MATH 317: Advanced Calculus of One Variable

Instructor:	Prof. Wolfgang Bangerth Weber 214 bangerth@colostate.edu
Lecture:	Monday, Wednesday, Friday, 9:00–9:50 am Engineering, room E 104
Office hours:	Monday, 10–11 am; or by appointment.

Catalog Description Convergence of sequences, series: limits, continuity, differentiation, integration of one-variable functions.

Motivation and Background for this Course You have learned much mathematics in your studies so far, but most of it was invented before around 1800. In particular, mathematicians had a pretty good idea about calculus by around that point.

What you have learned so far is true, but sometime in the 1800s, mathematicians came to understand that what they intuitively knew to be true was not put on a solid foundation: Many statements mathematicians made could not be explained in a rigorous way. This led to a revolution in which mathematics tried to go back to the basics to understand what could actually rigorously be proved, and what one simply needs to assume. As a consequence, we have today a very different understanding of what, for example, the real numbers are or how we define convergence, than what mathematicians 200 years ago would have thought about these topics.

MATH 317 is a course that takes you through the sort of discussions that mathematicians had in the 1800s. You will learn what numbers are, what convergence means, and what differentiation and integration really are. As a by-product (one might say that that is actually the point of the class), you will also learn not only how to *do* math, but *how to think like a mathematician.*

Course Topics The course will cover six large areas:

1. *Mathematical statements and proofs:* In order to concisely state that something is true, you need to learn the "language of proofs" and become fluent in it. The language we use today is a bit like shorthand: It uses the same words as regular language, but uses symbols (such as $\forall$ or $\exists$, but also $+$ or $\sum$) to abbreviate many words. It also typically uses a formalized structure for statements – a little bit like the "grammar" of the language. We'll get to see many examples.

2. *Numbers:* You are familiar with the natural numbers $\mathbb{N}$, the integers $\mathbb{Z}$, and the rationals $\mathbb{Q}$. You also probably think that you know the reals $\mathbb{R}$, but can you say how they are defined *precisely*, and how we know that that encompasses *all* numbers? We will work through their definition and how one arrives at it.

3. *Sequences, limits, convergence:* If you want to think about continuity of functions, derivatives, or integrals, you need to think about limits of sequences of numbers. And then you have to think about whether that sequence actually "converges". We will talk about what convergence means, how we can check whether a sequence converges, and how to define limits.

4. *Functions of one variable, continuity:* Ultimately, analysis is about *functions*. It was invented to concisely state *qualitative properties* of functions such as whether it is continuous, differentiable, etc. So we will have to talk about how continuity is defined, which kinds of functions are continuous, and how we can test a function for it.

5. *Differentiation of functions of one variable:* The next step after continuity is to see whether a function is differentiable. Again, we will explore the definition of the derivative and its consequences.

6. *Integration of functions of one variable:* The final topic for this class is integration, i.e., to compute the area under the curve of a function. The concept we will explore is Riemann integration (the simplest version of defining what an integral represents), its definition, and its consequences.

Textbook I do not require you to get any particular book, and in particular will not pose homework that references a book. That said, if you want to read up on some of the material we discuss in class, the following book covers essentially everything we do over the course of this semester:

- Brian S. Thomson, Judith B. Bruckner, Andrew M. Bruckner: *Elementary Real Analysis*. Second edition, 2008.

This book is available at http://classicalrealanalysis.info/com/Elementary-Real-Analysis.php.

Prerequisites (MATH 161) and (MATH 230 or MATH 235).

Webpage Homework assignments and other course information will be posted at the course webpage http://www.math.colostate.edu/~bangerth/teaching.html

Exams + Grading Final grades will be determined based on the following components:

- 30%: Homework assignments.
- 40%: 3 or 4 exams throughout the semester. The first two exams will be on Wednesday, February 12, 2020; and on Monday, March 6, 2020. The remaining at dates to be determined.
- 30%: Final exam, on May 13, 2020 (4:10-6:10pm).

Your minimum grade will be A, B, C, or D, for a score of 90%, 80%, 70%, and 60% over the course of the semester, respectively.
You must make arrangements in advance if you expect to miss an exam or quiz. Exam absences due to recognized University-related activities, religious holidays, verifiable illness, and family/medical emergencies will be dealt with on an individual basis. In all cases of absence from exams a written excuse is required. Ignorance of the time and place of an exam will not be accepted as an excuse for absence.

Learning Outcomes and Course Objectives Analysis is *the* foundation for one of the two big branches of mathematics: continuous mathematics. (The other is discrete mathematics, and deals with enumerable things.) As such, the goal is that you learn the language of analysis, how to prove things in this realm, and understand the basics of the theory of functions.
At the end of the semester, you should have (i) a basic level of literacy in the theory of analysis, (ii) a solid foundation in reading and writing mathematical proofs, (iii) an understanding of the difficulties of dealing with sequences and their convergence, (iv) an overview of qualitative properties of functions such as continuity and differentiability. You will also have a working knowledge of how continuity, differentiability, and integration are defined.

Policies *Academic integrity:* Academic integrity is integral to the success of the University and to you as a learner. Academic integrity is conceptualized as doing and taking credit for one's own work. Academic dishonesty undermines the educational experience at Colorado State University. Examples of academic dishonesty include (but are not limited to) cheating, plagiarism, and falsification. Plagiarism includes the copying of language, structure, images, ideas or thoughts of others and is related only to work submitted for credit. Cheating or any form of academic dishonesty will not be tolerated. The use of material from

improperly cited or credited sources will be considered plagiarism. You are encouraged to collaborate with your classmates, unless otherwise directed, but all work intended for a grade must clearly be your work as an individual. Ignorance of the rules does not exclude any member of the CSU community from the requirements or the processes meant to ensure academic integrity.

Disabilities: Colorado State University, in compliance with state and federal laws and regulations, does not discriminate on the basis of disability in administration of its education related programs and activities. We have an institutional commitment to provide equal educational opportunities for disabled students who are otherwise qualified. Students with documented disabilities must contact The Office of Resources for Disabled Students (RDS; 970-491-6385) to make arrangements for class accommodations. It is the responsibility of the student to obtain accommodation letters from RDS and to make arrangements for the implementation of accommodations with faculty in advance. Students who believe they have been denied access to services or accommodations required by law should contact RDS (970-491-6385). Students who believe they have been subjected to discrimination on the basis of disability should contact the Office of Equal Opportunity (970-491-5836). For more information regarding disability grievance procedures, visit http://oeo.colostate.edu.