 32, 34, 35\}\}$
Line 9 = b_4 has 5 points: $\{P_i \mid i \in \{6, 38, 40, 41, 43\}\}$
Line 10 = b_5 has 5 points: $\{P_i \mid i \in \{13, 21, 26, 31, 37\}\}$
Line 11 = b_6 has 5 points: $\{P_i \mid i \in \{0, 2, 8, 9, 10\}\}$
Line 12 = c_{12} has 5 points: $\{P_i \mid i \in \{14, 18, 28, 30, 39\}\}$
Line 13 = c_{13} has 5 points: $\{P_i \mid i \in \{12, 16, 23, 35, 42\}\}$
Line 14 = c_{14} has 5 points: $\{P_i \mid i \in \{3, 11, 24, 33, 43\}\}$
Line 15 = c_{15} has 5 points: $\{P_i \mid i \in \{1, 15, 17, 19, 21\}\}$
Line 16 = c_{16} has 5 points: $\{P_i \mid i \in \{9, 20, 27, 36, 44\}\}$
Line 17 = c_{23} has 5 points: $\{P_i \mid i \in \{4, 11, 15, 34, 44\}\}$
Line 18 = c_{24} has 5 points: $\{P_i \mid i \in \{12, 17, 22, 36, 41\}\}$
Line 19 = c_{25} has 5 points: $\{P_i \mid i \in \{5, 23, 24, 26, 27\}\}$
Line 20 = c_{26} has 5 points: $\{P_i \mid i \in \{10, 19, 25, 33, 42\}\}$
Line 21 = c_{34} has 5 points: $\{P_i \mid i \in \{14, 19, 27, 29, 40\}\}$
Line 22 = c_{35} has 5 points: $\{P_i \mid i \in \{7, 30, 31, 33, 36\}\}$
Line 23 = c_{36} has 5 points: $\{P_i \mid i \in \{8, 17, 24, 32, 39\}\}$
Line 24 = c_{45} has 5 points: $\{P_i \mid i \in \{6, 37, 39, 42, 44\}\}$
Line 25 = c_{46} has 5 points: $\{P_i \mid i \in \{2, 15, 23, 30, 38\}\}$
Line 26 = c_{56} has 5 points: $\{P_i \mid i \in \{0, 11, 12, 13, 14\}\}$

Points on surface but on no line

The surface has 0 points not on any line:

	0	1	2	3	4	5	6	7	8	9

The points on the surface but not on lines are:

Tritangent planes

The 45 tritangent planes are:

$$\pi_{12} = \pi_0 = 79 = \begin{bmatrix} 1 & 0 & 0 & \alpha^2 \\ 0 & 1 & 0 & \alpha^2 \\ 0 & 0 & 1 & 1 \end{bmatrix} = V\big(\alpha^2 X_0 + \alpha^2 X_1 + X_2 + X_3\big)$$

dual pt rank $= 52 = (3, 3, 1, 1)$.

$$\pi_{21} = \pi_1 = 46 = \begin{bmatrix} 1 & 0 & \alpha & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(\alpha X_0 + X_2\big)$$

dual pt rank $= 9 = (2, 0, 1, 0)$.

$$\pi_{13} = \pi_2 = 50 = \begin{bmatrix} 1 & 0 & 0 & \alpha \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & \alpha^2 \end{bmatrix} = V\big(\alpha X_0 + X_1 + \alpha^2 X_2 + X_3\big)$$

dual pt rank $= 75 = (2, 1, 3, 1)$.

$$\pi_{31} = \pi_3 = 25 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(X_0 + X_2\big)$$

dual pt rank $= 8 = (1, 0, 1, 0)$.

$$\pi_{14} = \pi_4 = 72 = \begin{bmatrix} 1 & 0 & \alpha^2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(\alpha^2 X_0 + X_1 + X_2\big)$$

dual pt rank $= 14 = (3, 1, 1, 0)$.

