MATH 651: Optimization I

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Homework assignment 6 – due Thursday 10/22/2009

Problem 1 (Some matrix inequalities). In the BFGS method, we update the matrix according to

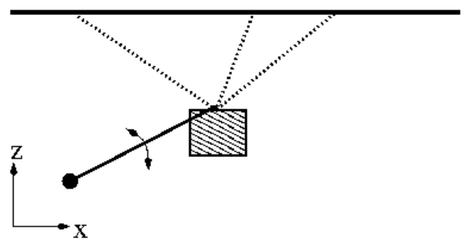
$$B_{k+1} = B_k - \frac{B_k s_k s_k^T B_k}{s_k^T B_k s_k} + \frac{y_k y_k^T}{y_k^T s_k},$$

given a symmetric and positive definite matrix B_k from the previous step. Prooving that the BFGS method converges is somewhat awkward and lengthy (take a look at chapter 8.4 of the book) and involves a few technical and not immediately intuitive parts. As intermediate steps in this proof, one needs to show the following two identies:

$$\text{trace } B_{k+1} = \text{trace } B_k - \frac{\|B_k s_k\|^2}{s_k^T B_k s_k} + \frac{\|y_k\|^2}{y_k^T s_k},$$
$$\det B_{k+1} = \frac{y_k^T s_k}{s_k^T B_k s_k} \det B_k.$$

Prove these two identities. As a remark, note that while it helps us prove convergence of the method, the first of these identities doesn't make any sense when considering physical units since it sums over the diagonal elements of B_{k+1}, which can have different units. **(5 points)**

Problem 2 (A modeling exercise, revisited once more). Consider the following system of three springs suspended from the ceiling at positions $(x, z) = (-20\text{cm}, 0\text{cm})$, $(5\text{cm}, 0\text{cm})$ and $(15\text{cm}, 0\text{cm})$ and a rod of fixed length 20cm that is attached at one end at $(-25\text{cm}, -20\text{cm})$ and at the other end to the same point where the springs meet each other:



Each spring has a rest length of $L_0 = 20\text{cm}$, and extending (or compressing) spring $i, i = 1 \ldots 3$ to a length L_i requires an energy of $E_{\text{spring},i} = \frac{1}{2}D(L_i - L_0)^2$ where the spring constant for all three springs equals $D = 300\frac{N}{m}$. On the other hand, the potential energy of the body is $E_{\text{pot}} = mgz$ where the body's mass is $m = 500g$, the gravity constant is $g = 9.81\frac{\text{m}}{\text{s}^2}$, and z is the vertical coordinate of the body's position.

State this as an optimization problem in standard form (i.e. what is the objective function and what are the constraints). Use the quadratic penalty method to find the solution of the problem that minimizes the energy subject to the constraints. In your answer, show this location as well as the value of the total energy function at this location. Is ther only a single energy minimum?

(6 points)

If you have comments on the way I teach – in particular suggestions how I can do things better, if I should do more or less examples, powerpoint slides vs whiteboard, etc – or on other things you would like to critique, feel free to hand those in with your homework as well. I want to make this as good a class as possible, and all comments are certainly much appreciated!