MATH 652: Optimization II

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Homework assignment 6 – due Thursday 3/11/2010

Problem 1 (Optimal control 1). Consider the following problem that gave rise to the term *shooting method*: We want to shoot a shell from location $x_0 = (0,0)^T \in \mathbb{R}^2$ to a target location $x_t = (1,0)^T$. The trajectory follows Newton's law which reads as follows if the only acting force is gravity:

$$\begin{aligned} \ddot{x}(t) &= \begin{pmatrix} 0 \\ -1 \end{pmatrix} \\ x(0) &= x_0 \\ \dot{x}(0) &= v_0. \end{aligned}$$

We assume that we can control the initial velocity $v_0 \in \mathbb{R}^2$ as the shell exits the cannon.

Formulate the following problems:

- The minimum time problem: The shell should get from x_0 to x_t in the least amount of time.
- The minimum initial velocity problem: The shell should still get from x_0 to x_t, but we want to minimize the exit velocity of the shell to minimize stress on the cannon.
- The fixed initial velocity problem: Oftentimes the magnitude of the exit velocity of the shell is fixed and given by the explosive charge. Formulate a problem whose solution is a trajectory from x_0 to x_t with fixed magnitude of the initial velocity. By choosing a suitable parameterization of the control variables, try to state this problem as a linear one, i.e. objective function and all constraints should be linear.

For each of these three problems, indicate the conditions under which you think that a solution might exist and whether you think that this solution is unique. **(6 points)**

Problem 2 (Optimal control 2). In the example above, the ODE that describes $x(t)$ can be integrated analytically (the trajectory is a parabola). State solutions (i.e. state the optimal values of the control variables and the corresponding trajectory) to the three problems you derived above.

(4 points)

Problem 3 (Integer programming). Consider the following problem:

$$
\begin{aligned}
\min_x \quad & x_1 + 2x_2 + x_3 \\
& \sqrt{1} \le x_1 \le \sqrt{10} \\
& \sqrt{2} \le x_2 \le \sqrt{20} \\
& \sqrt{3} \le x_3 \le \sqrt{30} \\
& x_i, i = 1, 2, 3 \quad \text{is integer.}
\end{aligned}
$$

Explain in words how the branch-and-bound algorithm can be applied to this problem. Demonstrate how it works by performing its steps on this relatively simple problem.

(4 points)

If you have comments on the way I teach – in particular suggestions how I can do things better, if I should do more or less examples, powerpoint slides vs whiteboard, etc – or on other things you would like to critique, feel free to hand those in with your homework as well. I want to make this as good a class as possible, and all comments are certainly much appreciated!