$$\pi_{41} = \pi_5 = 84 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(X_0\big)$$

dual pt rank $= 0 = (1, 0, 0, 0)$.

$$\pi_{15} = \pi_6 = 23 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \alpha \end{bmatrix} = V\big(X_0 + \alpha X_2 + X_3\big)$$

dual pt rank $= 54 = (1, 0, 2, 1)$.

$$\pi_{51} = \pi_7 = 4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(X_2\big)$$

dual pt rank $= 2 = (0, 0, 1, 0)$.

$$\pi_{16} = \pi_8 = 10 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \alpha \\ 0 & 0 & 1 & 0 \end{bmatrix} = V\big(\alpha X_1 + X_3\big)$$

dual pt rank $= 30 = (0, 2, 0, 1)$.

$$\pi_{61} = \pi_9 = 67 = \begin{bmatrix} 1 & 0 & \alpha^2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(\alpha^2 X_0 + X_2\big)$$

dual pt rank $= 10 = (3, 0, 1, 0)$.

$$\pi_{23} = \pi_{10} = 39 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & \alpha^2 \\ 0 & 0 & 1 & \alpha^2 \end{bmatrix} = V\big(X_0 + \alpha^2 X_1 + \alpha^2 X_2 + X_3\big)$$

dual pt rank $= 82 = (1,3,3,1)$.

$$\pi_{32} = \pi_{11} = 27 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = V\big(X_0 + X_1 + X_2 + X_3\big)$$

dual pt rank $= 4 = (1,1,1,1)$.

$$\pi_{24} = \pi_{12} = 58 = \begin{bmatrix} 1 & 0 & 0 & \alpha \\ 0 & 1 & 0 & \alpha^2 \\ 0 & 0 & 1 & 1 \end{bmatrix} = V\big(\alpha X_0 + \alpha^2 X_1 + X_2 + X_3\big)$$

dual pt rank $= 51 = (2,3,1,1)$.

$$\pi_{42} = \pi_{13} = 41 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(X_0 + X_1\big)$$

dual pt rank $= 5 = (1,1,0,0)$.

$$\pi_{25} = \pi_{14} = 80 = \begin{bmatrix} 1 & 0 & 0 & \alpha^2 \\ 0 & 1 & 0 & \alpha^2 \\ 0 & 0 & 1 & \alpha \end{bmatrix} = V\big(\alpha^2 X_0 + \alpha^2 X_1 + \alpha X_2 + X_3\big)$$

dual pt rank $= 68 = (3,3,2,1)$.

$$\pi_{52} = \pi_{15} = 1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} = V\big(X_2 + X_3\big)$$

dual pt rank $= 38 = (0,0,1,1)$.

$$\pi_{26} = \pi_{16} = 15 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \alpha^2 \\ 0 & 0 & 1 & 0 \end{bmatrix} = V\big(\alpha^2 X_1 + X_3\big)$$

dual pt rank $= 34 = (0,3,0,1)$.

$$\pi_{62} = \pi_{17} = 53 = \begin{bmatrix} 1 & 0 & 0 & \alpha \\ 0 & 1 & 0 & \alpha \\ 0 & 0 & 1 & 1 \end{bmatrix} = V\big(\alpha X_0 + \alpha X_1 + X_2 + X_3\big)$$

dual pt rank $= 47 = (2,2,1,1)$.

$$\pi_{34} = \pi_{18} = 71 = \begin{bmatrix} 1 & 0 & 0 & \alpha^2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & \alpha^2 \end{bmatrix} = V\big(\alpha^2 X_0 + X_1 + \alpha^2 X_2 + X_3\big)$$

dual pt rank $= 76 = (3,1,3,1)$.

$$\pi_{43} = \pi_{19} = 62 = \begin{bmatrix} 1 & \alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(\alpha X_0 + X_1\big)$$

dual pt rank $= 6 = (2,1,0,0)$.

