MATH 620
Variational Methods and Optimization I

Instructor:	Prof. Wolfgang Bangerth Weber 214 bangerth@colostate.edu
Lectures:	Engineering E 206, Mondays/Wednesdays/Fridays, 12-12:50am
Office hours:	Wednesdays, 1-2pm; or by appointment.

Course outline

The syllabus description for this course is pretty short:

> Unconstrained and constrained infinite dimensional optimization, calculus of variations, applications.

What I care about are essentially the following topics:

1. Infinite optimization problems are generally used to describe problems such as "Find that displacement field $\vec{u}(\vec{x})$ for every point $\vec{x}$ of a body that is subject to some external force, so that it minimizes some kind of energy functional E that assigns to every displacement field $\vec{u}$ a scalar energy. They also appear in fluid mechanics, electrodynamics, and many other areas. In other words, they are immensely practical and underly many areas of the sciences and engineering. An important part of this class to me is therefore to make this connection: from application to mathematical formulation.

2. Variational problems can be thought of (in many cases) as infinite dimensional equivalents of finite dimensional optimization problems: instead of minimizing a function $E(\vec{x})$ where the argument $\vec{x} = \{x_i\}_{i=1}^n$ is indexed by a discrete and finite index i, one now needs to minimize a functional $E(u)$ where u is a function indexed by a spatial variable $u(x)$ with $x \in \Omega \subset \mathbb{R}^d$ and where Ω has uncountably many elements. It turns out that going from finitely to infinitely many dimensions introduces *fundamental* difficulties that make the problem substantially more difficult. We will explore many of these kinds of issues, first by considering how finite dimensional optimization works, how this would apply to infinite dimensional problems, and what properties fail in the transition from one to the other.

3. The optimality conditions for these variational problems typically turn out to be (partial) differential equations. In other words, the function $u(\vec{x})$ that minimizes the energy has to satisfy some kind of differential equation – or, conversely, we can find the minimizer by solving a differential equation. I will demonstrate how this happens, and we will discuss the derivation process in great detail.

4. From a mathematical point of view, one is then left to ask questions such as "Is there a solution?" and "Is the solution unique?". Answering these kinds of questions requires detailed studies of the spaces, sets, and operators in question – essentially, one needs to study *functional analysis*. A significant part of this semester will deal with this.

5. All of these questions can be made more complicated if one introduces *constraints*. This leads to the theory of Lagrange multipliers, but again the situation is substantially more complicated in infinite dimensional spaces. We will again consider both the finite and infinite dimensional situations, and how they differ.

Time permitting, I may also cover other areas, for example based on student interest.

Learning objectives

At the end of the semester, this is what you should have learned:

- Where variational and infinite dimensional problems appear.
- How we think of finite and infinite dimensional optimization, and where the difficulties are in transitioning from one to the other. The important point here is to build *intuition* through examples and counter-examples.
- How one solves variational problems by deriving the optimality conditions in the form of (partial) differential equations.

Prerequisites

MATH 570 (Topology I) or MATH 517 (Introduction to Real Analysis).

Literature

I will be losely inspired by the books by D. G. Luenberger: "Optimization by Vector Space Methods" (Wiley-Interscience) and by B. Dacorogna: "Introduction To The Calculus Of Variations" (ICP Publishers). However, I will make the course self-contained and not reference particular parts or exercises in these books: you can just use them as backup material if you did not understand something or need to read up on material you may have missed.
In other words: You are not required to buy or use any of these books.

Webpage

Homework assignments and other course information will be posted at the course webpage
http://www.math.colostate.edu/~bangerth/teaching.html

Exams and grading

Final grades will be determined based on the following components:

- Biweekly homework and programming assignments: 50%
- Midterm exam, at a date in October still to be determined: 20%
- Final exam, Tuesday December 11, 11:50am–1:50pm: 30%

We may discuss replacing the final exam by a final project as the semester progresses.

Your minimum grade will be A, B, C, or D, for a score of 90%, 80%, 70%, and 60% over the course of the semester, respectively.

You must make arrangements in advance if you expect to miss an exam or quiz. Exam absences due to recognized University-related activities, religious holidays, verifiable illness, and family/medical emergencies will be dealt with on an individual basis. In all cases of absence from exams a written excuse is required. Ignorance of the time and place of an exam will not be accepted as an excuse for absence.

Incompletes: I will consider giving an incomplete if you have successfully completed all but a small portion of the work of the course, and are prevented from completing the course by a severe, unexpected event. Simply being behind work is not a reason for an Incomplete, though; in that case you should consider dropping the course.

S/U grades: If you are registered S/U your grade will be 'S' if your letter grade is C or above, and 'U' otherwise.

Policies *Academic integrity:* Academic integrity is integral to the success of the University and to you as a learner. Academic integrity is conceptualized as doing and taking credit for one's own work. Academic dishonesty undermines the educational experience at Colorado State University. Examples of academic dishonesty include (but are not limited to) cheating, plagiarism, and falsification. Plagiarism includes the copying of language, structure, images, ideas or thoughts of others and is related only to work submitted for credit. Cheating or any form of academic dishonesty will not be tolerated. The use of material from improperly cited or credited sources will be considered plagiarism. You are encouraged to collaborate with your classmates, unless otherwise directed, but all work intended for a grade must clearly be your work as an individual. Ignorance of the rules does not exclude any member of the CSU community from the requirements or the processes meant to ensure academic integrity.

Disabilities: Colorado State University, in compliance with state and federal laws and regulations, does not discriminate on the basis of disability in administration of its education related programs and activities. We have an institutional commitment to provide equal educational opportunities for disabled students who are otherwise qualified. Students with documented disabilities must contact The Office of Resources for Disabled Students (RDS; 970-491-6385) to make arrangements for class accommodations. It is the responsibility of the student to obtain accommodation letters from RDS and to make arrangements for the implementation of accommodations with faculty in advance. Students who believe they have been denied access to services or accommodations required by law should contact RDS (970-491-6385). Students who believe they have been subjected to discrimination on the basis of disability should contact the Office of Equal Opportunity (970-491-5836). For more information regarding disability grievance procedures, visit http://oeo.colostate.edu.