MATH 651: Optimization I

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Homework assignment 8 – due Thursday 11/12/2009

Problem 1 (Lagrange multipliers and Lagrangians). Consider the following, somewhat boring, equility-constrained problem over $x \in \mathbb{R}$:

$$\min_x \quad x^2$$
$$x - 1 = 0.$$

Answer the following questions: (i) Does it have a solution? (ii) Is the solution unique? (iii) Do the LICQ conditions hold? (iv) Can we derive necessary conditions for the solution that contain a Lagrange multiplier, and if so how do they look? (v) What is the solution to the problem above, and what is the value of the Lagrange multiplier if one exists? (vi) Verify that the Lagrangian of this problem describes a quadratic function in x, λ that has only a single, unique stationary point equal to the solution of the constrained minimization problem. (vii) Verify that the matrix of second derivatives of the Lagrangian is indefinite, i.e. that the stationary point is a saddle point, not a minimum or maximum. For the last two parts, it may be useful to plot the Lagrangian. **(4 points)**

Problem 2 (Quadratic programming). Numerically find the solution of the following problem in $x \in \mathbb{R}^4$:

$$\min_{x\in\mathbb{R}^4} \frac{1}{2}x^T G x + d^T x + e$$
$$Ax - b = 0,$$

where

$$G = \begin{pmatrix} 4 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 \\ 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 4 \end{pmatrix}, \qquad d = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}, \qquad e = 0,$$
$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 5 \\ 1 & 2 & 4 & 6 \end{pmatrix}, \qquad b = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$

Explain how the solution and the values of the Lagrange multipliers are going to change if the constraint matrix was changed to

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 5 \\ 1 & 2 & 3 & 6 \end{pmatrix}.$$

(4 points)

Problem 3 (SQP). Use the SQP method with full step length to find the solution of the following problem:

$$\min_{x\in\mathbb{R}^2} -\frac{1}{1+x_1^2+x_2^2}$$
$$x_1 + x_2 - 1 = 0$$

You may have to start close enough to the solution of this problem to converge.

(3 points)

If you have comments on the way I teach – in particular suggestions how I can do things better, if I should do more or less examples, powerpoint slides vs whiteboard, etc – or on other things you would like to critique, feel free to hand those in with your homework as well. I want to make this as good a class as possible, and all comments are certainly much appreciated!