$$\pi_{35} = \pi_{20} = 49 = \begin{bmatrix} 1 & 0 & 0 & \alpha \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & \alpha \end{bmatrix} = V\big(\alpha X_0 + X_1 + \alpha X_2 + X_3\big)$$

dual pt rank $= 59 = (2,1,2,1)$.

$$\pi_{53} = \pi_{21} = 3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \alpha^2 \end{bmatrix} = V\big(\alpha^2 X_2 + X_3\big)$$

dual pt rank $= 69 = (0,0,3,1)$.

$$\pi_{36} = \pi_{22} = 5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = V\big(X_1 + X_3\big)$$

dual pt rank $= 26 = (0,1,0,1)$.

$$\pi_{63} = \pi_{23} = 76 = \begin{bmatrix} 1 & 0 & 0 & \alpha^2 \\ 0 & 1 & 0 & \alpha \\ 0 & 0 & 1 & \alpha^2 \end{bmatrix} = V\big(\alpha^2 X_0 + \alpha X_1 + \alpha^2 X_2 + X_3\big)$$

dual pt rank $= 80 = (3,2,3,1)$.

$$\pi_{45} = \pi_{24} = 83 = \begin{bmatrix} 1 & \alpha^2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(\alpha^2 X_0 + X_1\big)$$

dual pt rank $= 7 = (3,1,0,0)$.

$$\pi_{54} = \pi_{25} = 2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \alpha \end{bmatrix} = V\big(\alpha X_2 + X_3\big)$$

dual pt rank $= 53 = (0,0,2,1)$.

$$\pi_{46} = \pi_{26} = 20 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(X_1\big)$$

dual pt rank $= 1 = (0,1,0,0)$.

$$\pi_{64} = \pi_{27} = 31 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & \alpha \\ 0 & 0 & 1 & 0 \end{bmatrix} = V\big(X_0 + \alpha X_1 + X_3\big)$$

dual pt rank $= 31 = (1,2,0,1)$.

$$\pi_{56} = \pi_{28} = 0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = V\big(X_3\big)$$

dual pt rank $= 3 = (0,0,0,1)$.

$$\pi_{65} = \pi_{29} = 12 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \alpha \\ 0 & 0 & 1 & \alpha \end{bmatrix} = V\big(\alpha X_1 + \alpha X_2 + X_3\big)$$

dual pt rank $= 61 = (0,2,2,1)$.

$$\pi_{12,34,56} = \pi_{30} = 18 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \alpha^2 \\ 0 & 0 & 1 & \alpha^2 \end{bmatrix} = V\big(\alpha^2 X_1 + \alpha^2 X_2 + X_3\big)$$

dual pt rank $= 81 = (0,3,3,1)$.

$$\pi_{12,35,46} = \pi_{31} = 36 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & \alpha^2 \\ 0 & 0 & 1 & 0 \end{bmatrix} = V\big(X_0 + \alpha^2 X_1 + X_3\big)$$

dual pt rank $= 35 = (1,3,0,1)$.

$$\pi_{12,36,45} = \pi_{32} = 59 = \begin{bmatrix} 1 & 0 & 0 & \alpha \\ 0 & 1 & 0 & \alpha^2 \\ 0 & 0 & 1 & \alpha \end{bmatrix} = V\big(\alpha X_0 + \alpha^2 X_1 + \alpha X_2 + X_3\big)$$

dual pt rank $= 67 = (2,3,2,1)$.

$$\pi_{13,24,56} = \pi_{33} = 6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = V\big(X_1 + X_2 + X_3\big)$$

dual pt rank $= 42 = (0,1,1,1)$.

$$\pi_{13,25,46} = \pi_{34} = 26 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = V\big(X_0 + X_1 + X_3\big)$$

dual pt rank $= 27 = (1,1,0,1)$.

$$\pi_{13,26,45} = \pi_{35} = 70 = \begin{bmatrix} 1 & 0 & 0 & \alpha^2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & \alpha \end{bmatrix} = V\big(\alpha^2 X_0 + X_1 + \alpha X_2 + X_3\big)$$

dual pt rank $= 60 = (3,1,2,1)$.

