MATH 652: Optimization II

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Homework assignment 3 – due Thursday 2/11/2010

Problem 1 (A little proof). Let $\{v_i \in \mathbb{R}^n, i = 1 \dots V\}$ be the set of vertices of a bounded polyhedron $P \subset \mathbb{R}^n$, $V \geq n$. Let

$$C = \left\{ x \in \mathbb{R}^n : x = \sum_{i=1}^{V} \lambda_i v_i, \sum_{i=1}^{V} \lambda_i = 1, \lambda_i \geq 0 \ \forall i = 1 \dots V \right\}$$

be the *convex hull* of these points v_i.

- Prove that $C = P$.
- Prove that in particular every vector $x \in P$ can be represented as

  $$x = \sum_{i=1}^{V} \lambda_i v_i, \sum_{i=1}^{V} \lambda_i = 1, \lambda_i \geq 0$$

  where in particular *at most* $n+1$ among the total of V coefficients λ_i are nonzero. In other words, show that there exist λ_i so that

  $$\#\{i : \lambda_i \neq 0\} \leq n+1.$$

(6 points)

Problem 2 (The simplex algorithm). Let $n = 10$. Consider the linear problem for $x \in \mathbb{R}^n$

$$\min_x \ c^T x$$
$$0 \leq x_i \leq 1, \qquad \forall 1 \leq i \leq n,$$

where $c = (-1, -2, \dots, -n)^T$. Solve this problem using the simplex algorithm starting with the feasible solution $(0, 0, \dots, 0)^T$. Show your sequence of iterates, i.e. the feasible basic solutions that your algorithm visits. How many iterations do you need? How many iterations would the algorithm you implemented for last week's homework have needed? **(6 points)**