MATH 442: Mathematical Modeling

Lecturer: Prof. Wolfgang Bangerth
Blocker Bldg., Room 507D
(979) 845 6393
bangerth@math.tamu.edu
http://www.math.tamu.edu/~bangerth

Homework assignment 3 – due 9/23/2010

Problem 1 (More estimating parameters in models). In class we saw the Gompertz growth model,

$$y'(t) = r \ln\left(\frac{K}{y(t)}\right) y(t),$$
$$y(1800) = y_{1800},$$

where here we have used the year 1800 as the arbitrary start time from which on we are interested in solutions, and y_{1800} the population at that time. Use Maple to find the general solution of these equations. **(2 points)**

Go back to the US Census Bureau website and obtain the total population in the United States for each of the years 1800, 1810, 1820, ..., 2000 (that is further back by one century than last week). Show these values graphically and in numbers. **(2 points)**

Find the best parameters r, K, y_{1800} that describe the population data you have, using the same procedure as last week. If you just call Maple's `solve` or `fsolve` function on the equations that describe the best fit, it will give you nonsensical values like $r = 1$ and y_0, K close to zero. Rather, you will have to tell Maple a *range* of possible values for these parameters in which they most likely lie. Describe for each of r, K, y_0 a range within which you believe the best fit values should lie. Explain your reasoning. (Hint: Think about what each of these parameters mean, intuitively, and what you would expect their values to be given the discussions we have had in class.) **(3 points)**

Read up in Maple's help texts on how such parameter ranges can be passed to the `fsolve` function. Then use this to find the best such parameters. Plot the function $y(t)$ (where you take the best fits for the parameters obtained from the least squares fit) along with the actual population numbers you looked up before. **(4 points)**

Compared with the linear and exponential fits you got last week, is the fit qualitatively better or worse for the time between 1900 and 2000? Can you give a quantitative answer to this question and if so what is it? **(3 points)**

What does the model predict the US population will be in 2100, 2200 and 3000? How are these numbers related to the parameter K you have identified? Speculate how likely you think it is that the actual numbers in these years are close to what the best fit model predicts? (Hint: Use your general knowledge about population growth in the rest of the developed world over the last 100 years. You could, for example, also look up the population development over the last 50 years in countries like Germany, the U.K. or Russia to compare with the US and consider what these examples might mean for the US.) If you believe that the model's predictions are not likely to actually happen, can you speculate what may be wrong with the model? **(3 points)**

Problem 2 (Coming up with a model). Suppose a tank initially contains 200 liters of a 20% salt solution. Through a hose, 6 liters per minute of a 5% solution flow into the tank where they immediately mix with the rest of the solution. Through a hole in the bottom, 8 liters per minute of the solution flow out. Find a model that describes the salinity (i.e. the salt percentage) in the tank at time t. **(3 points)**

What are the coefficients of the model, what are their values, and what are their physical dimensions? What is the time interval on which the model makes sense? What happens after that? **(3 points)**

Plot the salinity in the tank as a function of time. **(2 points)**

I try to be as good a teacher as possible, but to succeed in this goal I need feedback from those who see me teach, i.e. you. If you have comments on the way I teach – in particular suggestions how I can do things better, if I should do more or less examples, group work vs. whiteboard, etc – or on other things you would like to critique, feel free to hand those in with your homework as well. I want to make this as good a class as possible, and all comments are certainly much appreciated!