$$\pi_{14,23,56} = \pi_{36} = 9 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(X_1 + X_2\big)$$

dual pt rank $= 11 = (0,1,1,0)$.

$$\pi_{14,25,36} = \pi_{37} = 30 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(X_0 + X_1 + X_2\big)$$

dual pt rank $= 12 = (1,1,1,0)$.

$$\pi_{14,26,35} = \pi_{38} = 51 = \begin{bmatrix} 1 & 0 & \alpha & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = V\big(\alpha X_0 + X_1 + X_2\big)$$

dual pt rank $= 13 = (2,1,1,0)$.

$$\pi_{15,23,46} = \pi_{39} = 21 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = V\big(X_0 + X_3\big)$$

dual pt rank $= 23 = (1,0,0,1)$.

$$\pi_{15,24,36} = \pi_{40} = 22 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} = V\big(X_0 + X_2 + X_3\big)$$

dual pt rank $= 39 = (1,0,1,1)$.

$$\pi_{15,26,34} = \pi_{41} = 24 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \alpha^2 \end{bmatrix} = V\big(X_0 + \alpha^2 X_2 + X_3\big)$$

dual pt rank $= 70 = (1,0,3,1)$.

$$\pi_{16,23,45} = \pi_{42} = 33 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & \alpha \\ 0 & 0 & 1 & \alpha \end{bmatrix} = V\big(X_0 + \alpha X_1 + \alpha X_2 + X_3\big)$$

dual pt rank $= 62 = (1,2,2,1)$.

$$\pi_{16,24,35} = \pi_{43} = 74 = \begin{bmatrix} 1 & 0 & 0 & \alpha^2 \\ 0 & 1 & 0 & \alpha \\ 0 & 0 & 1 & 1 \end{bmatrix} = V\big(\alpha^2 X_0 + \alpha X_1 + X_2 + X_3\big)$$

dual pt rank $= 48 = (3,2,1,1)$.

$$\pi_{16,25,34} = \pi_{44} = 55 = \left[\begin{array}{cccc} 1 & 0 & 0 & \alpha \\ 0 & 1 & 0 & \alpha \\ 0 & 0 & 1 & \alpha^2 \end{array}\right] = V\big(\alpha X_0 + \alpha X_1 + \alpha^2 X_2 + X_3\big)$$

dual pt rank $= 79 = (2,2,3,1)$.

