The point of this lab is to play around with some of the ideas from class using Maple (obviously). Nothing needs to be turned in. Please let me know if you uncover any bugs (they exist) or interesting solutions!

Also, please note that these do not need to be tackled in order, and you don’t need to worry about hitting them all. Just follow your nose.

# 1. Here are three polynomials:

\[ g := x^2 + 2 \cdot x + 3; \]
\[ h := 3 \cdot x^3 - 2 \cdot x^2 + 4 \cdot x + 1; \]
\[ q := 8 \cdot x^4 + 16 \cdot x^3 - 16 \cdot x^2 + 26 \cdot x - 1 - 9 \cdot x^5; \]

# Is \( q \) in \( \langle g, h \rangle \)? Think about how we did it in class. Find the GCD, then divide. Alternatively, check out the PolynomialIdeals package.

# Cook up something not in \( \langle g, h \rangle \). How do you spot that it isn’t in \( \langle g, h \rangle \)?

# 2. (to challenge Maple a bit) In a loop, choose 10 polynomials of any degree (random coefficients). Form something in the ideal (again, choosing random coefficients) and test whether it’s in the ideal. Now try 25 polynomials. Now 100....

# 3. (takes some programming) Write a short procedure to do sturm sequences and bisection on a given polynomial \( p \) in a given interval \([a, b]\). Note the commands "sturm" and "sturmseq." Try it on a few examples where you know the example. Also, note that you’ll need some way of deciding when to quit (some tolerance on the interval size maybe).

# 4. (takes less programming) Do the sturm sequence and bisection method on the following:

\[ g := x^3 - 3 \cdot x^2 + 2 \cdot x; \]
\[ h := x^4 - 4.001 \cdot x^3 + 5.003 \cdot x^2 - 2.002 \cdot x; \]
\[ k := x^4 - 4.00000001 \cdot x^3 + 5.00000003 \cdot x^2 - 2.00000002 \cdot x; \]
\[ Digits := 100 : \]
\[ p := x^4 - 4.000000000000000000000000000000001 \cdot x^3 + 5.000000000000000000000000000000003 \cdot x^2 - 2.000000000000000000000000000000002 \cdot x; \]
\[ q := x^3 - 700 \cdot x^2 + 100000 \cdot x; \]

# 5. Check out isPrime, maximal, isPrimary, etc. (in PolynomialIdeals). Try some examples from class.

# 6. Try to divide \( f \) by \( g \) and \( h \) and then by \( h \) and \( g \):

\[ f := x^2 \cdot y + x \cdot y^2 + y^2; \]
\[ g := y^2 - 1; \]
\[ h := x \cdot y - 1; \]
# Notice that you’ll have to get into term orders....

# 7. Play around with solve and fsolve. Can you break these? Try high degree and huge (rational) coefficients....