MATH 601: QUIZ 3 (9/19/2012)

NAME: UIN:

Problem 1: For the following two matrices, do the following: (i) State whether it is invertible. If it is in fact invertible, then also: (ii) Show the inverse A^{-1} of the matrix; (iii) verify that it is indeed the inverse by multiplying $A^{-1}A$ and showing that it is equal to the identity matrix I.

Part a (2 points): $A_1 = \begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix}$

Answer: As discussed in class, the general form of an inverse of a 2×2 matrix $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ is $A^{-1} = \frac{1}{a_{11}a_{22}-a_{12}a_{21}} \begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}$. It is easy to verify by just multiplying out that $A^{-1}A = I$. The factor $a_{11}a_{22} - a_{12}a_{21}$ is called the *determinant* of the matrix A.
The matrix A_1 is not invertible because its determinant is zero and the fraction in the formula for A^{-1} does not exist.

Part b (3 points): $A_2 = \begin{pmatrix} 1 & 2 \\ 2 & 6 \end{pmatrix}$

Answer: Using the formula above, the inverse of this matrix is $A_2^{-1} = \frac{1}{1 \cdot 6 - 2 \cdot 2} \begin{pmatrix} 6 & -2 \\ -2 & 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 6 & -2 \\ -2 & 1 \end{pmatrix}$. We find that

$$A_2^{-1}A_2 = \frac{1}{2} \begin{pmatrix} 6 & -2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & 6 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 6 \cdot 1 + (-2) \cdot 2 & 6 \cdot 2 + (-2) \cdot 6 \\ (-2) \cdot 1 + 1 \cdot 2 & (-2) \cdot 2 + 1 \cdot 6 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I.$$

Problem 2 (2 points): Define what it means for a matrix to be invertible. (I.e., give the *definition* of invertibility. You don't need to state *conditions* that guarantee that a matrix is in fact invertible.)

Answer: We call a square matrix A invertible if there is a different matrix B so that $BA = I$ where I is the identity matrix. If such a matrix B exists, then we typically denote it by the syntax A^{-1}.

Problem 3 (3 points): Show that the following statement is true: Let A be an invertible matrix. Then $B = A^2 = AA$ is also an invertible matrix and its inverse $B^{-1} = (A^2)^{-1}$ equals $B^{-1} = (A^{-1})^2$.

Answer: To show that $B = A^2 = AA$ is invertible, all we need to do is find another matrix C so that $CB = I$. This is easy, in fact, the question already gives it away: $C = (A^{-1})^2$. We can verify this by just multiplying out:

$$CB = (A^{-1})^2 A^2 = A^{-1}A^{-1}AA = A^{-1}(A^{-1}A)A = A^{-1}IA = A^{-1}A = I.$$

In other words, $C = (A^{-1})^2$ is indeed an inverse of $B = A^2$ and consequently B is invertible.