The Generalized Quadrangle

The lines in the tritangent planes are:
$\pi_0 = \pi_{12} = \{\ell_i \mid i = 0,7,12\} = \{a_1, b_2, c_{12}\}$
$\pi_1 = \pi_{21} = \{\ell_i \mid i = 1,6,12\} = \{a_2, b_1, c_{12}\}$
$\pi_2 = \pi_{13} = \{\ell_i \mid i = 0,8,13\} = \{a_1, b_3, c_{13}\}$
$\pi_3 = \pi_{31} = \{\ell_i \mid i = 2,6,13\} = \{a_3, b_1, c_{13}\}$
$\pi_4 = \pi_{14} = \{\ell_i \mid i = 0,9,14\} = \{a_1, b_4, c_{14}\}$
$\pi_5 = \pi_{41} = \{\ell_i \mid i = 3,6,14\} = \{a_4, b_1, c_{14}\}$
$\pi_6 = \pi_{15} = \{\ell_i \mid i = 0,10,15\} = \{a_1, b_5, c_{15}\}$
$\pi_7 = \pi_{51} = \{\ell_i \mid i = 4,6,15\} = \{a_5, b_1, c_{15}\}$
$\pi_8 = \pi_{16} = \{\ell_i \mid i = 0,11,16\} = \{a_1, b_6, c_{16}\}$
$\pi_9 = \pi_{61} = \{\ell_i \mid i = 5,6,16\} = \{a_6, b_1, c_{16}\}$
$\pi_{10} = \pi_{23} = \{\ell_i \mid i = 1,8,17\} = \{a_2, b_3, c_{23}\}$
$\pi_{11} = \pi_{32} = \{\ell_i \mid i = 2,7,17\} = \{a_3, b_2, c_{23}\}$
$\pi_{12} = \pi_{24} = \{\ell_i \mid i = 1,9,18\} = \{a_2, b_4, c_{24}\}$
$\pi_{13} = \pi_{42} = \{\ell_i \mid i = 3,7,18\} = \{a_4, b_2, c_{24}\}$
$\pi_{14} = \pi_{25} = \{\ell_i \mid i = 1,10,19\} = \{a_2, b_5, c_{25}\}$
$\pi_{15} = \pi_{52} = \{\ell_i \mid i = 4,7,19\} = \{a_5, b_2, c_{25}\}$
$\pi_{16} = \pi_{26} = \{\ell_i \mid i = 1,11,20\} = \{a_2, b_6, c_{26}\}$
$\pi_{17} = \pi_{62} = \{\ell_i \mid i = 5,7,20\} = \{a_6, b_2, c_{26}\}$
$\pi_{18} = \pi_{34} = \{\ell_i \mid i = 2,9,21\} = \{a_3, b_4, c_{34}\}$
$\pi_{19} = \pi_{43} = \{\ell_i \mid i = 3,8,21\} = \{a_4, b_3, c_{34}\}$
$\pi_{20} = \pi_{35} = \{\ell_i \mid i = 2,10,22\} = \{a_3, b_5, c_{35}\}$
$\pi_{21} = \pi_{53} = \{\ell_i \mid i = 4,8,22\} = \{a_5, b_3, c_{35}\}$
$\pi_{22} = \pi_{36} = \{\ell_i \mid i = 2,11,23\} = \{a_3, b_6, c_{36}\}$
$\pi_{23} = \pi_{63} = \{\ell_i \mid i = 5,8,23\} = \{a_6, b_3, c_{36}\}$
$\pi_{24} = \pi_{45} = \{\ell_i \mid i = 3,10,24\} = \{a_4, b_5, c_{45}\}$
