MATH 689: Numerical Optimization

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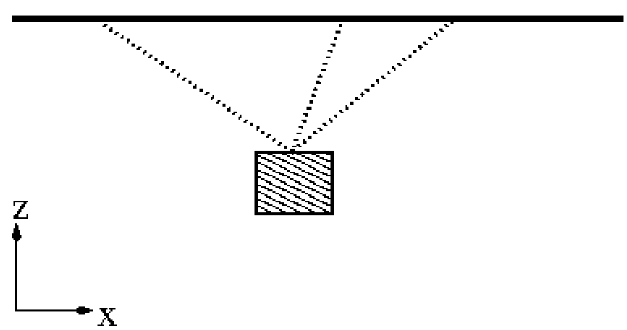
Homework assignment 4 – due Tuesday 2/18/2014

Problem 1 (Trust region algorithm using the dogleg strategy). In class we discussed the dogleg algorithm to determine the step $\tilde{p}_k$ in a trust region algorithm. The final slide discussed how to choose the step if Δ_k is small, medium, or large, in which case we either choose the "unconstrained minimizer" in the steepest descent direction, a point between the unconstrained minimizer and the quasi-Newton update, or the quasi-Newton update, respectively. However, this strategy wouldn't work if the dogleg would intersect the trust region sphere in more than one point.

Prove that that can't happen, i.e. that the dogleg curve has only a single intersection point with the trust region sphere. For the proof you will have to assume that B_k is chosen as a positive definite matrix.

(4 points)

Problem 2 (A modeling exercise). Consider the following system of three springs suspended from the ceiling at positions $(x,z) = (-20\text{cm}, 0\text{cm})$, $(5\text{cm}, 0\text{cm})$ and $(15\text{cm}, 0\text{cm})$ and that hold in place a body:



Each spring has a rest length of $L_0 = 20\text{cm}$, and extending (or compressing) spring $i, i = 1 \ldots 3$ to a length L_i requires an energy of $E_{\text{spring},i} = \frac{1}{2}D(L_i - L_0)^2$ where the spring constant for all three springs equals $D = 300\frac{N}{m}$. On the other hand, the potential energy of the body is $E_{\text{pot}} = mgz$ where the body's mass is $m = 500g$, the gravity constant is $g = 9.81\frac{\text{m}}{\text{s}^2}$, and z is the vertical coordinate of the body's position.

Express the total energy in the system (spring energies plus potential energy) as a function of the body's position (x,z). Make sure to convert all the quantities above into common units kg, m, and s. Nature likes to do things so that the energy is minimal, so implement a numerical method to find the location at which this energy is minimal. In your answer, show this location as well as the value of the total energy function at this location. Is there only a single energy minimum? **(4 points)**

Problem 3 (To converge or not to converge). Consider the function $f(x) = x_1^4 - x_1^2 + x_2^2$. It's minima lie at $x^* = (\pm\frac{1}{2}\sqrt{2}, 0)^T$. Explain in words, graphs, or numbers what is going to happen if we started Newton's method at the point $x_0 = (0,2)^T$ and did all computations exactly (i.e. not in floating point arithmetic), and explain why this leads to a somewhat unsatisfactory result.

(3 points)

A problem to think about (Where does Newton's method converge to?). In the previous problem, you have seen a situation where a function has more than one minimum. Imagine a function that has M points $x^*_m, m = 1 \ldots M$, to which a sequence of Newton steps x_k can converge. Typically, these x^*_m could be the minima of the function, but unless a Newton method is appropriately modified, these points could also include the maxima or saddle points of the function. Then we could ask what would happen if we started at a point x_0: which of the x^*_m will Newton's method converge to, assuming that it converges at all? We could graphically depict this for functions of two arguments by assigning to each possible starting point x_0 the color m of the point x^*_m that the sequence of Newton iterates started at x_0 converges to.

There are cases where the picture you get is very complicated – in fact, one can sometimes get fractals out of this, see the related examples shown in `http://en.wikipedia.org/wiki/Newton_fractal`. In simpler cases, one will typically just get irregularly shaped regions around each of the x^*_m where one converges to this particular x^*_m from each point within its corresponding region.

For the functions discussed in the previous two problems, generate color plots of the kind outlined above that show which point you converge to if you start from a particular point. **(No points)**

If you have comments on the way I teach – in particular suggestions how I can do things better, if I should do more or less examples, powerpoint slides vs whiteboard, etc – or on other things you would like to critique, feel free to hand those in with your homework as well. I want to make this as good a class as possible, and all comments are certainly much appreciated!