$\pi_{25} = \pi_{54} = \{\ell_i \mid i = 4,9,24\} = \{a_5, b_4, c_{45}\}$
$\pi_{26} = \pi_{46} = \{\ell_i \mid i = 3,11,25\} = \{a_4, b_6, c_{46}\}$
$\pi_{27} = \pi_{64} = \{\ell_i \mid i = 5,9,25\} = \{a_6, b_4, c_{46}\}$
$\pi_{28} = \pi_{56} = \{\ell_i \mid i = 4,11,26\} = \{a_5, b_6, c_{56}\}$
$\pi_{29} = \pi_{65} = \{\ell_i \mid i = 5,10,26\} = \{a_6, b_5, c_{56}\}$
$\pi_{30} = \pi_{12,34,56} = \{\ell_i \mid i = 12,21,26\} = \{c_{12}, c_{34}, c_{56}\}$
$\pi_{31} = \pi_{12,35,46} = \{\ell_i \mid i = 12,22,25\} = \{c_{12}, c_{35}, c_{46}\}$
$\pi_{32} = \pi_{12,36,45} = \{\ell_i \mid i = 12,23,24\} = \{c_{12}, c_{36}, c_{45}\}$
$\pi_{33} = \pi_{13,24,56} = \{\ell_i \mid i = 13,18,26\} = \{c_{13}, c_{24}, c_{56}\}$
$\pi_{34} = \pi_{13,25,46} = \{\ell_i \mid i = 13,19,25\} = \{c_{13}, c_{25}, c_{46}\}$
$\pi_{35} = \pi_{13,26,45} = \{\ell_i \mid i = 13,20,24\} = \{c_{13}, c_{26}, c_{45}\}$
$\pi_{36} = \pi_{14,23,56} = \{\ell_i \mid i = 14,17,26\} = \{c_{14}, c_{23}, c_{56}\}$
$\pi_{37} = \pi_{14,25,36} = \{\ell_i \mid i = 14,19,23\} = \{c_{14}, c_{25}, c_{36}\}$
$\pi_{38} = \pi_{14,26,35} = \{\ell_i \mid i = 14,20,22\} = \{c_{14}, c_{26}, c_{35}\}$
$\pi_{39} = \pi_{15,23,46} = \{\ell_i \mid i = 15,17,25\} = \{c_{15}, c_{23}, c_{46}\}$
$\pi_{40} = \pi_{15,24,36} = \{\ell_i \mid i = 15,18,23\} = \{c_{15}, c_{24}, c_{36}\}$
$\pi_{41} = \pi_{15,26,34} = \{\ell_i \mid i = 15,20,21\} = \{c_{15}, c_{26}, c_{34}\}$
$\pi_{42} = \pi_{16,23,45} = \{\ell_i \mid i = 16,17,24\} = \{c_{16}, c_{23}, c_{45}\}$
$\pi_{43} = \pi_{16,24,35} = \{\ell_i \mid i = 16,18,22\} = \{c_{16}, c_{24}, c_{35}\}$
$\pi_{44} = \pi_{16,25,34} = \{\ell_i \mid i = 16,19,21\} = \{c_{16}, c_{25}, c_{34}\}$
The tritangent planes through the 27 lines are:
$a_1 = \ell_0 \in \{\pi_i \mid i = 2,6,8,0,4\} = \{\pi_{13}, \pi_{15}, \pi_{16}, \pi_{12}, \pi_{14}\}$
$a_2 = \ell_1 \in \{\pi_i \mid i = 12,1,10,16,14\} = \{\pi_{24}, \pi_{21}, \pi_{23}, \pi_{26}, \pi_{25}\}$

$a_3 = \ell_2 \in \{\pi_i \mid i = 11, 3, 20, 22, 18\} = \{\pi_{32}, \pi_{31}, \pi_{35}, \pi_{36}, \pi_{34}\}$
$a_4 = \ell_3 \in \{\pi_i \mid i = 19, 13, 26, 24, 5\} = \{\pi_{43}, \pi_{42}, \pi_{46}, \pi_{45}, \pi_{41}\}$
$a_5 = \ell_4 \in \{\pi_i \mid i = 25, 7, 21, 15, 28\} = \{\pi_{54}, \pi_{51}, \pi_{53}, \pi_{52}, \pi_{56}\}$
$a_6 = \ell_5 \in \{\pi_i \mid i = 29, 17, 23, 9, 27\} = \{\pi_{65}, \pi_{62}, \pi_{63}, \pi_{61}, \pi_{64}\}$
$b_1 = \ell_6 \in \{\pi_i \mid i = 3, 1, 7, 9, 5\} = \{\pi_{31}, \pi_{21}, \pi_{51}, \pi_{61}, \pi_{41}\}$
$b_2 = \ell_7 \in \{\pi_i \mid i = 11, 17, 13, 0, 15\} = \{\pi_{32}, \pi_{62}, \pi_{42}, \pi_{12}, \pi_{52}\}$
$b_3 = \ell_8 \in \{\pi_i \mid i = 2, 19, 10, 23, 21\} = \{\pi_{13}, \pi_{43}, \pi_{23}, \pi_{63}, \pi_{53}\}$
$b_4 = \ell_9 \in \{\pi_i \mid i = 12, 25, 4, 18, 27\} = \{\pi_{24}, \pi_{54}, \pi_{14}, \pi_{34}, \pi_{64}\}$
$b_5 = \ell_{10} \in \{\pi_i \mid i = 29, 20, 6, 14, 24\} = \{\pi_{65}, \pi_{35}, \pi_{15}, \pi_{25}, \pi_{45}\}$
$b_6 = \ell_{11} \in \{\pi_i \mid i = 22, 8, 26, 16, 28\} = \{\pi_{36}, \pi_{16}, \pi_{46}, \pi_{26}, \pi_{56}\}$
$c_{12} = \ell_{12} \in \{\pi_i \mid i = 32, 1, 0, 30, 31\} = \{\pi_{12,36,45}, \pi_{21}, \pi_{12}, \pi_{12,34,56}, \pi_{12,35,46}\}$
$c_{13} = \ell_{13} \in \{\pi_i \mid i = 33, 34, 3, 2, 35\} = \{\pi_{13,24,56}, \pi_{13,25,46}, \pi_{31}, \pi_{13}, \pi_{13,26,45}\}$
$c_{14} = \ell_{14} \in \{\pi_i \mid i = 37, 38, 36, 4, 5\} = \{\pi_{14,25,36}, \pi_{14,26,35}, \pi_{14,23,56}, \pi_{14}, \pi_{41}\}$
$c_{15} = \ell_{15} \in \{\pi_i \mid i = 41, 6, 40, 39, 7\} = \{\pi_{15,26,34}, \pi_{15}, \pi_{15,24,36}, \pi_{15,23,46}, \pi_{51}\}$
$c_{16} = \ell_{16} \in \{\pi_i \mid i = 44, 8, 43, 9, 42\} = \{\pi_{16,25,34}, \pi_{16}, \pi_{16,24,35}, \pi_{61}, \pi_{16,23,45}\}$
$c_{23} = \ell_{17} \in \{\pi_i \mid i = 11, 39, 10, 36, 42\} = \{\pi_{32}, \pi_{15,23,46}, \pi_{23}, \pi_{14,23,56}, \pi_{16,23,45}\}$
$c_{24} = \ell_{18} \in \{\pi_i \mid i = 12, 33, 40, 13, 43\} = \{\pi_{24}, \pi_{13,24,56}, \pi_{15,24,36}, \pi_{42}, \pi_{16,24,35}\}$
$c_{25} = \ell_{19} \in \{\pi_i \mid i = 37, 44, 34, 14, 15\} = \{\pi_{14,25,36}, \pi_{16,25,34}, \pi_{13,25,46}, \pi_{25}, \pi_{52}\}$
$c_{26} = \ell_{20} \in \{\pi_i \mid i = 38, 17, 41, 35, 16\} = \{\pi_{14,26,35}, \pi_{62}, \pi_{15,26,34}, \pi_{13,26,45}, \pi_{26}\}$
$c_{34} = \ell_{21} \in \{\pi_i \mid i = 44, 41, 19, 30, 18\} = \{\pi_{16,25,34}, \pi_{15,26,34}, \pi_{43}, \pi_{12,34,56}, \pi_{34}\}$
$c_{35} = \ell_{22} \in \{\pi_i \mid i = 38, 20, 43, 31, 21\} = \{\pi_{14,26,35}, \pi_{35}, \pi_{16,24,35}, \pi_{12,35,46}, \pi_{53}\}$
$c_{36} = \ell_{23} \in \{\pi_i \mid i = 37, 32, 22, 40, 23\} = \{\pi_{14,25,36}, \pi_{12,36,45}, \pi_{36}, \pi_{15,24,36}, \pi_{63}\}$
$c_{45} = \ell_{24} \in \{\pi_i \mid i = 32, 25, 35, 42, 24\} = \{\pi_{12,36,45}, \pi_{54}, \pi_{13,26,45}, \pi_{16,23,45}, \pi_{45}\}$
$c_{46} = \ell_{25} \in \{\pi_i \mid i = 34, 39, 26, 31, 27\} = \{\pi_{13,25,46}, \pi_{15,23,46}, \pi_{46}, \pi_{12,35,46}, \pi_{64}\}$
$c_{56} = \ell_{26} \in \{\pi_i \mid i = 33, 29, 36, 30, 28\} = \{\pi_{13,24,56}, \pi_{65}, \pi_{14,23,56}, \pi_{12,34,56}, \pi_{56}\}$